Two-Jet Production in Hadron Collisions at Order α_s^3 in QCD

Stephen D. Ellis
Department of Physics, University of Washington, Seattle, Washington 98195

Zoltan Kunszt
Eidgenössische Technische Hochschule, CH-8093 Zürich, Switzerland

Davison E. Soper
Institute of Theoretical Science, University of Oregon, Eugene, Oregon 97403

(Received 9 July 1992)

We calculate the two-jet inclusive cross section at order α_s^3 in QCD. Because the one-loop perturbative corrections are included, there is considerable improvement in theoretical accuracy compared to the Born-level calculation. We compare the predicted dependence of the cross section on the jet-jet scattering angle $\eta_1 - \eta_2$ to data from the CDF Collaboration. We also discuss the dependence of the cross section on the jet-jet mass.

PACS numbers: 13.87.Ce, 12.38.Bx, 12.38.Qk

The experimental investigation of two-jet production in high-energy hadron collisions provides a direct view of the underlying process, parton-parton scattering. It thus gives us a chance to test quantum chromodynamics in some detail.

For instance, the gluons of QCD carry the color charge and thus interact strongly with one another. Two-jet production gives us the opportunity to examine experimentally the process in which a gluon from one hadron is scattered by means of a gluon exchange with a quark or gluon in the other hadron. This process involving the gluon self-coupling predominates in two-jet production when the jet-jet mass M_{JJ} is smaller than approximately $\sqrt{s}/5$ and the scattering angle in the parton-parton center-of-mass frame is small. One can investigate the details of the scattering by measuring the angular distribution of the jets that are the decay products of the outgoing partons.

Parton-parton scattering also provides a chance to look for a breakdown of the standard model at small distances. One can look at large M_{JJ} for deviations of the two-jet cross section $d\sigma/dM_{JJ}$ from the QCD prediction for pointlike quarks and gluons (cf. [1]).

In this Letter, we report on a calculation of the two-jet inclusive cross section at order α_s^3 in QCD. At this order the calculation involves final states with two or three partons and includes the one-loop virtual corrections for the two-parton final state. Because the one-loop perturbative corrections are included, there is a considerable improvement in theoretical accuracy compared to the Born-level calculation.

Our calculation begins with the order α_s^3 squared matrix elements provided by Ellis and Sexton [2]. We integrate, partially numerically and partially analytically, over the phase space of the final-state partons as required according to the jet definition used. Infinities occur in individual terms in the cross section, but these infinities cancel, and we compute the finite remainder. In earlier work [3] we have done this for the one-jet inclusive cross section. Now we have a computer program based on a more powerful algorithm that allows for the calculation of any "infrared safe" cross section for which the perturbation expansion begins at α_s^3. The first results using this algorithm were reported in Ref. [4] and the algorithm itself is described at some length in Ref. [5]. We have checked that the new program agrees with our earlier one-jet program when it is applied to the one-jet cross section. We have also checked that our one-jet program agrees with that of Aversa, Greco, Chiappetta, and Guillet [6], which is also based on this Ellis-Sexton matrix elements but uses a computational algorithm that is much different from ours.

The cross section that we calculate is $d\sigma/dM_{JJ}d\eta_1 d\eta_2$ where M_{JJ} is the invariant mass of the two-jet system, η_1 is the pseudorapidity ($-\ln\tan\Theta/2$) of jet 1, and η_2 is the pseudorapidity of jet 2. One uses pseudorapidity because of its simple transformation properties under Lorentz boosts along the beam direction. The jet definition is also chosen to display similar simplicity. In the definition [7] that we use, a jet consists of all the particles n whose momenta $\mathbf{p}_n$ lie within a cone centered on a jet axis (η_J,ϕ_J) in rapidity η and azimuthal angle ϕ, $[(\eta_n-\eta_J)^2+(\phi_n-\phi_J)^2]^{1/2}<R$, where the cone size used in this paper is $R=0.7$. The jet angles (η_J,ϕ_J) are the averages of the particles' angles,

$$\eta_J = \left(\sum_{n\in\text{cone}} p_{T,n}\eta_n\right)\Big/\left(\sum_{n\in\text{cone}} p_{T,n}\right),$$
$$\phi_J = \left(\sum_{n\in\text{cone}} p_{T,n}\phi_n\right)\Big/\left(\sum_{n\in\text{cone}} p_{T,n}\right). \quad (1)$$

The process is iterated so that the cone center matches the jet center (η_J,ϕ_J) computed using Eq. (1). We deal with jet overlaps as discussed in Ref. [3].

To define the two-jet inclusive cross section using events that may have more than two jets, we follow the definition of the CDF group [8] and use the two jets in the event that have the largest transverse energies $E_T=\sum p_{T,n}$. Having chosen the two jets of interest, we let η_1 and η_2 be the rapidities of these two jets and we define the jet-jet mass M_{JJ} as the invariant mass $[(\sum P_n^\mu)^2]^{1/2}$ of the particles in the two jets.

These definitions apply to final states containing any number of particles. In our calculation, we apply the definitions to final states consisting of either two or three partons, letting the p_n^μ label the momenta of the partons. This gives the order α_s^3 cross section.

The physics of the two-jet system can be most clearly seen if we use the variables η_{JJ} and η_*, defined by

$$\eta_{JJ}=(\eta_1+\eta_2)/2,\quad \eta_*=|\eta_1-\eta_2|/2\,. \tag{2}$$

If one performs a Lorentz boost in the z direction through a hyperbolic angle $\omega=-\eta_{JJ}$, then the rapidities η_1',η_2' in the boosted system are equal and opposite: $\eta_1'=-\eta_2'$. The variable η_* is the rapidity of the jets in this frame: $\eta_*=|\eta_1'|=|\eta_2'|$. In order to present graphs in which the expected angular distribution in QCD is rather flat, we often use instead of η_* the variable

$$\lambda=\tfrac{1}{2}\sinh(2\eta_*)\,. \tag{3}$$

In a Born-level 2 parton→2 parton calculation, the boosted frame is the parton-parton center-of-mass frame. The variable λ is related to the parton-parton scattering angle θ_* in this frame by $\lambda=\cos\theta_*/\sin^2\theta_*$. [An alternative variable is $\chi=(1+\cos\theta_*)/(1-\cos\theta_*)$ [8], so that $\lambda=(\chi-1/\chi)/4$.] Thus the λ dependence of the cross section tells us about the parton-parton scattering dynamics.

Consider the cross section

$$\Delta\sigma(M_{JJ},\lambda)=\int_{-0.75}^{0.75}d\eta_{JJ}\frac{d\sigma}{dM_{JJ}d\eta_{JJ}d\lambda}\,. \tag{4}$$

The behavior of the angular distribution $\Delta\sigma(M_{JJ},\lambda)/\Delta\sigma(M_{JJ},0)$ at large λ indicates how the invariant squared matrix element $\langle|\mathcal{M}(s,t,u)|^2\rangle$ behaves as $s/t\to\infty$, thus indicating the spin of the quantum exchanged between the two scattering partons. With spin-0 exchange, one has $\langle|\mathcal{M}|^2\rangle\sim1$ as $s/t\to\infty$, which gives $\Delta\sigma(M_{JJ},\lambda)\sim1/\lambda^2$ for $\lambda\to\infty$. With spin-$\frac{1}{2}$ exchange, one has $\langle|\mathcal{M}|^2\rangle\sim s/t$, which gives $\Delta\sigma(M_{JJ},\lambda)\sim1/\lambda$. Finally, for spin-1 exchange, one has $\langle|\mathcal{M}|^2\rangle\sim(s/t)^2$ as $s/t\to\infty$, which gives $\Delta\sigma(M_{JJ},\lambda)\sim1$ as $\lambda\to\infty$. These behaviors are illustrated in the "spin 0," "spin $\frac{1}{2}$," and "spin 1" curves in Fig. 1. In the final curve in Fig. 1, we show the cross section to make two jets with $M_{JJ}=300$ GeV in $p\bar{p}$ collisions at $\sqrt{s}=1800$ GeV as calculated in *Born-level* QCD with a fixed value of the scale parameter μ, namely, $\mu=M_{JJ}/4$. The cross section is a combination of spin-1 exchange, spin-$\frac{1}{2}$ exchange, and terms such as s-channel virtual-gluon production that look approximately like spin-0 exchange. The net result is a cross section that looks quite flat.

We now discuss the choice of the arbitrary factorization and renormalization scale μ. One might choose $\mu=AM_{JJ}$ where A is a constant of order 1, as we did

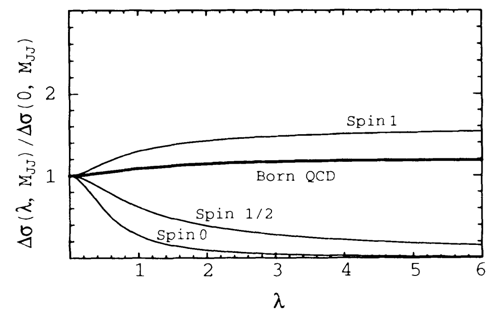

FIG. 1. The angular distribution $\Delta\sigma(M_{JJ},\lambda)/\Delta\sigma(M_{JJ},0)$ as a function of λ with three choices of exchanged quanta: Spin 0, $\langle|\mathcal{M}|^2\rangle\propto1$; spin $\frac{1}{2}$, $\langle|\mathcal{M}|^2\rangle\propto t/u+u/t$; spin 1, $\langle|\mathcal{M}|^2\rangle\propto(s^2+t^2)/u^2+(s^2+u^2)/t^2$. Also shown is the cross section as calculated in Born-level QCD with a fixed value of $\mu=M_{JJ}/4$ for $M_{JJ}=300$ GeV in $p\bar{p}$ collisions at $\sqrt{s}=1800$ GeV.

above. However, one suspects that a better measure of the "hardness" of the process might be the transverse momentum P_T of the one of the jets. In a Born-level calculation, $P_T=M_{JJ}/2\cosh(\eta_*)$. In order to allow ourselves to interpolate between these two possibilities, we choose a form for μ with two adjustable parameters, A and B: $\mu=AM_{JJ}/2\cosh(B\eta_*)$. We chose $A=0.5$ and $B=0.7$, so that the difference between the order α_s^3 calculation and the Born calculation is small over the angular region of interest. Thus our "standard" choice of μ is

$$\mu_0(M_{JJ},\lambda)=M_{JJ}/4\cosh(0.7\eta_*)\,. \tag{5}$$

This implies that for large λ, $\mu_0\sim0.3M_{JJ}/\lambda^{0.35}$.

We now turn to an estimate of the uncertainty in the theoretical prediction. The dependence of the cross section on the unphysical parameter μ provides a way to estimate the uncertainty in the calculation that results from leaving out order α_s^4 and higher-order contributions to the cross section. In a Born-level (α_s^2) calculation, μ appears as an argument to the coupling $\alpha_s(\mu)$ and the parton distribution functions $f_{a/A}(x,\mu)$. When one goes to order α_s^3, most of this μ dependence is caneled by explicit $\ln(\mu)$ terms. If higher-order contributions could be calculated, they would eliminate, successively, more of the μ dependence. Thus an estimate of the size of the uncalculated higher-order terms is provided by the change of the order α_s^3 cross section when one varies μ by, say, a factor of 2. This is illustrated in Fig. 2, where we show the cross section $\Delta\sigma(M_{JJ},\lambda)$ as a function of $\mu/\mu_0(M_{JJ},\lambda)$ for $M_{JJ}=300$ GeV and two choices for λ, $\lambda=0$ and $\lambda=14$. In each case, curves are shown both for a Born calculation and for a full order α_s^3 calculation. We see that the Born curves vary by some 40% as one varies μ by a factor of 2, suggesting that there is a 40% uncertainty in the Born-level calculation. However, the order α_s^3 cross section changes only by about 10% as one varies μ by a factor of 2, suggesting that only a 10% uncertainty from yet

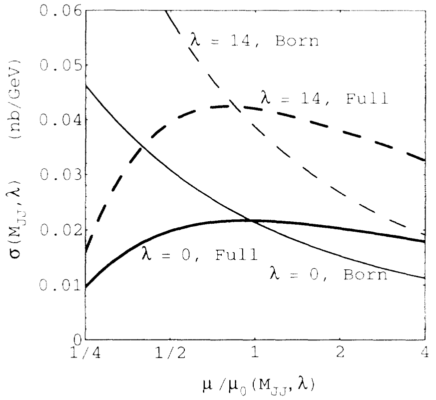

FIG. 2. The dependence of the cross section $\Delta\sigma(M_{JJ},\lambda)$, on the ratio of the scale parameter μ to our "standard" scale $\mu_0(M_{JJ},\lambda)$, Eq. (5). Both the full α_s^3 cross section and the Born cross section are shown for $\lambda=0$ and $\lambda=14$. The curves are for $p\bar{p}$ collisions at $\sqrt{s}=1800$ GeV with $M_{JJ}=300$ GeV.

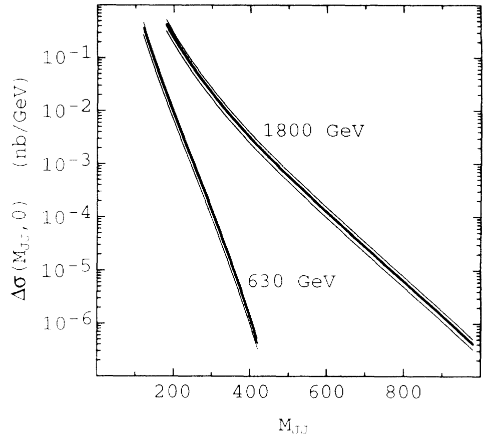

FIG. 3. The order α_s^3 prediction for the jet-jet mass distribution $\Delta\sigma(M_{JJ},0)$, as a function of M_{JJ}, for $p\bar{p}$ collisions at $\sqrt{s}=1800$ GeV (upper curve) and for $\sqrt{s}=630$ GeV (lower curve). The error band represents the estimated theoretical uncertainty.

higher-order contributions remains in the α_s^3 result.

There is also a theoretical error because the perturbative QCD formula used does not include contributions to the cross section that are suppressed at high momenta by powers of (1 GeV)/M_{JJ}. One contribution of this type arises when energy carried by low momentum (~ 300 MeV) particles from the "underlying event" is included in the jet cones. In addition, nonperturbative fragmentation of a hard parton in a jet can produce low momentum particles at wide angles to the parton momentum. Some of these low momentum particles may fall outside of the jet cone. A rough estimate is that a net 1 GeV of transverse energy could be gained or lost by a jet in this way, thus changing the jet-jet mass by $\delta M_{JJ}\sim(2\ \text{GeV})\cosh(\eta_*)$. Since the cross section falls with increasing M_{JJ} roughly like M_{JJ}^{-8}, this uncertainty translates into an uncertainty in the cross section $\delta\Delta\sigma/\Delta\sigma \sim 8\delta M_{JJ}/M_{JJ}\sim(16\ \text{GeV})\cosh(\eta_*)/M_{JJ}$. For instance, for $M_{JJ}=300$ GeV and $\eta_*=1$ this is an 8% uncertainty. For much smaller values of M_{JJ} or larger values of η_*, these nonperturbative effects could be significant, and should be carefully estimated.

A final source of uncertainty in the calculation arises from the fact that the parton distribution functions are only imperfectly known. In this paper we have used HMRS-B parton distributions [9]. On the basis of trying some of the modern parton distributions [9,10], we estimate a 20% uncertainty in $\Delta\sigma(M_{JJ},\lambda)$ from this source.

We are now ready to look at the cross section as predicted by order α_s^3 QCD. In Fig. 3, we plot $\Delta\sigma(M_{JJ},0)$ as a function of M_{JJ}. Also shown are error bands representing the theoretical errors as estimated above, combined in quadrature. We see that the cross section falls quite steeply. A deviation from the standard model might be seen as a resonance bump in the data as compared to the theoretical curve or as a deviation from the theory at the highest values of M_{JJ}. The deviation could be either positive or negative, depending on whether the "new physics" interferes constructively or destructively with the QCD amplitude.

Finally, in Fig. 4 we compare the predicted angular distribution to data from the CDF Collaboration [8]. We define

$$\Delta\sigma(\lambda)=\int_{240\ \text{GeV}}^{475\ \text{GeV}} dM_{JJ}\int_{-0.75}^{0.75} d\eta_{JJ}\frac{d\sigma}{dM_{JJ}d\eta_{JJ}d\lambda} \quad (6)$$

and plot $\Delta\sigma(\lambda)/\Delta\sigma_T$, where $\Delta\sigma_T=\int_0^6 d\lambda\Delta\sigma(\lambda)$. The theoretical uncertainty is represented as an error band. To a certain extent, the uncertainty will cancel between the numerator and denominator of $\Delta\sigma(\lambda)/\Delta\sigma_T$. Thus we take the uncertainty arising from the parton distributions

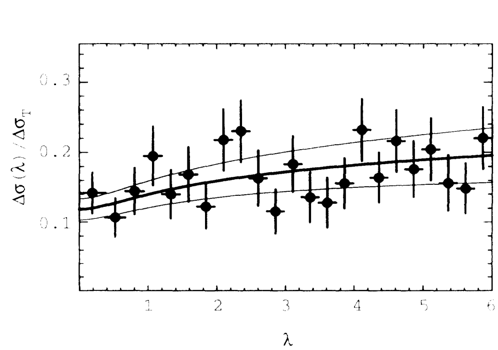

FIG. 4. Comparison of the order α_s^3 prediction for the jet-jet angular distribution to data from the CDF Collaboration [8]. We show $\Delta\sigma(\lambda)/\Delta\sigma_T$ for 240 GeV $< M_{JJ} <$ 475 GeV, for $p\bar{p}$ collisions at $\sqrt{s}=1800$ GeV. The error band represents the estimated theoretical uncertainty, while the error bars on the data include the statistical experimental errors only.

to be only 5%, instead of the 20% that we estimated for either the numerator or the denominator separately. For the uncertainties arising from higher-order contributions to the cross section and from power-suppressed nonperturbative contributions, we conservatively use the uncertainties in the numerator that were estimated earlier, ignoring any possible cancellations.

We notice that the order α_s^3 prediction is not as flat as was the Born-level prediction in Fig. 1. This effect, which is seen in the data, is due to higher-order QCD. We see that the data are in good agreement with QCD. By comparing with Fig. 1, we also see that the data are good enough to clearly distinguish between QCD, characterized mainly by spin-1 gluon exchange, and models using only spin-$\frac{1}{2}$ or spin-0 exchange.

We thank the members of the CDF Collaboration's QCD group, and in particular B. Flaugher, for advice on the measurement of jets. One of the authors (D.E.S.) thanks the SSC theory group for its hospitality during the final stage of the preparation of this paper. This work was supported in part by U.S. Department of Energy Grants No. DE-FG06-85ER-40224 and No. DE-FG06-91ER-40614.

[1] E. Eichten, I. Hinchliffe, K. Lane, and C. Quigg, Rev. Mod. Phys. **56**, 579 (1984).

[2] R. K. Ellis and J. Sexton, Nucl. Phys. **B269**, 445 (1986).

[3] S. D. Ellis, Z. Kunszt, and D. E. Soper, Phys. Rev. D **40**, 2188 (1989); Phys. Rev. Lett. **64**, 2121 (1990).

[4] S. D. Ellis, Z. Kunszt, and D. E. Soper, in Proceedings of the 1991 International Symposium on Lepton and Photon Interactions at High Energies, Geneva, July, 1991 (to be published).

[5] Z. Kunszt and D. E. Soper, Phys. Rev. D (to be published).

[6] F. Aversa, M. Greco, P. Chiappetta, and J. Ph. Guillet, Phys. Rev. Lett. **65**, 401 (1990); Z. Phys. C **49**, 459 (1991); J. Ph. Guillet (private communication).

[7] J. E. Huth, N. Wainer, K. Meier, N. Hadley, F. Aversa, M. Greco, P. Chiappetta, J. Ph. Guillet, S. D. Ellis, Z. Kunszt, and D. E. Soper, in Proceedings of the Summer Study on High Energy Physics, Research Directions for the Decade, Snowmass, Colorado, 24 June–13 July 1990 (to be published).

[8] CDF Collaboration, Report No. FERMILAB-PUB-92/182-E (to be published).

[9] P. N. Harriman, A. D. Martin, R. G. Roberts, and W. J. Stirling, Phys. Rev. D **42**, 798 (1990).

[10] J. G. Morfin and W.-K. Tung, Z. Phys. C **52**, 13 (1991).