Classical Hall Plateaus in Ballistic Microjunctions

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We demonstrate that nonlinear dynamics is able to elucidate many details of Hall experiments in ballistic multiprobe conductors in semiconductor heterojunctions, if soft boundaries with a realistic confining potential are assumed. This gives rise to two additional plateaus above the last Hall plateau, which were observed experimentally, but had remained unexplained. We show that all three plateaus are caused by trajectories leaving the junction in the *forward* direction and that the generally accepted explanation of the last Hall plateau is invalid.

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In recent years transport in semiconductor microstructures has been investigated intensely and a variety of magnetotransport anomalies were found [1–4]. Among the phenomena observed in ballistic multiprobe conductors (cross junctions) in a two-dimensional (2D) electron gas are a quenched or negative Hall resistance [5–10], bend resistances [11–13], and a feature known as the last Hall plateau [5–8, 14, 15]. The physical origin of these anomalies may be understood in the scattering approach developed by Landauer [16] and Büttiker [17], which expresses the resistances in terms of transmission probabilities. As the size of the cross junctions is less than the mean free path, transport is ballistic. Beenakker and van Houten [14] simulated the transmission probabilities in a hard-wall billiard model and showed that many experimental effects, e.g., the last Hall plateau, can be reproduced based on classical trajectories. This billiard model, however, fails to reproduce additional plateau-like anomalies [5, 18] that are observed in the magnetic field range above the last Hall plateau and below the fractional quantum Hall plateaus [19].

In the present Letter we demonstrate that the latter features [5, 18] have a classical origin and pertain to classes of classical trajectories that cannot arise in ideal (hard wall) billiards but require a soft confining potential. An appropriate choice for this potential gives rise to two additional Hall plateaus besides exhibiting the last Hall plateau and the quenching of the Hall resistance. The plateaus are associated with the disappearance of classes of trajectories leaving the junction in the forward direction after a different number of loops. Furthermore, we demonstrate that the so far generally accepted explanation of the last Hall plateau must be revised. The large number of trajectories guided around a corner of the junction (see, e.g., lowest inset in Fig. 3) are not responsible for the last Hall plateau. Instead it originates from trajectories leaving the junction in the forward direction. These results underline the relevance of nonlinear dynamics for ballistic transport in semiconductor microstructures and illustrate the extent to which details of transport experiments can be understood, if details of the potential and of the dynamics are taken into account.

We have previously shown that a series of peaks observed in the magnetoresistance of antidot superlattices [20] can be explained by taking into account details of the antidot potential [21]. In a cross junction sample, the confining electrostatic potential for an electron has a minimum at the center of the junction, as there the distance to the confining walls is larger than in the narrow incoming leads. The exact shape of this potential is not known, but, as will turn out, the occurrence of additional plateaus is determined by qualitative properties of the potential, i.e., the existence or not of a potential minimum. Under these aspects it is appropriate to qualitatively model this minimum and the smooth potential walls by the function

$$V(x,y) = \cos x + \cos y + A\,\cos x \cos y + A \tag{1}$$

with $x, y \in [0, 2\pi]$ (see Fig. 1, inset). A magnetic field B is applied perpendicular to the x-y plane. The potential minimum distinguishes our work from previous simulations [4, 14], where potentials without a central well were assumed. The main geometric features of this potential can be adjusted by varying the Fermi energy E_F and A ($-1 < A \leq 1$) and are described by two parameters $\hat{r}$ and $\hat{f}$. The ratio $\hat{r} = r/W$ is the radius r of the corners divided by the width W of the leads at E_F and the ratio $\hat{f} = (2A-2)/E_F$ is the depth of the potential minimum (at $x = y = \pi$) divided by E_F. We mention that Eq. (1) when extended as a periodic potential in the x-y plane also describes lateral surface superlattices on semiconductor heterojunctions where chaotic dynamics occurs in the form of diffusion and anomalous diffusion [22, 23].

The current I_i in lead i of a multiprobe conductor with chemical potentials $\mu_j = eV_j$ attached to leads j can be expressed in terms of the transmission probabilities T_{ij} across the junction from reservoir j to lead i by [17]

$$I_i = \frac{e^2}{h} \sum_j T_{ij} N_j (V_i - V_j), \tag{2}$$

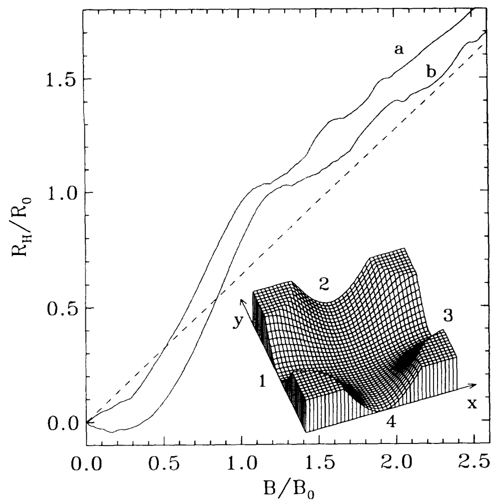

FIG. 1. Hall resistance R_H vs magnetic field for a smooth model potential of a cross junction (inset with lead numbers) with $\hat{r} = 1/2$ and $\hat{f} = -1/3$ (curve a) showing the last Hall plateau and two additional plateaus. For $\hat{f} = -1$ (curve b) the additional plateaus are shifted to higher values of R_H and still lie above the 2D Hall resistance of free electrons (dashed line). The latter is approached asymptotically for higher magnetic fields.

where N_j is the number of modes in lead j and normalization requires $\sum_j T_{ij} = \sum_i T_{ij} = 1$. In a fourfold symmetric junction the Hall resistance $R_H = (V_2 - V_4)/I$ for a current $I = I_1$ from lead 1 to lead 3 ($I_3 = -I$) and zero current $I_2 = I_4 = 0$ in the voltage leads 2 and 4 is then given by

$$R_H = R_0 \frac{T_{21} - T_{41}}{T_{21}^2 + T_{41}^2 + 2T_{31}(T_{21} + T_{31} + T_{41})} \tag{3}$$

with $R_0 = h/e^2N$ and $N = N_j$ in all leads. In the semiclassical regime ($N \gg 1$) the transmission probabilities T_{ij} can be determined from classical dynamics of electrons at the Fermi energy starting in lead 1. Assuming zero magnetic field in the leads, the electrons have a uniform angular distribution, as they come from a reservoir in thermal equilibrium. Their contribution to the current, however, is proportional to their velocity component $v_{||}$ parallel to the lead [4] and thus we choose an injection probability into the cross junction following an angular and spatial distribution

$$p(y, \theta) \propto v_{||} = \sqrt{2[E_F - V(x = 0, y)]/m} \cos\theta \tag{4}$$

where θ is the angle between $\mathbf{v}$ and the x axis. The magnetic field can be scaled by $B_0 = mv_F/eW_{\text{eff}}$, the magnetic field where the cyclotron radius of a free electron with Fermi velocity v_F in the lead equals the effective width $W_{\text{eff}} = \int dy\, v(y)/v_F$ of the lead. For comparison the scaled 2D free electron result is $R_H/R_0 = (2/\pi)B/B_0$ where the number of modes N is given by $N = mv_FW_{\text{eff}}/\hbar\pi$.

We have calculated the Hall resistance equation (3) from transmission probabilities T_{ij} determined by numerical simulations of classical trajectories injected into the junction potential $V(x, y)$ according to the distribution $p(y, \theta)$ (Fig. 1). Studying two different parameter sets $\hat{r}, \hat{f}$ we find that, besides the last Hall plateau near $B \approx B_0$ with $R_H/R_0 \approx 1$, there are two additional plateaus for $B > B_0$. A similar sequence of Hall plateaus was observed in experiments [5, 18] and has as yet remained unexplained. In agreement with experiment there is an enhancement of the Hall resistance in this magnetic field range compared to the free electron case and a dependence of the location of the additional plateaus on geometry (Fig. 1). For small magnetic fields the Hall resistance R_H is quenched or even negative. This phenomenon is already well understood in ideal billiards in terms of collimation and rebound trajectories [4, 9, 24–26].

We can explain the occurrence of the three Hall plateaus by analyzing the transmission probabilities T_{ij} in more detail. For magnetic fields $B > B_0$ the electrons roughly skip along the corners of the junction in a clockwise sense and eventually leave the junction through the kth possible lead with a probability P_k ($k \in \mathbb{N}$). We introduce these quantities P_k in order to distinguish escapes through a lead after different numbers of revolutions along the corners. Thus P_1 is the probability to return into the injecting lead without reflections, whereas P_5 is the probability to leave through the incoming lead after skipping along all four corners of the junction. The total transmission probability T_{i1} from lead 1 to lead i is given by the sum $T_{i1} = \sum_{k=0}^{\infty} P_{4k+i}$. As the probabilities P_k decay rapidly with index k [27], a suitable normalization helps to display more details. We therefore consider the conditional probabilities $\tilde{P}_k = P_k / \sum_{i=k}^{\infty} P_i$ that the trajectory leaves the junction at the kth lead if the first $k - 1$ possible leads were avoided.

In Fig. 2 the magnetic field dependence of these probabilities is compared with the calculated Hall resistance. $\tilde{P}_1$ and surprisingly $\tilde{P}_2$ have no anomalies in the magnetic field range of the three plateaus. Thus the trajectories going directly from lead 1 to lead 2 —although they do give the largest contribution to the Hall conductance—are not at all responsible for the Hall plateaus. This is in clear contrast to the widespread opinion on the origin of the last Hall plateau where this is assumed. In fact, all three Hall plateaus set in at prominent minima of $\tilde{P}_3$, the conditional probability to leave the junction through lead 3 after going along two corners (see Fig. 2 insets). The minima of $\tilde{P}_3$ can be understood by the disappearance of different classes of trajectories illustrated in the four insets in the top of Fig. 2. They all leave the junction upon their first arrival at lead 3 after a different number of loops in the vicinity of lead 2. Dashed and solid lines for $\tilde{P}_3$ correspond with trajectories indicated by dashed and solid lines. Whenever

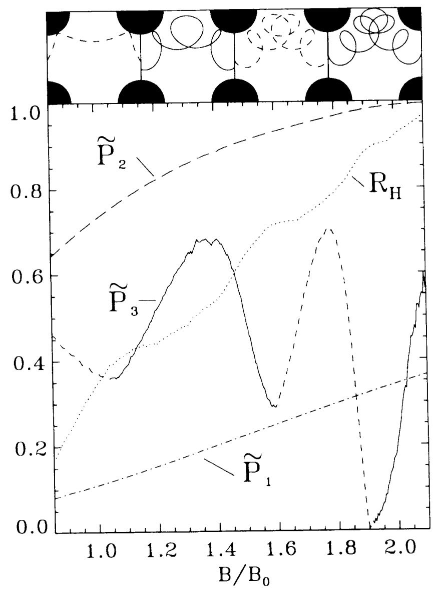

FIG. 2. Conditional transmission probabilities $\tilde{P}_1$, $\tilde{P}_2$, and $\tilde{P}_3$ for exits through leads 1, 2, and 3, respectively, after injection from lead 1 for $\hat{r} = 1/2$ and $\hat{f} = -1/3$. No anomalies can be seen in $\tilde{P}_2$, the conditional probability for direct exits around a single corner of the junction. The prominent minima in $\tilde{P}_3$, however, correspond with the onset of the three plateaus in the Hall resistance R_H (dotted line). They are caused by the extinction of the first three classes of trajectories connecting lead 1 with lead 3, which are shown in the first three insets on top where dashed and solid lines correspond with the respective regimes of $\tilde{P}_3$.

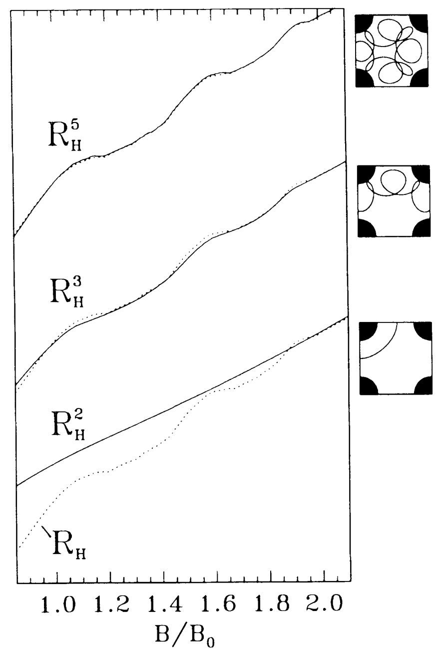

FIG. 3. Partial Hall resistances R_H^k for k =2, 3, and 5 (solid lines) calculated from all probabilities P_i with $i \leq k$ compared with the full Hall resistance R_H (dotted line) and shifted vertically for clarity. R_H^2 does not show any anomalies, whereas in R_H^3 the Hall plateaus appear corresponding with the minima of $\tilde{P}_3$. Still longer trajectories (R_H^5) only add minor contributions to the full Hall resistance. The insets on the right illustrate some typical trajectories associated with the partial Hall resistances.

the dominant class of trajectories ceases to exist with increasing magnetic field a minimum appears in $\tilde{P}_3$. The three minima of $\tilde{P}_3$ corresponding with the onset of the three Hall plateaus are due to the vanishing of the first three classes of trajectories in the insets of Fig. 2, respectively. These trajectories with a varying number of loops without escape near lead 2 are possible only due to the existence of the potential minimum in the center of the well. The increasing potential in the direction of lead 2 causes the particle to return and to perform the loops shown in the insets of Fig. 2. For a flat potential along the direction of the lead and considering the small cyclotron radius, the particle would escape through lead 2. Thus the additional Hall plateaus can show up neither in hard-wall billiard models nor in smooth confining potentials lacking a minimum.

To support the conclusion that the plateaus are mainly caused by these trajectories we calculated partial Hall resistances R_H^k, shown in Fig. 3 (solid lines), where all trajectories leaving the junction through one of the first k leads were included correctly, whereas longer trajectories were included for completeness with an equal distribution of exit through the four leads. In R_H^2 no anomalies can be seen, whereas R_H^3 already anticipates most of the three Hall plateaus in the full Hall resistance R_H (shown by the dotted lines). This substantiates the conclusion that trajectories going from lead 1 to lead 3 are responsible for the Hall plateaus. Still longer trajectories only add minor contributions to R_H, as can be seen from R_H^5.

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