Effects of Dephasing and Dissipation on Quantum Noise in Conductors

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We consider the nonequilibrium quantum noise of electrical current in one-dimensional conductors, and clarify the mechanism that distinguishes macroscopic conductors from mesoscopic conductors. The noise does *not* become of the macroscopic type even when a strong dephasing is introduced. Macroscopic noise is obtained only when the energy of the total electron system is maximally dissipated when each electron traverses the conductor. Energy transfer from the electron system to other systems is therefore essential for noise suppression and for the noise to be macroscopic.

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Current fluctuations at *thermal equilibrium* are related to the linear conductivity by the fluctuation-dissipation theorem, and this equilibrium noise becomes negligible at low temperatures. In the presence of transport, however, a *nonequilibrium* noise appears. This noise increases with current and does *not* vanish even at zero temperature [1]. The nonequilibrium noise has in general no simple relationship to the linear transport coefficients, and a noise formula has been obtained only for systems with perfect quantum coherence [1]. To compare the noise level among conductors of different conductivities, we characterize the noise level by a "noise figure" W defined as the ratio of the actual noise level to the shot-noise level [see Eq. (3)]. In terms of W, the nonequilibrium noise for a one-dimensional mesoscopic conductor at zero temperature [1] is characterized by

$$W=1-T, \tag{1}$$

where T is the transmittance determined by potential scatterers in the conductor. This formula assumes that the conductor is perfectly coherent and that the reservoirs connected to it are ideal [see Eq. (21)]. If the length L of the conductor is increased, T eventually goes to zero and W approaches unity, i.e., Eq. (1) predicts that the nonequilibrium noise would approach the full shot noise. It is well known, however, that $W\ll 1$ for *macroscopic* conductors [2]. In this Letter, we resolve this apparent contradiction by analyzing how dephasing and dissipation affect the nonequilibrium noise, and we clarify the mechanism that distinguishes macroscopic conductors from mesoscopic conductors [3].

Macroscopic regime.—Suppose that we have a (statistically) uniform conductor of infinite length, and that by cutting the conductor we obtain conductors of various lengths L. There must exist some characteristic length $L_{\rm cr}$ that distinguishes mesoscopic conductors ($L\ll L_{\rm cr}$) from macroscopic conductors ($L\gg L_{\rm cr}$). First, let us derive the L dependence of W in the macroscopic regime. Consider two conductors of length L_1 and L_2 ($L_1,L_2\gg L_{\rm cr}$). By putting them together, we obtain a conductor of length L_1+L_2. Since both the voltage fluctuations and the resistance are additive for macroscopic conductors, we immediately find

$$W(L_1+L_2)=\frac{L_1^2W(L_1)+L_2^2W(L_2)}{(L_1+L_2)^2}\quad (L_1,L_2\gg L_{\rm cr}). \tag{2}$$

Hence, if L is doubled, W is reduced by a factor of 2. The same conclusion was obtained previously [4]. We thus find that in the macroscopic regime W *decreases* monotonically with increasing L, which is consistent with the fact that $W\ll 1$ for macroscopic conductors. However, as we will see later [Eq. (18)], Eq. (2) does *not* hold in most regions of interest, where W is not very small and $L/L_{\rm cr}$ is not so large. Moreover, as mentioned above, Eq. (1) predicts the contradictory behavior of $W\to 1$ as $L\to\infty$. To resolve these problems, we shall derive a formula for W in the intermediate region where $L\sim L_{\rm cr}$. We shall also clarify the mechanism by which W is suppressed, and thereby find the characteristic length $L_{\rm cr}$. To simplify the discussion, we will restrict ourselves to a one-dimensional two-terminal conductor at zero temperature.

Model.—Equation (1) was derived by neglecting any interactions of electrons with other electrons, or with phonons, photons, or magnetic impurities, etc. When these interactions are taken into account, the lifetime $\tau_{\rm life}$ of a one-body electron state becomes finite. For mesoscopic conductors, however, a more important time scale is the phase relaxation time τ_ϕ, which is usually longer than $\tau_{\rm life}$ [5]. We will show later that there is another important time scale $\tau_{\rm rlx}$, which is the time spent by an electron of energy $\Delta\mu$ above the Fermi energy before it relaxes onto the Fermi surface [see Eqs. (16) and (17)]. To demonstrate that the dephasing process plays a role completely different from this energy-relaxation process, we employ a model in which τ_ϕ can be shortened while keeping $\tau_{\rm rlx}$ long. That is, we consider conductors with magnetic impurities and electron-phonon interactions. As the concentration of the magnetic impurities is increased, the spin of an electron will be flipped by the impurities. This disturbs the electron interference, so that τ_ϕ is shortened

while the energy of the electron is preserved. On the other hand, the electron-phonon interaction induces phonon-emission processes and thereby determines $\tau_{\rm rlx}$.

Following the standard technique [6], we suppose that perfect leads 1 and 2 of equal length L_τ are connected to the conductor of length L. We decompose the Hamiltonian as $\hat{H}=\hat{H}_e+\hat{H}_p+\hat{V}_{ep}+\hat{H}_m+\hat{V}_{em}$, where $\hat{H}_e$ and $\hat{H}_p$ are the Hamiltonians of *renormalized* electrons (polarons) and renormalized phonons, respectively, and $\hat{V}_{ep}$ is the electron-phonon interaction between these renormalized particles. $\hat{H}_m$ denotes the Hamiltonian of the magnetic impurities, and $\hat{V}_{em}$ the sum of interactions between electrons and the magnetic impurities, each of which is assumed to be weak and ferromagnetic [7] so that the pure-dephasing condition is ideal [8]. That is, to increase $\hat{V}_{em}$, we have to increase the concentration of the impurities rather than individual coupling strengths [7]. For example, $\hat{H}_e$ is the Hamiltonian of noninteracting polarons under an elastic-scattering potential $\hat{V}_s$, which determines the renormalized transmittance T of the polaron. Hence, if both $\hat{V}_{ep}$ and $\hat{V}_{em}$ are absent, W would be given by Eq. (1) with this renormalized T. Our task is to evaluate W for nonzero $\hat{V}_{ep}$ and/or $\hat{V}_{em}$. Since leads 1 and 2 are hypothetically perfect, we assume that $\hat{V}_s=\hat{V}_{ep}=\hat{V}_{em}=0$ in these leads. In the following we will loosely call a polaron an electron.

We consider a steady state, and assume that the chemical-potential difference is much smaller than the Fermi energy. Then W can be conveniently evaluated from

$$W=\frac{\langle\delta N_\alpha^2\rangle}{\langle N_\alpha\rangle},\quad \hat{N}_\alpha\equiv\frac{1}{v_F}\int_{\text{lead }\alpha}\hat{J}_\alpha(z)\,dz\,, \tag{3}$$

where v_F denotes the Fermi velocity, $\hat{J}_\alpha$ the current operator in lead α ($=1,2$), and z the coordinate along the lead. $\hat{N}_\alpha$ gives the number of electrons that carry the *net* current during the period $\tau\equiv L_\tau/v_F$. Its average and variance are proportional to those of the current. Let $\hat{a}_{\alpha\sigma}^{(0)\dagger}(\epsilon)$ be the creation operator, in the absence of $\hat{V}\equiv\hat{V}_{ep}+\hat{V}_{em}+\hat{V}_s$, of an electron with energy ϵ, spin σ, and unit flux (in an appropriate unit) flowing from lead α to the opposite lead. When $\hat{V}$ is turned on, the electron will undergo energy relaxations due to $\hat{V}_{ep}$, spin flips due to $\hat{V}_{em}$, and elastic scattering due to $\hat{V}_s$. At zero temperature, the dominant effect of $\hat{V}_{ep}$ should be spontaneous phonon-emission processes, without the creation of additional electron-hole pairs. This means that $\hat{a}_{\alpha\sigma}^{(0)\dagger}(\epsilon)$ evolves into an operator $\hat{a}_{\alpha\sigma}^{\dagger}(\epsilon)$, which creates the incoming electron wave in lead α plus outgoing waves in both leads, simultaneously creating some phonons and inverting spins of some impurities. Since $\hat{V}$ is localized in the conductor, the incoming electron wave should be the same as that of $\hat{a}_{\alpha\sigma}^{(0)\dagger}(\epsilon)$ and should thus be of unit flux. Hence, the magnitude of the incoming flux is proportional to $\hat{a}_{\alpha\sigma}^{\dagger}\hat{a}_{\alpha\sigma}$. On the other hand, we assume that by superposing the operators $\hat{a}_{\alpha\sigma}^{\dagger}(\epsilon)$ an operator $\hat{b}_{\alpha\sigma}^{\dagger}(\epsilon)$ can be constructed, which creates an outgoing electron wave of energy ϵ, spin σ, and unit flux, plus some ingoing waves in both leads, while simultaneously modifying phonon and impurity-spin states. That is,

$$\hat{b}_{\alpha\sigma}(\epsilon)=\sum_{\epsilon',\sigma'}\sum_{\beta=1}^{2}\hat{s}_{\alpha\sigma\beta\sigma'}(\epsilon,\epsilon')\hat{a}_{\beta\sigma'}(\epsilon')\,, \tag{4}$$

where $\hat{s}_{\alpha\sigma\beta\sigma'}$ is a matrix that includes phonon and/or impurity-spin operators. When $\hat{V}_{ep}=\hat{V}_{em}=0$, the matrix reduces to $\delta_{\sigma,\sigma'}\delta_{\epsilon,\epsilon'}$ (Kronecker's delta) times the usual one-body scattering matrix [6]. By expressing the incoming and outgoing electron fluxes using $\hat{a}_{\alpha\sigma}$ and $\hat{b}_{\alpha\sigma}$, respectively, we obtain

$$\hat{N}_\alpha=\sum_{\epsilon,\sigma}[\hat{a}_{\alpha\sigma}^{\dagger}(\epsilon)\hat{a}_{\alpha\sigma}(\epsilon)-\hat{b}_{\alpha\sigma}^{\dagger}(\epsilon)\hat{b}_{\alpha\sigma}(\epsilon)]\,. \tag{5}$$

Since electrons are constantly supplied from ideal reservoirs, the incoming electrons obey the Fermi-Dirac (FD) distribution: $\langle\hat{a}_{\alpha\sigma}^{\dagger}(\epsilon)\hat{a}_{\beta\sigma'}(\epsilon')\rangle=\delta_{\alpha,\beta}\delta_{\sigma,\sigma'}\delta_{\epsilon,\epsilon'}f_\alpha(\epsilon)$, where f_α is the FD function. Here, the average of $\hat{N}_\alpha$ is given by

$$\langle N_\alpha\rangle=2\sum_{\epsilon}[f_\alpha(\epsilon)-\tilde{f}_\alpha(\epsilon)]\,, \tag{6}$$

where $\tilde{f}_\alpha(\epsilon)\equiv\langle\hat{b}_{\alpha\sigma}^{\dagger}(\epsilon)\hat{b}_{\alpha\sigma}(\epsilon)\rangle$ is the distribution function of outgoing electrons. By approximating the expectation value of the product of a function of phonon (and/or impurity-spin) operators and a function of electron operators by the product of the expectation values of each function, we find

$$\tilde{f}_\alpha(\epsilon)\simeq\sum_{\epsilon',\sigma'}\sum_{\beta}\langle\hat{s}_{\alpha\sigma\beta\sigma'}^{\dagger}(\epsilon,\epsilon')\hat{s}_{\alpha\sigma\beta\sigma'}(\epsilon,\epsilon')\rangle f_\beta(\epsilon')\,. \tag{7}$$

To calculate the variance of $\hat{N}_\alpha$, we also use the Hartree-Fock approximation, and find, at zero temperature, that

$$\langle\delta N_\alpha^2\rangle\simeq2\sum_{\epsilon,\epsilon'}\{[\delta_{\epsilon,\epsilon'}-\langle\hat{b}_{\alpha\sigma}^{\dagger}(\epsilon')\hat{b}_{\alpha\sigma}(\epsilon)\rangle]\langle\hat{b}_{\alpha\sigma}^{\dagger}(\epsilon)\hat{b}_{\alpha\sigma}(\epsilon')\rangle - |\langle\hat{b}_{\alpha\sigma}^{\dagger}(\epsilon)\hat{b}_{\alpha-\sigma}(\epsilon')\rangle|^2\}\,. \tag{8}$$

We note that the last term is zero because, by symmetry, the outgoing electrons are not spin polarized. We also note that correlations between different modes are in general small: $\langle\hat{b}_{\alpha\sigma}^{\dagger}(\epsilon)\hat{b}_{\alpha\sigma}(\epsilon')\rangle\simeq0$ for $\epsilon\neq\epsilon'$ (which becomes exact in both the coherent and damped limits). Thus, we finally obtain the very simple formula

$$\langle\delta N_\alpha^2\rangle\simeq2\sum_{\epsilon}[1-\tilde{f}_\alpha(\epsilon)]\tilde{f}_\alpha(\epsilon)\,. \tag{9}$$

Before proceeding further, let us confirm that in the coherent regime ($L\gg L_\phi,L_{\rm rlx}$) our general results, Eqs. (6), (7), and (9), reproduce the previous results. When $\hat{V}_{ep}=\hat{V}_{em}=0$, Eq. (7) yields, for $\alpha=1$,

$$\tilde{f}_1(\epsilon)=\tilde{f}_1^0(\epsilon)\equiv\begin{cases}0 & (\mu_1<\epsilon)\,,\\ 1-T & (\mu_2<\epsilon\le\mu_1)\,,\\ 1 & (\epsilon\le\mu_2)\,,\end{cases} \tag{10}$$

where μ_α ($\alpha=1,2$) denotes the chemical potential of reservoir α. If we neglect a weak ϵ dependence of T for $\mu_2<\epsilon\le\mu_1$, then Eqs. (6), (9), and (10) indeed reproduce the results of Ref. [1]:

$$\langle N_\alpha\rangle=CT\Delta\mu,\quad\langle\delta N_\alpha^2\rangle=CT(1-T)\Delta\mu\,. \tag{11}$$

Here $C\equiv g\tau/2\pi\hbar$, where $g=2$ is the spin degeneracy, and

$\Delta\mu \equiv \mu_1 - \mu_2 \geq 0$. Substituting this result into Eq. (3) yields Eq. (1).

Dephasing.—Most quantum effects characteristic of mesoscopic conductors disappear when the transit time τ_{tr} of an electron through the conductor exceeds τ_ϕ [5,6]. It is therefore worthwhile to investigate whether W is suppressed when $\tau_\phi < \tau_{tr} \ll \tau_{rlx}$. This is equivalent, in length scale, to $L_\phi < L \ll L_{cr}$, since we will later show that $L_{rlx} = L_{cr}$. Our model yields the last inequality ($L \ll L_{cr}$) when $\hat{V}_{ep} = 0$, which leads to $\hat{s}_{\alpha\sigma\beta\sigma'} \propto \delta_{\epsilon,\epsilon'}$. In this case, we find that $\tilde{f}_1$ takes the same functional form as Eq. (10), but we should replace T with a generalized transmittance. That is, since the electron spin is not conserved, T can no longer be defined as the square of the usual one-body scattering amplitude. Instead, we define T simply as the probability that an electron passes through the conductor, disregarding whether or not its spin is conserved. With this T, $\tilde{f}_1$ is given by Eq. (10), and $\langle N_\alpha \rangle$ and $\langle \delta N_\alpha^2 \rangle$ are therefore given by Eq. (11). Hence,

$$W \simeq 1 - T \quad (\text{for any } L \ll L_{cr}) . \tag{12}$$

That is, although Eq. (1) was obtained for $L \ll L_\phi, L_{cr}$ the same *form* approximately holds even when $L > L_\phi$ if the generalized transmittance is used for T. (The *value* of this T is of course different from that for $\hat{V}_{em} = 0$.) As L is increased, this T eventually goes to zero. Hence, we see that the dephasing without energy relaxation does *not* lead to $W \ll 1$ as $L \to \infty$ [9]. This is in sharp contrast to other fluctuations in mesoscopic systems, such as the universal conductance fluctuations, most of which are suppressed by the dephasing [5,6].

Dissipation.—We now consider the energy dissipations of electrons by taking $\hat{V}_{ep} \neq 0$ and $\hat{V}_{em} = 0$. Let us first consider the damped limit where $\hat{V}_{ep}$ is very large. In this limiting case, all outgoing electrons are relaxed to states of minimum allowable energy, so

$$\tilde{f}_\alpha(\epsilon) = \tilde{f}_\alpha^\infty(\epsilon) \equiv \begin{cases} 0 & (\tilde{\mu}_\alpha < \epsilon), \\ 1 & (\epsilon \leq \tilde{\mu}_\alpha), \end{cases} \tag{13}$$

where $\tilde{\mu}_\alpha$ ($\alpha = 1,2$) denotes the Fermi energy of outgoing electrons in lead α, whose energy can be defined in the damped limit. Substituting this form into Eqs. (6), (9), and (3), we get

$$\langle N_\alpha \rangle = C(\mu_1 - \tilde{\mu}_1), \quad \langle \delta N_\alpha^2 \rangle = 0, \quad W = 0 \quad (L \gg L_{cr}) . \tag{14}$$

Note that in this case the Landauer formula [6] for the average current is no longer valid. We see that the nonequilibrium noise is completely suppressed in this damped limit. That is, the energy transfer from the electron system to the phonon system (or photon system [8]) is essential for noise suppression.

To interpolate between the two limiting (and apparently contradicting) formulas, Eq. (14) and Eq. (1) or (12), we calculated W in the intermediate region by taking the following model for the electron-phonon interaction [8]: $\hat{V}_{ep} = \sum_{k,q} \hbar g_{kq} \hat{a}_k^\dagger \hat{a}_k (\hat{c}_q + \hat{c}_q^\dagger)$, where $\hat{c}_q$ is the annihilation operator of phonons, and g_{kq} is the coupling constant of the electron-phonon interaction. The result is

$$W = (1 - \kappa)(1 - T) , \tag{15}$$

$$\kappa \equiv \frac{1}{\Delta\mu} \int EP(E)dE , \tag{16}$$

where $P(E)$ denotes the phonon excitation spectra of the coupled electron-phonon system, and κ represents the average energy transfer, per traversal of one electron, from the electron system to the phonon system [8]. That is, $0 \leq \kappa \leq 1$, and κ increases with increasing $\hat{V}_{ep}$ or L. Note that the mean free path l for momentum relaxation processes of an electron is smaller than L_{rlx}. Hence, when $L \gg l$, an electron would undergo diffusive motions, going back and forth in the conductor until it escapes into a lead. The τ_{tr} is therefore proportional to L^2 for a given $\hat{V}_{ep}$, and we can write

$$\kappa \simeq 1 - \exp[-\tau_{tr}/\tau_{rlx}] \simeq 1 - \exp[-(L/L_{rlx})^2] \quad (l \ll L \sim L_{rlx}) . \tag{17}$$

We see that $W \ll 1$ when $L \gg L_{rlx}$, the characteristic length at which an electron of excess energy $\Delta\mu$ relaxes onto the Fermi surface by emitting phonons. Hence, by definition, $L_{cr} \sim L_{rlx}$. Since L_{cr} can be taken arbitrarily up to a factor of order unity, we simply take $L_{cr} = L_{rlx}$. We thus obtain

$$W \simeq e^{-(L/L_{cr})^2}(1 - T) \quad (l \ll L \sim L_{cr}) . \tag{18}$$

Equations (1), (2), (12), (14), and (15) or (18) constitute a set of formulas that gives W in most regions of interest.

Although Eqs. (15) and (18) have been derived for the specific form of $\hat{V}_{ep}$, we can supplement them with the following general argument. Suppose that we have many conductors of different magnitudes of $\hat{V}_s$ and $\hat{V}_{ep}$, and we apply the same chemical-potential difference $\Delta\mu$. We choose a set of conductors that exhibit the same magnitude of $\langle N_\alpha \rangle$. By comparing two conductors in this set, one with no damping and the other with strong damping, we find from Eqs. (11) and (14) that

$$\tilde{\mu}_1 = \mu_1 - T\Delta\mu , \tag{19}$$

where $\tilde{\mu}_1$ and T are for the conductors with strong damping and no damping, respectively. The two conductors have the limiting forms of $\tilde{f}_1$, Eqs. (10) and (13), respectively. For other conductors in our set, $\tilde{f}_1$ must be an interpolated function of these limiting forms. We note that the definition of κ, Eq. (16), is general, and its L dependence, Eq. (17), is also general. Thus, we can generally use κ as the parameter for the interpolation: $\tilde{f}_1$ should gradually change from $\tilde{f}_1^0$ to $\tilde{f}_1^\infty$ as κ is increased from 0 to 1. We can then see from Eqs. (6), (9), and (19) that in general W becomes a decreasing function of κ, and our conclusions based on Eqs. (15) and (18) are qualitatively correct. In particular, when $\tilde{f}_\alpha$ can be approximated by

$$\tilde{f}_\alpha(\epsilon) \simeq (1 - \sqrt{\kappa})\tilde{f}_\alpha^0(\epsilon) + \sqrt{\kappa}\tilde{f}_\alpha^\infty(\epsilon) , \tag{20}$$

then Eqs. (15) and (18) can be reproduced.

Nonideal reservoirs and remarks.—We have assumed so far that the reservoirs are ideal. In real samples under a finite $\Delta\mu$, however, the reservoirs (particularly their boundaries between the conductor) would be excited by a finite current. If the energies of excited electrons are smaller than $\Delta\mu$, we find—assuming perfect coherence in the conductor—that [8]

$$W=\eta(\Delta\mu)T+(1-T)\,, \tag{21}$$

where $0\leq\eta(\Delta\mu)\leq 1$ measures the degree of the excitation, the detailed form of which is irrelevant to the present discussion. On the right-hand side, the first term is the scaled emission noise [10] that appears because the reservoirs are nonideal, whereas the second term is the granularity noise [10] caused by the random transmission processes through the conductor. This formula explains why the W observed for a quantum point contact [11] does not vanish when $T=1$, where Eq. (1) predicts $W=0$. However, it predicts, as Eq. (1) does, that $W=1$ as $L\rightarrow\infty$. Hence, imperfectness of reservoirs does *not* lead to $W\ll 1$.

We finally make two remarks. The first concerns electron-electron (e-e) interactions in the conductor. Equation (9) indicates that the e-e interactions do *not* lead to $W\ll 1$ either, because they cannot lead to the zero-noise distribution $\tilde{f}_1^{\infty}$ since the energy of the *total* electron system is conserved. That is, the energy transfer from the *total* electron system to other systems (such as phonons and photons) is essential for the noise suppression. However, we do not know whether Eq. (9) is valid in the presence of e-e interactions, and future study will therefore be needed to reach a definite conclusion on the e-e interactions. Second, throughout this Letter we have assumed one-dimensional conductors. Our formulas may therefore be experimentally confirmed by measuring the noise of currents in high-quality quantum wires. For higher-dimensional or multimode quasi-one-dimensional conductors, our formulas may be modified. This is also a subject for future study.

In summary, we have analyzed the nonequilibrium quantum noise of the current in one-dimensional conductors at zero temperature. To compare the noise levels among conductors of different conductivities, we introduced the noise figure W [Eq. (3)]. In the coherent limit, W is given by Eq. (1), which approaches *unity* as the length L of the conductor is increased. In the macroscopic regime, on the other hand, W varies according to Eq. (2), approaching *zero* as $L\rightarrow\infty$. To resolve this apparent contradiction, we considered the effects of dephasing and dissipation processes in the conductor, and of imperfect reservoirs connected to it. We showed that *neither* a strong dephasing in the conductor [Eq. (12)] *nor* imperfectness of the reservoirs [Eq. (21)] can lead to the noise of a macroscopic type, $W\ll 1$. It is found that $W\ll 1$ is obtained only when the energy of the total electron system is maximally dissipated when each electron traverses the conductor [Eq. (15)]. That is, energy transfer from the electron system to other systems, such as a phonon system or photon system, is essential for noise suppression and for the noise to be macroscopic.

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[9] We can also confirm this last statement by the following model calculation, which does not rely on the mean-field approximations used to derive Eq. (12). Suppose that a pair of electrons of spin up and spin down, and of the same energy, enter a conductor from lead 1. The final states include the following possibilities: Both electrons are in lead 2 (with probability T_2); both are in lead 1 (R_2); one electron of spin up is in lead 1 and the other of spin down in lead 2 (T_1); one electron of spin down is in lead 1 and the other of spin up in lead 2 (T_1); an up-spin electron is in each lead (T_1'); and a down-spin electron is in each lead (T_1'). Hence, the average $\langle N_\alpha\rangle=2T$, where $T\equiv T_2+T_1+T_1'$, and the variance $\langle\delta N_\alpha^2\rangle=2(T+T_2-2T^2)$. Since we are assuming a large number of weak ferromagnetic impurities, the correlations they induce among electrons of different energies are negligible. Thus, by summing over states in the energy range of $\Delta\mu$, we finally find $W=1-2T+T_2/T$. As $T\rightarrow 0$, we see that $W\rightarrow 1+T_2/(T_2+T_1+T_1')\geq 1$. That is, we can indeed confirm that W cannot be suppressed—although the form of W in this model would differ from Eq. (12) when the usual relationship $T_2=T^2$ is violated.

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