Acoustic Waveguide Modes Observed in Electrically Heated Metal Wires

J. Seyler and M. N. Wybourne

Department of Physics, University of Oregon, Eugene, Oregon 97403 (Received 13 April 1992)

The steady-state resistance of small metal wires subject to an electric field is found to exhibit periodic resonances that scale with the width of the wire. The energies of the resonances have been determined using the low-temperature corrections to the resistance of the wires, and are shown to correspond to acoustic modes of the wire. We conclude that the resonances originate from increased energy relaxation that occurs when the electron system gains enough energy from the electric field to emit phonons in a higher acoustic subband of the wire.

PACS numbers: 73.50.Rb, 66.90.+r, 68.60.Bs

Nonequilibrium experiments performed on metallic films and wires have been of interest as a way to study modifications of the phonon spectrum in systems where the dominant phonon wavelength becomes comparable with one or more of the sample's dimensions [1,2]. Recently, it has been suggested that the density of phonon states in electrically heated Au-Pd films on silicon substrates can be reduced, provided the acoustic coupling between the metal and the substrate is weak [3]. In this Letter we present the results of a steady-state heating experiment on narrow wires with cross sections between 600 and 1800 nm². The data show periodic resonances in the heating characteristics that scale with the width of the wires and can be assigned to the energies of acoustic waveguide modes of the wires.

The samples used for the experiments were $Au_{60}Pd_{40}$ alloy wires that were defined by electron-beam lithography and thermally evaporated onto a silicon substrate at room temperature at a pressure of 5×10^{-6} Torr. The native oxide layer on the silicon was not removed prior to the thermal evaporation of the wires. Three sets of wires were studied: 45 parallel wires 20 nm thick, 90 nm wide, and 30 µm long; 25 parallel wires 18 nm thick, 50 nm wide, and 10 μ m long; and 100 parallel wires 20 nm thick, 30 nm wide, and 85 µm long. The 90- and 30-nmwide wires were made concurrently on the same substrate, while the 50-nm-wide wires were fabricated at a different time using a different silicon substrate. At 4.2 K, the sheet resistance of the wires was $R_{\Box} = 12 \Omega$. The samples were designed for true four-terminal measurement. They were secured to the inside of an evacuated copper chamber that was thermally attached to a temperature-controlled helium bath regulated to better than 1 mK over the temperature range 1 to 20 K.

Resistance measurements were made using a lowfrequency (23 Hz) four-terminal bridge [4] and signal averaging. With this technique, a fractional resistance change of 1 part in 10^6 could be detected with sensing currents as low as 5 nA per wire. Electron heating was achieved by applying a dc electric field E of up to 13 V cm⁻¹ across the wires. The ac sensing bias was kept low enough that its contribution to the heating always remained negligible.

The temperature dependence of the equilibrium resistance change, $(R - R_0)/R_0 = \Delta R/R_0$, of all three sets of wires showed a minimum at $T_0 \approx 8$ K, below which the resistance increased as approximately $T^{-1/2}$, as shown in Fig. 1. The equilibrium resistance minimum R_0 has been taken to be the value at T_0 . Above T_0 the resistance change increased and was sample dependent. The 30nm-wide wires exhibited an almost linear temperature dependence between T_0 and 20 K, while the 50-nm-wide samples were best fitted by a T^2 dependence, and the 90nm-wide wires showed a T^3 power law. This variation in temperature dependence above T_0 has a range of powers similar to those reported previously [3,5]. The $T^{-1/2}$ dependence below the minimum is characteristic of interaction corrections in one dimension [6], as seen in wires when the diffusion length, $l_c = (D\hbar/k_BT)^{1/2}$, is less than the width W and thickness d of the wires. For the three samples, the three-dimensional electronic diffusion constant D is 2×10^{-3} m²s⁻¹. Thus $l_c \ge 44$ nm at temperatures below T_0 . Therefore, only the narrowest wires



FIG. 1. The equilibrium temperature dependence of $\Delta R/R_0$ for the wires of width 30 (\oplus), 50 (\oplus), and 90 (\triangle) nm. Inset: An expanded region of $\Delta R(E)/R_0$ for the 30-nm-wide wire in which the first two resonances are indicated by the arrows.

are strictly one dimensional. The electron dimensionality in the 50- and 90-nm-wide wires is expected to be reduced from two to one within the temperature range studied; consequently the temperature dependence of $\Delta R/R_0$ is expected to change from logarithmic to $T^{-1/2}$ [6]. This could not be detected in the experiments, and a $T^{-1/2}$ dependence was found to describe all three sets of data well. The resistance corrections we observe are similar to those reported by other workers for Au-Pd wires and films [5], where it was shown that the dominant resistance correction is due to electron-electron interactions.

In the presence of an applied electric field, the steadystate resistance change of the three wire samples, $[R(E) - R_0]/R_0 = \Delta R(E)/R_0$, was found to fall below R_0 at fields above $\sim 3 \text{ V cm}^{-1}$. Also, at certain values of the electric field the steady-state resistance exhibited distinct features, as illustrated in the inset to Fig. 1. Since these features are small compared with the overall change in resistance, to show them more clearly the background has been removed by fitting it with the form $\Delta R(E)/R_0$ $= \alpha E^{-1} + \beta E + \gamma E^2$, where α , β , and γ are constants, and then subtracting the fit from the original data. Following this procedure, the data for the three sets of wires are shown in Fig. 2. For each wire, certain features appear as a series of peaks that are close to being periodic in the electric field; these are labeled numerically in Fig. 2. Peaks having no obvious periodicity are also found, for example, those above peak 4 for the 90-nm wire. The resonances were found to broaden with increasing temperature, but to within experimental uncertainty their position was temperature independent.



FIG. 2. The change in resistance vs electric field for the wires of width 30 (\oplus), 50 (\oplus), and 90 (\blacktriangle) nm, after removing the background as described in the text. The numbers are the assigned values of the subband index *n*. The peak labeled *T* is believed to be associated with the transverse velocity of sound. For clarity the traces have been offset.

Previous electron heating experiments on thin metal films have reported a fall in the steady-state resistance below R_0 [3,7]. The origin of this effect can be understood by considering how a low-temperature resistance correction and a Drude component that make up the equilibrium resistance combine in the steady state. In the present samples the resistance correction arises from electron-electron interactions which are dependent on the electron temperature T_e . The Drude component describes the resistance arising from impurity and electronphonon scattering, where the impurity scattering is temperature independent and the electron-phonon scattering depends on both the electron and phonon distributions. Since the correction and Drude components of the resistance each have a different dependence on the electron and phonon distributions in the wires, when the electric field moves the electrons and phonons away from equilibrium, the relative contribution of the two components to the total resistance is changed. It has been demonstrated that such a shift in the relative contribution of each component can cause the steady-state resistance to fall below R_0 [3].

Resonant features arising from universal conductance fluctuations (UCF) are expected in electrically heated mesoscopic systems [8]. However, unlike UCF features, in the present experiments the position of the resonances was independent of the polarity of the electric field. The amplitude of the features observed in the present experiments appears to scale with the magnitude of the interaction resistance term, being the largest for the 30-nm-wide wires. Since this sample has the greatest number and length of parallel wires, it is expected to have the largest ensemble averaging of the three samples, and thus the smallest UCF structure. This is contrary to the present data which further suggests that the origin of the features is other than UCF.

To explain the structure observed in $\Delta R(E)/R_0$, we consider the acoustic spectrum in the small wires. The acoustic phonon spectrum is expected to be modified when the dominant phonon wavelength becomes comparable to the width and thickness of the wires. The temperature at which this occurs can be estimated by equating the dominant phonon wavelength, $\lambda_{ph} \approx V_s h/2k_B T$, to the largest cross-sectional dimension of the wires, which for all the present samples is W. At 1 K, using the estimated value of the longitudinal velocity of sound, $V_1 = 4.1 \times 10^3$ ms⁻¹ obtained from a weighted average of Au and Pd values [9], W is less than $\lambda_{ph} \approx 98$ nm for all three samples. Therefore, at the lowest temperature of the present experiments, the bulk longitudinal phonon spectrum is anticipated to be modified into a series of acoustic subbands, each corresponding to a lateral acoustic waveguide mode of the wire. It is worth mentioning that the acoustic subbands will be well defined up to energies at which the acoustic mean free path due to anharmonic processes becomes comparable with W and d. Thus the phonon-wavelength argument for the temperature at which this phenomenon should occur only provides an estimate of the lowest temperature at which waveguide modes will be resolved. The presence of the subbands is expected to affect the relaxation of the electrons heated by the electric field. As the electric field across the wires is increased, the electron system will gain energy until at sufficiently high field the electrons will have enough energy to relax by emitting phonons in a higher acoustic subband. Since the resistance change in the experiments was measured using an ac technique, we interpret the peaks as a differential increase in the electron-phonon scattering rate caused by the increased density of phonon states when additional acoustic subbands are accessed. The dispersion for uncoupled acoustic modes in a rectangular wire has the form [10]

$$\omega^2 = V_s^2 \left[q^2 + \left(\frac{n\pi}{W} \right)^2 + \left(\frac{m\pi}{d} \right)^2 \right], \qquad (1)$$

where V_s is the velocity of sound, q is the wave vector along the wire, and the integers n and m are the subband indices. Thus, the energy spacing of the subbands scales inversely with W and d. To estimate the energy of the resonances observed in $\Delta R(E)/R_0$, dT_e/dE was calculated for each sample from the equilibrium resistance correction and the slope of the nonequilibrium data. By this means a linear relationship between T_e and E was found with $dT_e/dE = 0.59$, 0.65, and 0.60 KV⁻¹ cm for the 30-, 50-, and 90-nm-wide wires, respectively. Using the values of dT_e/dE , $\Delta R(E)/R_0$ was plotted as a function of $T_e \times W$ and, as seen in Fig. 3, the resonances from all three samples were found to scale inversely with W. No scaling with d was found. This result shows that subbands associated with the width of the wire are responsible for the resonances observed in $\Delta R(E)/R_0$.



FIG. 3. The change in resistance vs the electron temperature T_e multiplied by the width of the wire, for the wires of width 30 (\bullet), 50 (\bullet), and 90 (\blacktriangle) nm.

When T_e is greater than the substrate temperature, the relationship between E and T_e can be written as $eE(D\tau)^{1/2} = k_B T_e$, where τ is the energy relaxation time of the electron system. The linear relationship found between E and T_e implies that τ is temperature independent, which is in contrast to previous results [11]. The difference between the present and earlier experiments is the strength of the acoustic coupling between the sample and the substrate. For Au-Pd on silicon the coupling is weak and we have previously shown that it creates a bottleneck for energy relaxation from the electrons which strengthens the dependence between E and T_e [3,12]. Therefore, the present experiments indicate that τ is dominated by the phonon escape time from the wires, which is temperature independent.

Acoustic coupling between the wire and the substrate will broaden the acoustic subbands. The broadening of the modes associated with W is expected to be less than the broadening of modes associated with d because the width of the wire is bounded by two free surfaces whereas the thickness is bounded by one free surface, the other being in direct contact with the substrate. From Eq. (1), the subband energy separation due to the wire thickness is $\delta \epsilon_d = V_s \pi \hbar/d$. Therefore, the phonon escape time from the wire can be estimated to be $\tau_{\rm esc} \leq 2\hbar/\delta \epsilon_d$. Assuming $V_s = V_1$, we calculate $\tau_{\rm esc} \leq 2d/V_1 \pi \approx 3 \times 10^{-12}$ s. It is interesting that the argument leading to the present formulation of $\tau_{\rm esc}$ is different from the usual ballistic escape argument used by many other authors [13], but the functional form of $\tau_{\rm esc}$ is the same in both cases.

For the three samples the half-widths of the resonances have been estimated from the data to be $\Delta \epsilon_W/\epsilon_W = 0.21$, 0.18, and 0.13 for the 30-, 50-, and 90-nm-wide wires, respectively. Several mechanisms are expected to contribute to $\Delta \epsilon_W/\epsilon_W$, for example, temperature and the width variation in the wire. From Eq. (1) the contribution from the width variation is simply $\Delta \epsilon_W/\epsilon_W = \Delta W/W$, where ΔW is the width variation. Electron micrographs of the three sets of wires show that $\Delta W = 8$, 8, and 10 nm, which gives $\Delta W/W$ to be 0.26, 0.16, and 0.11 for the 30-, 50-, and 90-nm-wide wires, respectively. Thus, the values of $\Delta W/W$ are consistent with the measured half-widths $\Delta \epsilon_W/\epsilon_W$ indicating that the width variation of the wires provides the dominant mechanism broadening the acoustic subbands.

The energy (T_e) of the resonances normalized with width are shown in Fig. 4 as a function of the subband index *n*. It is expected that the curve in Fig. 4 should pass through the origin because n=0 is also a solution to the wave equation describing acoustic modes in the wires [10]. A least-squares fit to the data gives a slope of 98 ± 5 nmK, from which $V_s = (4.2 \pm 0.2) \times 10^3$ ms⁻¹. This value is in excellent agreement with the estimated longitudinal velocity of sound in bulk Au-Pd [9].

In the 30-nm-wide sample, an additional peak in $\Delta R(E)/R_0$ was always observed below the n=1 resonance, indicated as peak T in Fig. 2. This feature is



FIG. 4. The position of the resonances plotted vs subband index.

thought to result from the coupling of electrons to transverse acoustic modes of the wire. From this resonance the transverse velocity of sound in Au-Pd is calculated to be $V_t = 1.9 \times 10^3$ ms⁻¹, which is very close to the predicted value of 1.8×10^3 ms⁻¹ for Au-Pd [9]. At present we do not understand why similar transverse modes are not clearly seen in the data from the wider wires. One possibility is that the broadening of the subbands is greater for transverse modes, as suggested in Fig. 2, peaks T and 1, for the 30-nm-wide wire. Since the transverse subbands are more closely spaced, the larger broadening most probably causes them to be unresolved in the wider wires. When coupling between the transverse and longitudinal phonons is taken into account, the dispersion near the zone center has regions where $|d\omega/dq|$ is reduced, in some instances to zero [10]. We suggest the consequent increase in the density of states may be responsible for the nonperiodic peaks.

In summary, periodic features in the electron heating characteristics of narrow wires have been shown to scale with the wire width. We conclude that the resonances originate from a change in the electron relaxation that occurs when they gain enough energy to emit phonons in a higher acoustic subband. From the subband spacing an estimate has been made of the longitudinal and transverse velocity of sound in the wires, and in both instances it has been shown to be close to the bulk value.

This work was supported by the National Science Foundation under Grant No. DMR-90-19525, and a grant from NATO.

- G. Bergmann, Wei Wei, Yao Zou, and R. M. Mueller, Phys. Rev. B 41, 7386 (1990).
- [2] Y. K. Kwong, K. Lin, M. S. Isaacson, and J. M. Parpia, J. Low Temp. Phys. (to be published).
- [3] J. C. Nabity and M. N. Wybourne, Phys. Rev. B 44, 8990 (1991).
- [4] M. V. Moody, J. L. Peterson, and R. L. Ciali, Rev. Sci. Instrum. 50, 903 (1979).
- [5] J. J. Lin and N. Giordano, Phys. Rev. B 35, 545 (1987).
- [6] B. L. Al'tshuler, D. Khmel'nitzkii, A. I. Larkin, and P. A. Lee, Phys. Rev. B 22, 5142 (1980).
- [7] J. Liu, T. L. Meisenheimer, and N. Giordano, Phys. Rev. B 40, 7527 (1989).
- [8] R. A. Webb, S. Washburn, and C. P. Umbach, Phys. Rev. B 37, 8455 (1988).
- [9] W. C. McGinnis and P. M. Chaikin, Phys. Rev. B 32, 6319 (1985).
- [10] B. A. Auld, Acoustic Fields and Waves (Wiley, New York, 1973), Vol. 2, Chap. 10.
- [11] M. L. Roukes, M. R. Freeman, R. S. Germain, and R. C. Richardson, Phys. Rev. Lett. 55, 422 (1985).
- [12] J. C. Nabity and M. N. Wybourne, J. Phys. Condens. Matter 2, 3125 (1990).
- [13] For example, W. E. Bron and W. Grill, Phys. Rev. B 16, 5303 (1977).