

CP and Other Gauge Symmetries in String Theory

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We argue that *CP* is a gauge symmetry in string theory. As a consequence, *CP* cannot be explicitly broken either perturbatively or nonperturbatively; there can be no nonperturbative *CP*-violating parameters. String theory is thus an example of a theory where all θ angles arise due to spontaneous *CP* violation, and are in principle calculable.

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There are two standard suggestions for solving the strong *CP* problem. The most popular is the Peccei-Quinn symmetry, and its associated axion [1]. Another possibility, which has been pursued by several authors, is to suppose that the underlying laws of nature are *CP* conserving, and *CP* is spontaneously broken [2]. In particular, one assumes that the "bare θ " is zero; the observable θ is then "calculable," i.e., it is determined in terms of the fundamental parameters of the theory. The main difficulty with this program is to understand why the observed θ is in fact so small. Usually one tries to arrange that, as a consequence of (other) symmetries, θ vanishes at tree level, and that loop corrections are suppressed by powers of small Yukawa couplings and the like.

Witten noted some time ago that string theory possesses axions, and that as a result it has the potential to solve the strong *CP* problem [3]. Since then, there has been much discussion as to whether this axion can remove all θ angles, whether there exist other axions, whether the axions have suitable decay constants, and whether the minimum of the axion potential is necessarily at $\theta=0$. But little or no attention has been paid to the question of whether or not string theory might in fact be a theory of the second kind, i.e., one where the underlying, microscopic theory preserves *CP*, and the bare θ vanishes.

It has been known for some time that in string perturbation theory, *CP* is a good symmetry [4,5]. Strominger and Witten observed that there are four-dimensional string vacua in which classically *CP* is unbroken. They pointed out that in generic vacua *CP* is broken and that this can be thought of as arising spontaneously through expectation values for *CP*-odd moduli. Clearly one can also contemplate the possibility that *CP* is broken at lower energies by expectation values for other *CP*-odd fields. These observations, however, are strictly perturbative and do not establish whether or not the bare θ vanishes. For example, there has been much speculation as to the possible existence of nonperturbative parameters in string theory [6]. *A priori*, if such parameters exist (in the case of critical strings), some could be *CP* violating; θ angles might then arise as functions of these parameters. Indeed, θ angles are in some sense the paradigms of nonperturbative parameters. For example, in the case of the $E_8 \times E_8$ theory, compactified to four dimensions, it is natural to ask whether one could obtain two (or more) θ an-

gles, only one of which could be removed by the model-independent axion.

In this Letter, we argue that this cannot happen: If string theory has nonperturbative parameters, they are necessarily *CP* conserving. String theory, as a result, is a perfect example of a theory in which the bare θ vanishes as a consequence of symmetry. The basis of this argument is a very simple observation: In string theory, four-dimensional *CP* transformations are *gauge transformations*. As a result, provided simply that the theory *exists*, no explicit breaking of the symmetry is possible, perturbatively or nonperturbatively. In the course of this discussion, we will encounter some other amusing facts. For example, we will see that the \mathbb{Z}_2 symmetry of the $E_8 \times E_8$ theory which interchanges the two E_8 's is itself a gauge symmetry, and again is not susceptible to explicit breaking.

To understand in what sense *CP*, in four-dimensional compactifications of string theory, can be thought of as a gauge symmetry, consider some features of the ten-dimensional heterotic string theory. This is a theory which violates *P* and conserves *C*. In particular, the Gliozzi-Scherk-Olive (GSO) condition, which requires that spinors be (say) left handed, violates parity. It is perhaps helpful to understand this statement from a world-sheet viewpoint. The two-dimensional field theory which describes the ten-dimensional theory has a symmetry under which one changes the signs of the nine space-like coordinates x^i , and separately those of their right-handed fermionic partners, ψ^i . However, the separate transformations do not commute with the Becchi-Rouet-Stora-Tyutin (BRST) operator (the physical state conditions), and thus cannot be symmetries in space-time. Simultaneously changing the signs of both the x^i 's and the ψ^i 's does respect the BRST symmetry, but this condition does not respect the GSO condition, which involves a product of all the ψ 's.

Charge conjugation, on the other hand, is a good symmetry. If we choose a Majorana basis for the Dirac matrices, *C* is just the instruction to take the complex conjugate of (space-time) spinor fields. The reality condition on these fields is obviously invariant under this operation, as are the GSO and physical state conditions. In the left-moving sector, the effect of *C* is most easily understood in the bosonic formulation. There it is just the in-

struction to take $X^I \rightarrow -X^I$, $I=1, \dots, 16$. This is obviously a symmetry of the world-sheet Lagrangian. It has the effect on the lattice of taking $p^I \rightarrow -p^I$. Since this is a symmetry of both the $O(32)$ and $E_8 \times E_8$ lattices, it corresponds to a good symmetry in space-time.

It is very easy to see that C is in fact a gauge transformation, in either the $E_8 \times E_8$ or $O(32)$ theories. Consider, for definiteness, the $O(32)$ case; the argument is virtually identical for $E_8 \times E_8$. Indeed the transformation C can be viewed as a set of rotations in sixteen two-dimensional subspaces through an angle π , a transformation that is obviously contained in $O(32)$. In the fermionic formulation, if one works with sixteen complex λ^i 's, $X^I \rightarrow -X^I$ corresponds to $\lambda^i \rightarrow \lambda^{\bar{i}}$; this is just the rotation we have described.

If the theory is compactified toroidally, C must also reverse the signs of *both* the left- and right-moving momenta associated with the compact dimensions. For compactification of an even number of dimensions, however, this is obviously a proper Lorentz transformation.

What about P in lower dimensions? It is well known that for toroidal compactifications in string theory (and Kaluza-Klein theory), even when one starts with a higher-dimensional theory which is P violating, the four-dimensional theory is P conserving. We do not expect that this symmetry arises "out of the air;" it must be one of the symmetries of the original ten-dimensional theory—indeed, it must be a (proper) Lorentz transformation in that theory. To see explicitly what it is, it is helpful to consider a toroidal compactification of the theory (without background gauge or antisymmetric tensor fields), and to group the six internal coordinates as three complex ones, $y^1 = x^4 + ix^5$, etc. Then consider the transformation which reverses the signs of x^1, x^2, x^3, x^5, x^7 , and x^9 (and ψ^1, ψ^2 , etc.). This is a combination of ordinary parity in four dimensions, times complex conjugation of the y^i 's (and ψ^i 's); it commutes with the GSO condition. From a ten-dimensional perspective, it is a proper Lorentz transformation. It is easy to verify that on massless fermions it has precisely the correct effect. The reader who wishes to check this point may find it convenient to adopt the following basis for the ten-dimensional Dirac matrices:

$$\Gamma^\mu = \gamma^\mu \otimes 1, \quad \mu = 0, \dots, 3,$$

$$\Gamma^I = \gamma^5 \otimes \gamma^I, \quad I = 1, \dots, 6.$$

Write the four-dimensional γ matrices in a Weyl basis, and the $O(6)$ γ matrices in terms of creation and annihilation operators (see, e.g., Ref. [5]). Then the sixteen-component, ten-dimensional spinors break up into pieces $u_{aa}, u_{\bar{a}\bar{a}}$, where a, \bar{a} denote left- and right-moving spinors, and a, \bar{a} are indices referring to the $\mathbf{4}$ and $\bar{\mathbf{4}}$ representations of $O(6) \sim SU(4)$. Grouping the u_{aa} 's and $u_{\bar{a}\bar{a}}$'s into four four-component spinors, Ψ^a , P takes left-handed fermions to their right-handed counterparts. It is straight-

forward to show, in addition, that the corresponding vertex operators are also suitably mapped into one another; similarly, bosonic vertex operators have well-defined transformation properties (e.g., scalars transform as scalars or pseudoscalars, and gauge bosons transform appropriately).

Of course, since both C and P are gauge transformations, it follows that CP is as well. So far, however, we have only illustrated these statements for toroidal compactifications. Many other types of compactifications violate P and C separately in four dimensions, while conserving CP . A good example is provided by conventional Calabi-Yau compactifications, with the so-called "standard embedding of the gauge group" [5]. In these theories, while the C and P symmetries which have been defined above are spontaneously broken by the expectation values of the graviton and gauge fields, the combination is conserved, for suitable values of the moduli. Indeed, at the level of the σ model which describes such compactifications, $P \times C$ is precisely the CP symmetry of Ref. [4]. The same construction also works for symmetric orbifolds. Thus once again CP can (almost certainly) be thought of as a gauge symmetry. While the complete space of four-dimensional string theories is not known, and it is by no means clear that all such theories can be obtained (for some limiting value of some moduli) by solution of ten-dimensional field (or β -function) equations, it is quite natural to suppose that this result is completely general: CP is always a gauge symmetry in string theory.

If string theory describes the real world, CP must be spontaneously broken. This might occur at very high energies, through expectation values for some moduli fields. In this case, the theory at low energies will be essentially indistinguishable from theories with explicit CP violation. Alternatively, CP might be broken by expectation values of matter fields at lower energies. Before speculating on how these phenomena might occur with sufficiently small effective θ , it is instructive to understand the absence of multiple θ parameters in other, rather similar, ways. Consider, for example, toroidal compactifications of the heterotic string. We have already remarked that one might worry that there are different θ 's for each low-energy gauge group, only one of which can be removed by the model-independent axion. That this is not the case follows from our discussion of CP , but it can be seen another way. Consider first the $E_8 \times E_8$ theory. In this case, it is tempting to say that one can add two θ 's, one for each E_8 . But in perturbation theory there is a \mathbb{Z}_2 which relates the two E_8 's. It is not hard to show that this is a gauge symmetry [7], and thus the two θ 's are necessarily the same. For example, in Ref. [8] it was shown that by turning on a background expectation value for certain gauge fields, one can map the $E_8 \times E_8$ theory continuously to the $O(32)$ theory. But under this mapping, it is a straightforward matter to check that the \mathbb{Z}_2 is

mapped into a particular $O(32)$ gauge transformation. That is, one obtains the Z_2 transformation by turning on background fields such that one obtains the $O(32)$ theory, rotating the lattice by an $O(32)$ transformation, and then returning to the (Z_2 transformed) $E_8 \times E_8$ theory by again turning on certain background fields.

More generally, one can ask whether different θ 's might appear as one moves around in the moduli space, e.g., at points of enhanced symmetry. Again, the answer is no, as a consequence of gauge invariance. We are worried, here, about terms which do not change as one moves around in the moduli space. In particular, then, we can ask about the coefficient of $F\tilde{F}$ for each of the 22 $U(1)$ gauge bosons which exist everywhere in the moduli space, associated with left-moving fields [9]. There is a point in the moduli space where all 22 of these symmetries are unified in a single non-Abelian group. At this point, gauge invariance requires that all θ 's be equal.

In view of these observations, one can envisage several scenarios for solving the strong CP problem in string theory. Our comments here will be rather preliminary. The fact that CP is spontaneously broken and θ_{QCD} is in principle calculable does not mean that it is small. (We should stress that we are using "calculable" in the technical sense that it is not an independent parameter; we certainly do not know what dynamics, if any, determines the string ground state, etc.) For example, it could be that in string theory there are several strong gauge groups, and that the effective θ 's for each of these groups is large. In this situation, one must make sure that there are enough axions to cancel these θ 's. In addition to the model-independent axion, some further approximate Peccei-Quinn symmetries must appear "by accident," e.g., as a consequence of discrete symmetries [10,11]. This is the conventional view we referred to in the first paragraph. Alternatively, perhaps the various θ 's are simply small with string theory realizing some version of the ideas of Nelson and Barr [2]; the low-energy structure of string theory is sufficiently rich that this might occur. One probably does not want CP broken at too low an energy; otherwise one cannot hope to inflate away the associated domain walls. Implementing variants of these schemes at high energies may require additional discrete symmetries; fortunately these are common in string theory. These ideas will be explored in a subsequent publication, where details of the analyses reported here will also be presented.

The observation that CP is a gauge symmetry raises an obvious question, about which we will only make some timid speculations: What about T invariance, and CPT ? We cannot use precisely the same sort of reasoning to argue that T is a gauge symmetry as we did for parity. The problem is that the would-be Lorentz transformation in ten dimensions is not part of the proper Lorentz group. However, from a stringy viewpoint, the similarities be-

tween the four-dimensional T and P are so striking that it is natural to speculate that T , also, is a gauge transformation. Whether this would have profound consequences we do not presently know. Unlike the case of field theory, it is not easy to make a general statement about CPT in string theory. In perturbation theory, CPT appears to hold, basically because the space-time theory inherits CPT from the world-sheet theory. But it is not yet clear that CPT need hold nonperturbatively. Of course, if T as well as CP is a gauge transformation, this would ensure that CPT is as well.

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- [1] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); Phys. Rev. D **16**, 1791 (1977).
 - [2] A. Nelson, Phys. Lett. **136B**, 387 (1984); S. M. Barr, Phys. Rev. Lett. **53**, 329 (1984); Phys. Rev. D **30**, 1805 (1984); P. H. Frampton and T. W. Kephart, Phys. Rev. Lett. **66**, 1666 (1991).
 - [3] E. Witten, Phys. Lett. **149B**, 351 (1984).
 - [4] A. Strominger and E. Witten, Commun. Math. Phys. **101**, 341 (1985).
 - [5] M. B. Green, J. Schwarz, and E. Witten, *Superstring Theory* (Cambridge Univ. Press, New York, 1986).
 - [6] M. Douglas and S. Shenker, Nucl. Phys. **B355**, 635 (1990).
 - [7] Implications of discrete gauge symmetries have been considered extensively in the recent literature. See, for example, L. Krauss and F. Wilczek, Phys. Rev. Lett. **62**, 1221 (1989); J. Preskill and L. Krauss, Nucl. Phys. **B341**, 50 (1990); S. Coleman, J. Preskill, and F. Wilczek, Phys. Rev. Lett. **67**, 1975 (1991); L. E. Ibanez and G. Ross, Phys. Lett. B **260**, 291 (1991); Nucl. Phys. **B368**, 3 (1992); M. Dine and T. Banks, Phys. Rev. D **45**, 1424 (1992).
 - [8] P. Ginsparg, Phys. Rev. D **35**, 648 (1987).
 - [9] Note such θ 's are physically meaningful even at points where the $U(1)$'s are not unified in non-Abelian groups. To illustrate the point, consider an $O(3)$ gauge theory with a scalar triplet. If one gives the scalar a small expectation value, the $O(3)$ is broken to $U(1)$, but θ -dependent effects are still present, albeit suppressed.
 - [10] G. Lazarides, C. Panagiotakopoulos, and Q. Shafi, Phys. Rev. Lett. **56**, 432 (1986); J. A. Casas and G. G. Ross, Phys. Lett. B **192**, 119 (1987).
 - [11] M. Dine, in Proceedings of the Cincinnati Symposium in Honor of the Retirement of Louis Witten [SCIPP report (to be published)].
 - [12] K. Choi, D. Kaplan, and A. Nelson, UCSD Report No. PTH 92-11, 1992 (to be published).