Cosmic Microwave Background Probes Models of Inflation

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(Received 22 June 1992)

Inflation creates both scalar (density) and tensor (gravity wave) metric perturbations. We find that the tensor-mode contribution to the cosmic microwave background anisotropy on large-angular scales can only exceed that of the scalar mode in models where the spectrum of perturbations deviates significantly from scale invariance (e.g., extended and power-law inflation models and extreme versions of chaotic inflation). If the tensor mode dominates at large-angular scales, then the value of $\Delta T/T$ predicted on 1° is less than if the scalar mode dominates, and, for cold-dark-matter models, bias factors $b>1$ can be made consistent with Cosmic Background Explorer (COBE) DMR results.

PACS numbers: 98.70.Vc, 04.30.+x, 98.80.Cq

The recent measurements [1] of large-angular-scale anisotropy in the cosmic microwave background (CMB) by the Cosmic Background Explorer (COBE) Differential Microwave Radiometers (DMR) provide important experimental support for the hot big bang model. Perhaps the most striking conclusion to be drawn from the COBE DMR data is that they are consistent with a scale-invariant spectrum of primordial density (scalar) perturbations.

A scale-invariant spectrum is consistent with inflation, which predicts perturbations generated by quantum fluctuations [2], and also with models that generate perturbations by classical effects, such as theories with cosmic strings, textures, global monopoles, and nontopological excitations. Inflation also produces a spectrum of gravity waves (tensor metric fluctuations) with wavelengths extending beyond the horizon, providing a possible means for distinguishing it from the other scenarios. Recently it was speculated that the anisotropy detected by the COBE DMR might be largely due to inflation-induced tensor rather than scalar perturbations [3]. In this Letter, we show that tensor dominance of the CMB quadrupole anisotropy is indeed possible for a class of inflationary models. We find that the ratio of tensor to scalar contributions is directly tied to the rate of inflationary expansion and the "tilt" of the spectrum of density perturbations away from scale invariance. Models that permit tensor dominance include extended inflation, power-law inflation, and extreme versions of chaotic inflation. While the COBE DMR results alone cannot distinguish tensor from scalar perturbations, we show how additional measurements on small-angular scales may distinguish the two. We also discuss the implications for large-scale structure.

CMB temperature anisotropies on large-angular scales ($\gtrsim 1°$) are produced by metric fluctuations through the Sachs-Wolfe effect [4]. These temperature fluctuations can be decomposed into spherical-harmonic amplitudes; for scale-invariant scalar-mode fluctuations, the quadrupole is given by [5]

$$S\equiv\langle a_2^2\rangle_S\equiv\left\langle\sum_{m=-2}^{m=2}|a_{2m}|^2\right\rangle=\frac{1}{60\pi}\frac{H^4}{\dot\phi^2}=\frac{128\pi^2}{45}\frac{V^3}{V'^2m_{\rm Pl}^6}, \tag{1}$$

where H is the Hubble parameter, ϕ is the scalar field that rolls during inflation, $V(\phi)$ is its potential, $m_{\rm Pl}=1.22\times10^{19}$ GeV is the Planck mass, and the final expression follows from the slow-roll equation of motion, $3H\dot\phi=-V'$. Here $\langle\cdots\rangle$ indicates an ensemble average over all observers in the universe. The right-hand side is to be evaluated $N\sim60$ e-foldings before the end of inflation, when fluctuations on CMB length scales crossed outside the horizon [6]. The corresponding formula for tensor fluctuations is

$$T\equiv\langle a_2^2\rangle_T=7.7V/m_{\rm Pl}^4, \tag{2}$$

obtained by evaluating Eq. (2.15) in [7]. The ratio of tensor to scalar quadrupole anisotropies is, therefore,

$$\frac{T}{S}\equiv\frac{\langle a_2^2\rangle_T}{\langle a_2^2\rangle_S}\approx0.28\left(\frac{V'm_{\rm Pl}}{V}\right)^2\Bigg|_{N\sim60}, \tag{3}$$

where, more precisely, this is the ratio of the ensemble averages for T and S. To compare with experiment, one must take into account the variance from this ensemble average associated with a single measurement (so-called "cosmic variance" [1,8]).

Note that the coefficients in Eqs. (1) and (2) were derived assuming strict scale invariance. Since we will find below that models with $T/S\gtrsim1$ deviate from scale invariance, we have numerically computed the coefficients in

Eqs. (1) and (2) for "tilted" spectra and find that the numerical coefficient in Eq. (3) changes very little ($\lesssim 10\%$) for the tilts consistent with the COBE DMR results.

Extended [9] and power-law [10] inflation models can be described in terms of a potential of the form $V(\phi) = V_0 \exp(-\beta\phi/m_{\rm Pl})$, where β is constant or slowly time dependent. In extended inflation ϕ is related to a field that is coupled to the scalar curvature (e.g., a dilaton or Brans-Dicke field), which leads to a modification of Einstein gravity. The modified gravity action can be reexpressed via a Weyl transformation as the usual Einstein action plus a minimally coupled scalar field (ϕ) with an exponential potential. In the simplest example of extended inflation [9], $\beta = \sqrt{64\pi/(2\omega+3)}$, where ω is the Brans-Dicke parameter. For an exponential potential, Eq. (3) implies

$$\frac{T}{S} \approx 0.28\beta^2 = \frac{56}{2\omega+3}. \tag{4}$$

The ratio $T/S \gtrsim 1$ for $\omega \lesssim 26$ ($\beta \gtrsim 1.9$). Interestingly, $\omega \lesssim 26$ is almost precisely what is required to avoid unacceptable inhomogeneities from big bubbles in extended inflation [11]. (Though $\omega \lesssim 26$ is inconsistent with solar-system limits for Brans-Dicke theory, these constraints are evaded by giving the Brans-Dicke field a mass.)

Chaotic inflation models [12] typically invoke a potential of the form $V(\phi) = \lambda\phi^p$, where $\phi \gg m_{\rm Pl}$ initially, and rolls to $\phi = 0$. The ratio of tensor to scalar anisotropies can be expressed in terms of ϕ_N, the value of the scalar field $N \sim 60$ e-foldings before the end of inflation. Using the relation

$$N(\phi) = \int_{t_{\rm end}}^{t_N} H\,dt = \frac{8\pi}{m_{\rm Pl}^2}\int_{\phi_{\rm end}}^{\phi_N} \frac{V}{V'}d\phi = \frac{4\pi}{p}\frac{\phi^2}{m_{\rm Pl}^2} - \frac{p}{12}, \tag{5}$$

where $\phi_{\rm end}^2 = p^2 m_{\rm Pl}^2/48\pi$, we find that [13]

$$\frac{T}{S} \approx \frac{p}{17}\left[1 + \frac{p}{720}\right]^{-1}, \tag{6}$$

where we have set $N = 60$. For the chaotic inflation models usually discussed, $p = 2$ and 4, the scalar mode dominates: $T/S = 0.11$ and 0.23; however, for extreme models, $p \gtrsim 18$, the tensor mode could dominate.

New-inflation models [14] entail a slow roll from $\phi \approx 0$ to $\phi = \sigma$ down flat potentials of the Coleman-Weinberg form, $V(\phi) = B\sigma^4/2 + B\phi^4[\ln(\phi^2/\sigma^2) - \frac{1}{2}]$, where $B \simeq 10^{-15}$ for density perturbations of an acceptable size. In new inflation T/S also depends upon ϕ_N; paralleling the previous analysis,

$$N(\phi) = \frac{8\pi}{m_{\rm Pl}^2}\int_{\phi_{\rm end}}^{\phi_N} \frac{V}{V'}d\phi \approx \frac{\pi}{2|\ln(\phi_N^2/\sigma^2)|}\frac{\sigma^4}{\phi_N^2 m_{\rm Pl}^2}, \tag{7}$$

$$\frac{T}{S} \approx \frac{3.2\times 10^{-4}}{|\ln(\phi_N^2/\sigma^2)|}\left(\frac{\sigma}{m_{\rm Pl}}\right)^4. \tag{8}$$

Scalar dominates tensor for $\sigma \lesssim 10 m_{\rm Pl}$, and, naively, it would appear that T/S can be made greater than unity for $\sigma \gtrsim 10 m_{\rm Pl}$. However, one finds that ϕ_{60} is very close to σ for $\sigma \gtrsim 10 m_{\rm Pl}$, violating the implicit assumption, $\phi_{60} \ll \sigma$. That is, for $\sigma \gg m_{\rm Pl}$, ϕ rolls down the steeper (harmonic) part of the potential close to the minimum, so that $V(\phi) \simeq 4B\sigma^2(\phi-\sigma)^2$, just as in chaotic inflation with $p = 2$. In this case, the tensor mode does not dominate ($T/S \simeq 0.11$).

We will now show that T/S cannot be arbitrarily large by deriving model-independent relations between T/S, the rate of inflation, and the tilt of the density perturbation spectrum away from scale invariance [15]. The ratio of tensor to scalar perturbations is controlled by the steepness of the potential, $V' m_{\rm Pl}/V$; cf. Eq. (3). During inflation, this quantity also determines the ratio of the kinetic to potential energy of the scalar field [16], $\frac{1}{2}\dot\phi^2/V \simeq (V' m_{\rm Pl}/V)^2/48\pi$, which in turn determines the effective equation of state ($p = \gamma\rho$) and the evolution of the cosmic-scale factor ($R \propto t^m$): $\gamma = [\frac{1}{2}\dot\phi^2 - V]/[\frac{1}{2}\dot\phi^2 + V]$ and $m = 2/3(1+\gamma)$ (during inflation γ and m can vary). It is simple to show that the tensor perturbations are characterized by a power spectrum $|\delta_k^T|^2 \propto k^{n_T - 1}$ and the scalar (density) perturbations by $|\delta_k^S|^2 \propto k^n$ [17], where $n_T = (m-3)/(m-1)$. For the models considered in this paper, the difference between the scalar exponent n and the tensor exponent n_T is not significant, and we will henceforth use n to represent both [18]. In the limit of exponential inflation, i.e., $\frac{1}{2}\dot\phi^2/V \to 0$, $m \to \infty$ and the tensor perturbations are scale invariant ($n_T \to 1$), and in most models [19] the scalar perturbations are also scale invariant ($n \to 1$).

The above relationships together with Eq. (3) allow us to express the expansion-rate index m and the power-spectrum index n (for $N \sim 60$) in terms of T/S:

$$m = 14\left(\frac{S}{T}\right) + \frac{1}{3} \simeq 14\left(\frac{S}{T}\right),$$
$$n = 1 - \frac{3(T/S)}{21 - (T/S)} \simeq 1 - \frac{1}{7}\left(\frac{T}{S}\right). \tag{9}$$

[We remind the reader that the numerical coefficients here depend upon that in Eq. (3), which depends weakly on the ratio T/S for $n \gtrsim 0.5$.] If the tensor mode is to dominate—i.e., $T/S \gtrsim 1$—then m must be less than about 14 and n must be less than about 0.85. (Conversely, in models where the expansion is exponential and the spectrum is scale invariant, the ratio of tensor to scalar is very small.) From the fact that inflation must be "superluminal" ($m > 1$), we can use Eq. (9) to derive an approximate *upper* bound, $T/S \lesssim 20$ [20]. However, the COBE DMR [1] bound on the power-spectrum index n, $n = 1.1 \pm 0.6$, which implies that $n \gtrsim 0.5$ when $T/S \gtrsim 1$, leads to the stronger limit $T/S \lesssim 3$ (and $m \gtrsim 5$). (Doubtless, there are yet stronger bounds on n based upon structure formation.)

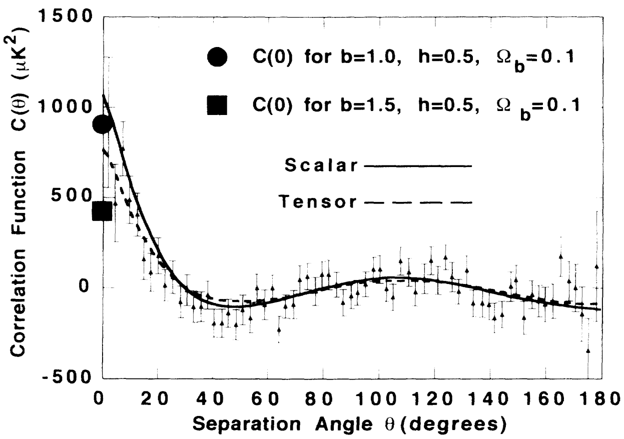

FIG. 1. Temperature autocorrelation function (from the Sachs-Wolfe effect) for tensor and scalar modes each normalized to the COBE DMR quadrupole anisotropy using a scale-invariant spectrum and the COBE DMR window function [1]. Tensor and scalar modes are distinguishable at small angles but the COBE DMR results (data superimposed) are unable to resolve the difference. We have not shown the cosmic variance which leads to additional uncertainty. (The cosmic variance for the scalar contribution is shown in Fig. 3 of Ref. [1].) CDM predictions [23] for the scalar contribution to $C(0)$ ($h=0.5$ and $\Omega_b=0.1$) is $C(0)\approx 980\ \mu K^2$ for $b=1$ and $C(0)\approx 460\ \mu K^2$ for $b=1.5$.

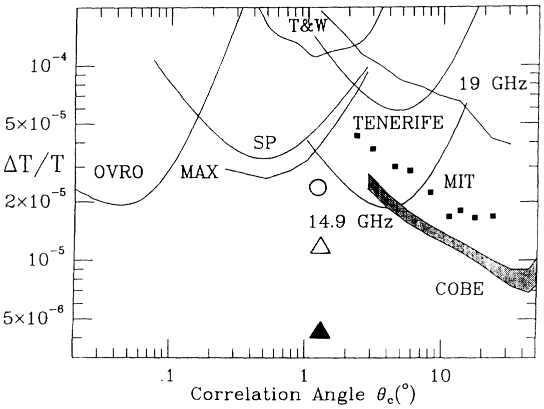

FIG. 2. Constraints to the CMB anisotropy from various experiments and predictions for the South Pole anisotropy experiment on 1° for CDM models ($\Omega=1$, $\Omega_B=0.1$, $h=0.5$), using the filter function from [24]: Open circle, CDM with $b=1$, the best-fit CDM model to the COBE DMR if $T/S \ll 1$; open triangle, CDM with $b=2$, consistent with the COBE DMR only if $T/S \gtrsim 1$; closed triangle, upper bound if COBE DMR were detecting the Sachs-Wolfe effect from pure tensor mode ($T/S \gg 1$).

We can now apply these results for the specific models for which we found $T/S \gtrsim 1$, extended and chaotic inflation. In extended (or power-law) inflation, the power spectrum is tilted according to $n \simeq (2\omega-9)/(2\omega-1)$ and $m=(2\omega+3)/4$. Using the COBE DMR limit, $n \gtrsim 0.5$, we find a plausible range, $26 \gtrsim \omega \gtrsim 9$. For chaotic inflation, $n \approx 1-p/120$ and $m \approx 240/p$, leading to a somewhat extreme range, $60 \gtrsim p \gtrsim 18$.

Tensor contributions have significant implications for CMB measurements. First, the COBE DMR results alone do not distinguish scalar from tensor contributions to the anisotropy; see Fig. 1. However, the COBE DMR results, combined with measurements on smaller-angular scales, might distinguish the two. The COBE DMR measurement implies $\langle a_2^2 \rangle = (4.53 \pm 2.5)\times 10^{-10}$, where we should keep in mind that this is a measurement of $\langle a_2^2\rangle_T + \langle a_2^2 \rangle_S$. Going to smaller-angular scales, the scalar contribution to the CMB anisotropy grows relative to the tensor, but the net contribution to small-angle measurements is diminished compared to no tensor mode at all; see Fig. 2. (We are assuming that no late reionization washes out fluctuations on small-angular scales.) Hence, comparing large- and small-angle anisotropy measurements can, in principle, separate the scalar and tensor contributions. Of course, a careful analysis would have to include the uncertainties due to cosmic variance.

The tensor mode can seriously affect the interpretation of CMB measurements for large-scale structure, regardless of the form of dark matter. As an example, the best-fit cold-dark-matter (CDM) model to the COBE DMR results assuming $T/S \ll 1$ has a bias factor $b \simeq 1$. (The bias factor $b \equiv 1/\sigma_8$, where σ_8 is the rms mass fluctuation on the scale $8h^{-1}$ Mpc.) If, however, the tensor contribution to the CMB quadrupole is significant, then the extrapolated density perturbation amplitude at $8h^{-1}$ Mpc is reduced, and the best-fit CDM model has $b>1$; see Fig. 2. Two related effects combine to increase b: The power spectrum is tilted (less power on small scales for fixed quadrupole anisotropy), and scalar perturbations only account for a fraction of the quadrupole anisotropy. We find, very roughly,

$$b \simeq 100^{(1-n)/2}\sqrt{1+T/S} \simeq 10^{(T/S)/7}\sqrt{1+T/S}\,, \quad (10)$$

where "100" is the ratio of the scale relevant to the quadrupole anisotropy, $\lambda \sim 1000h^{-1}$ Mpc, to the scale $8h^{-1}$ Mpc. For $T/S=0.53$, 1.4, 2.5, and 3.3, the bias factor is $b=1.4$, 2.4, 4.6, and 7.8 (and $n=0.92$, 0.78, 0.59, and 0.44). While these numbers should only be taken as rough estimates, the trend is clear: Larger T/S permits larger bias.

In sum, if small-angular-scale measurements find $\Delta T/T$ significantly lower than that extrapolated from the COBE DMR quadrupole (see, e.g., [21]), there are now at least two possible explanations consistent with inflation. Either reionization has washed out the small-angle fluctuations, or tensor fluctuations contribute significantly to the COBE DMR observations [22]. In the latter case, what can CMB studies tell us about inflation? Our analysis suggests a remarkable conclusion—COBE DMR combined with small-angular-scale measurements can directly relate the key cosmological parameters that govern large-scale structure, such as the bias factor b in CDM models and the power-spectrum index n, to the microphysical parameters that control inflation.

We gratefully acknowledge discussions with D. Bennett, R. Holman, E. W. Kolb, and S.-H. Rhie. We thank L. Kofman for bringing Ref. [22] to our attention and P. Lubin for sharing some of his data prior to publication. This research was supported by the DOE at Penn (DOE-EY-76-C-02-3071), at Berkeley (DOE-AC-03-76SF0098), and at Chicago and Fermilab, by the NSF (PHY89-04035), and by the NASA at Fermilab (NAGW-2381) and at the Harvard-Smithsonian Center for Astrophysics (NAGW-931).

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[18] The scalar perturbations have additional scale dependence if the steepness of the potential, $m_{\mathrm{Pl}}V'/V$, varies during inflation. In most models this additional scale dependence is not significant. For example, in power-law inflation the steepness is constant and $n\equiv n_T$; in chaotic inflation and new inflation n is slightly smaller than the tensor exponent n_T: $(1-n)=(1+m/120)(1-n_T)$, where $m=240/p$ (chaotic); and $(1-n)=(1+m/40)(1-n_T)$, where $m=4\times10^4|\ln(\phi_N/\sigma^2)|/(\sigma/m_{\mathrm{Pl}})^4$ (new). It is possible to construct models where n and n_T differ significantly—and even to have $n>1$ and $n_T<1$. However, we find that our general bound, $T/S\lesssim3$, still holds, and it remains true that $T/S\gtrsim1$ implies $n_T\lesssim0.85$.

[19] We have examined a variety of other models, e.g., cosine and polynomial potentials. The cosine potential has the unusual feature that the tensor perturbations remain scale invariant while the scalar perturbations tilt away from scale invariance ($n>1$). Of all of these other models, the only example that we found that permits $T/S\gtrsim1$ is, $V(\phi)=\lambda(\phi^2-\sigma^2)^2$. For $\sigma\lesssim0.8m_{\mathrm{Pl}}$, $T/S\gtrsim1$; since a necessary condition for sufficient inflation is $\sigma\gtrsim0.5m_{\mathrm{Pl}}$, this example is a marginal one at best [16].

[20] As T/S approaches this upper bound, $m\to1$ and $n\to-\infty$. In this limit, our assumptions, slow roll ($\dot\phi=-V'/3H$) and the coefficient in Eq. (3), break down.

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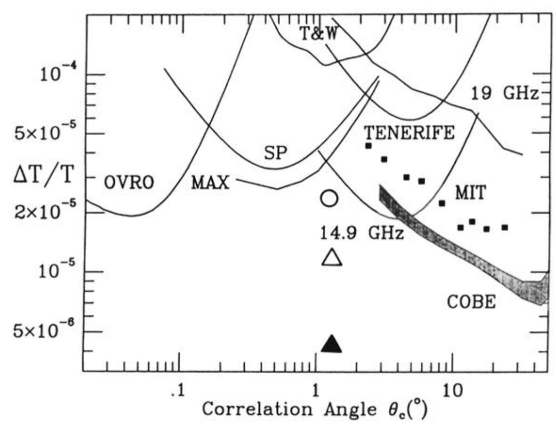

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