New Origin of a Convective Motion: Elastically Induced Convection in Granular Materials

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Convection and fluidization in a vibrated bed of powder are reproduced in a numerical simulation. In the simulation, each particle of the powder, during a collision, has a viscoelastic interaction with the other colliding particle. Because of the discreteness of the particles, this elasticity causes convection. The critical values of fluidization and convection agree with experiments.

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Recently, many physicists [1,2] have studied the dynamics of granular materials, because these materials behave differently from continuous media like rigid, elastic, or viscous bodies. This difference is because granular materials behave in two distinct ways: as a set of particles and as continuous media. For example, granular materials under gravity can have a slope with nonzero angle (angle of repose) in the quasistationary state. Avalanches are also characteristic features in the dynamics of granular materials.

One of the stranger phenomena is convection in a fluidized bed under vertical vibration [3–7]. A schematic figure of a typical experimental setup is shown in Fig. 1. When a vessel containing a large number of small particles is shaken strongly, heaping of the surface starts spontaneously. At the same time, convection of particles starts. Experimentalists claim that these phenomena are dynamical phase transitions.

Moreover, convection is localized near the surface (surface fluidization). The depth of the convection region increases as the strength of the vibration increases. The purpose of this paper is to propose a numerical modeling to reproduce the convective motion and surface fluidization effect.

Our model is a very simple one [1,8]. However, no one has pointed out that this model can reveal an instability in the fluidized bed [9]. In this model, each particle is regarded as a sphere with definite diameter d. When particles collide with one another, they penetrate into each other. During the collision, there is a viscoelastic interaction between them. The equation the particles obey is

$$\ddot{\mathbf{x}}_i = -\sum_{j=1}^{N} \theta(d - |\mathbf{x}_i - \mathbf{x}_j|) \left[k \left(\mathbf{x}_i - \mathbf{x}_j - d\frac{\mathbf{x}_i - \mathbf{x}_j}{|\mathbf{x}_i - \mathbf{x}_j|} \right) + \eta(\mathbf{v}_i - \mathbf{v}_j) \right] - \mathbf{g},$$

where $\theta(x)$ is a step function, N is the total number of spheres, $\mathbf{x}_i$ is the position vector of the ith sphere, and k and η are the elastic constant and the viscosity coefficient, respectively. $\mathbf{g}$ is the acceleration of gravity and $\mathbf{v}$ is velocity. The step function restricts the interaction between particles to periods when the distance between two particles is less than d.

In order to understand the physical meanings of the two interaction parameters k and η, we consider the case where two spheres collide head-on with each other. In this case, they have an effective coefficient of restitution $e = \exp(-\eta\pi/\omega)$. The time interval during a collision is also a function of k and η. We call this interval the collision time, $t_{\rm col} = \pi/\omega$ $[\omega = (2k - \eta^2)^{1/2}]$. Therefore, fixing k and η determines the effective coefficient of restitution and the collision time.

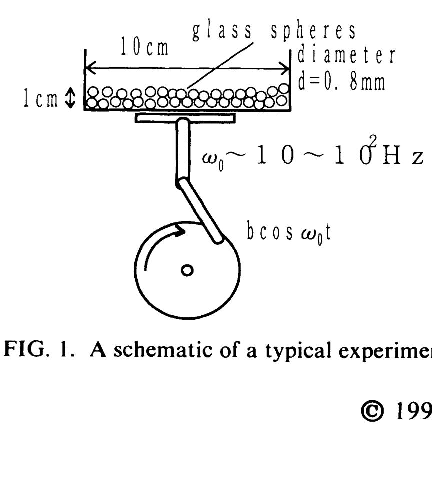


FIG. 1. A schematic of a typical experiment.

In our simulation, we neglect the effect of rotation for simplicity. We employ Euler's scheme to integrate the equation. The time step Δt is modified at each step such that Δx_i, the displacement of the ith particle during Δt, does not exceed some value α.

A hundred such particles are put into a vessel having a horizontal width of 26 and being semi-infinite in the vertical direction.

We introduce a larger friction ($\eta = 20$) between spheres and side walls than that between spheres. This is because the direction of convection is opposite to that observed in experiments when there is no friction between side walls and particles [10]. Particles also have an elastic interaction with both side and bottom walls. The strength of the elasticity is the same as that between par-

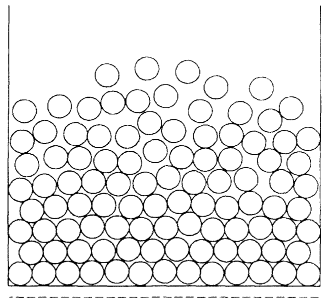


FIG. 2. A snapshot of simulations. $\Gamma = 1.44$. Solid lines show the vessel. The dashed line indicates the location of the bottom of the vessel at $\omega_0 t = \pi/2$. The vessel is going down.

ticles.

Such a vessel containing powder particles is shaken vertically in the simulation; as a function of time, the position of the bottom of the vessel is $b\cos\omega_0 t$. Experimentalists report [4] that the control parameter which determines the critical point (the beginning of the heaping) is the acceleration amplitude of vibration, $\Gamma \equiv b\omega_0^2$. (Although $\Gamma = b\omega_0^2/g$ in this experiment, we set $g = 1.0$.) We set $b = 1.1$ and change ω_0 in order to control Γ. Other parameters are fixed; $d = 2.0$, $e = 0.9$, $\alpha = 0.01$, and $t_{\rm col} = 0.1$. A snapshot of the simulation is shown in Fig. 2.

When Γ is increased to a large enough value, convective motion starts. In order to measure the strength of the flow of convection, we introduce the cell-to-cell flow $\mathcal{J}$. First the whole space is divided into cells of $d\times d$ squares. We introduce integer coordinates X and Y in order to label the cells. This means the cell (X,Y) spans the space $Xd < x < (X+1)d$ and $Yd < y < (Y+1)d$, where x is the horizontal coordinate and y is the vertical one.

The convection we are interested in should be measured by the coordinates fixed to the vessel. Therefore, we record the positions of particles at times $t = \frac{3}{4}T + nT$ (n is an integer), when the vessel is at the same position. Here T is the period of the vibration of the bottom, $2\pi/\omega_0$.

By counting the number of particles which go out of and come into a cell, we get the cell-to-cell flow averaged over time steps t,

$$\langle \mathcal{J}(X,Y)\rangle = \left\langle \sum_i |n_i(X,Y;t) - n_i(X,Y;t-T)|\,(X_i(t) - X_i(t-T), Y_i(t) - Y_i(t-T))\right\rangle_t ,$$

where X_i and Y_i are integers describing the cell containing the ith particle. If the ith particle is in the cell having coordinates X and Y, $n_i(X,Y,;t)$ equals 1, otherwise it is 0. In Fig. 3, the averaged cell-to-cell flow $\langle \mathcal{J}\rangle$ is displayed. Convection is clearly observed in these figures.

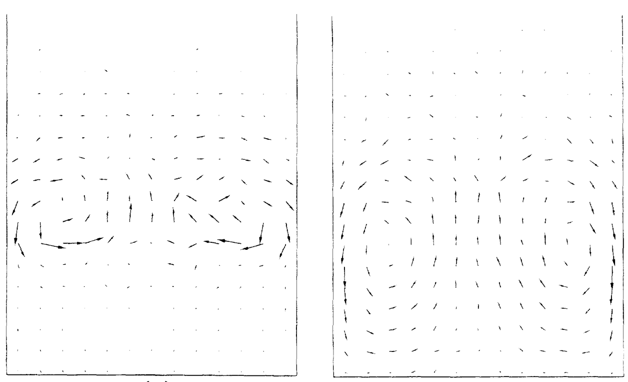
Next, in order to see whether or not a critical value Γ_c exists, we show $\mathcal{J} = [\sum_{X,Y}\mathcal{J}(X,Y)^2]^{1/2}$ as a function of Γ in Fig. 4. $\mathcal{J}$ is almost 0 when $\Gamma < 0.9$. On the other hand, $\mathcal{J}$ takes on nonzero values when $\Gamma > 1.2$. Therefore there should be a critical value Γ_c between 0.9 and 1.2.

Here we define Γ_c as a threshold value for the start of convective motion. On the other hand, in the experiment described in Ref. [4], Γ_c is defined as the threshold value for the onset of heaping. Although it is not clear if both definitions are identical, it is confirmed that heaping is maintained by convective motion [3–5]. Since the heaping will not occur without convective motion, our Γ_c is a lower bound of that obtained in the experiment. This consideration agrees with the fact that the experimental value of Γ_c [4], $\Gamma_c \simeq (12.5\pm 1)/9.8 \simeq 1.2$, is a little bit larger than 1.0.

In addition, the convective motion concentrates near the surface region in Fig. 3. This corresponds to the surface fluidization reported in the experiment by Evesque, Szmatula, and Denis [3]. In order to see the existence of the critical value observed in this experiment, we estimate the depth of the region where the convective motion exists. We define the vertical radius of the convection $\mathcal{R}$ as

$$\mathcal{R}^2 = \langle (Y - \langle Y\rangle_{\langle \mathcal{J}(Y)\rangle_t})^2\rangle_{\langle \mathcal{J}(Y)\rangle_t} ,$$

where



(a) (b)

FIG. 3. Cell-to-cell flow lines. Solid lines indicate the vessel. (a) $\Gamma = 1.44$. (b) $\Gamma = 2.71$.

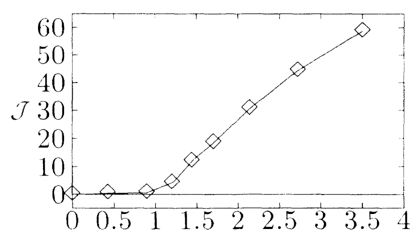


FIG. 4. Total flow $\mathcal{J}$ as a function of Γ.

$$\langle O\rangle_{\langle \mathcal{J}(Y)\rangle_l} \equiv \frac{\sum_{Y\geq \bar{Y}} \langle \mathcal{J}(Y)\rangle_l O}{\sum_{Y\geq \bar{Y}} \langle \mathcal{J}(Y)\rangle_l}$$

and

$$\langle \mathcal{J}(Y)\rangle_l \equiv \left[\sum_X \langle \mathcal{J}\rangle (X,Y)^2\right]^{1/2}.$$

$\bar{Y}$ is the Y coordinate of a box which intersects the surface in the static state. As shown in Fig. 5(a), in the fluidized phase with convection, the density of powder is smaller than that in the solid phase [7]. Therefore the total volume of powder becomes larger. This makes the position of the surface higher. We estimate this change of surface with $\mathcal{R}$ and use it as a depth of fluidization.

The dependence of $\mathcal{R}$ upon Γ is shown in Fig. 5(b). The existence of Γ_c is less clear than that for $\langle \mathcal{J}\rangle$, because $\mathcal{R}$ takes on nonzero values even when Γ is small. However, a drastic increase of $\mathcal{R}$ is observed just above $\Gamma = 1.0$. Therefore there is a critical value of Γ for surface fluidization too. Although changing e and t_{col} changes the absolute values of $\mathcal{R}$ and $\mathcal{J}$, Γ_c still remains at $\Gamma \simeq 1$.

In the following we will discuss the origin of this convective motion (see Fig. 6). For $\Gamma > g$, we can separate each oscillation cycle into intervals corresponding to either $-g \leq \Gamma \cos\omega_0 t$ or $-g \geq \Gamma \cos\omega_0 t$. The difference between the two conditions is whether particles can slide past each other or not. When $-g \leq \Gamma \cos\omega_0 t$, the (downward) acceleration of gravity is greater than the downward acceleration of the bottom. Therefore particles are pressed toward the bottom, are almost always in contact with each other, and cannot slide past each other.

On the other hand, when $-g \geq \Gamma \cos\omega_0 t$, the downward acceleration of the bottom is larger than the acceleration of gravity. Therefore, the bottom does not affect the movement of particles; the particles are in free fall and have enough space to slide past one another.

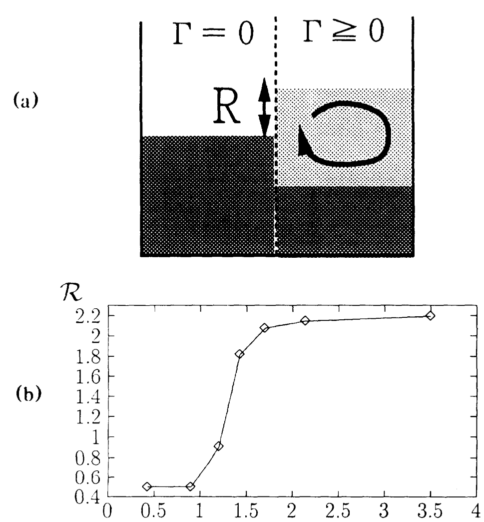


FIG. 5. (a) A schematic of the explanation of the meaning of $\mathcal{R}$. (b) Vertical radius $\mathcal{R}$ of convective motion as a function of Γ.

Now we consider what happens during these intervals. During the intervals when $-g \leq \Gamma \cos\omega_0 t$, the particles are tightly packed and behave like a continuous viscoelastic body. Then both vertical stress and horizontal stress are induced (Fig. 6, left).

In the intervals when $-g \geq \Gamma \cos\omega_0 t$, gravity cannot push the spheres against the bottom of the vessel. Then the spheres start to recover their relaxed state. First, vertical stress is quickly released because there is no restriction on vertical motion. However, because of the finite width of the vessel, horizontal stress remains unreleased.

The remaining horizontal stress causes a horizontal flow, because particles slide past each other as mentioned above. This motion is not induced when $-g \leq \Gamma \cos\omega_0 t$, because the particles have no space to move. This nonreversibility is the main reason that elasticity induces convection in granular materials.

Since the induced horizontal flow collides at the center of the vessel, it must become vertical. However, particles cannot flow downwards because of the existence of the bottom. The flow turns upwards, pushes up the surface, and makes a bump (Fig. 6, right). Although some vacancy appears at the bottom, the surrounding particles flow down and fill it. This whole process constitutes a convective motion.

Since the vibration destroys the bump on the surface, we cannot see steady existence of heaping in our simulation. But it may have some relationship with the heaping observed in the experiment.

Rajchenbach has proposed another explanation of heaping [6]. He explained the convection by the instability of the surface. This instability is caused by an effect like the Seebeck effect. In his theory, $\mathcal{J} \propto -\mathrm{grad}\, v(\mathbf{r})$. $v(\mathbf{r})$ is the mean quadratic velocity of the particles and will be inversely proportional to the pressure. If the surface is higher, the depth of the layer of powder is larger; then the pressure is large and $v(\mathbf{r})$ will be small. Therefore the direction of $\mathcal{J}$ is from the region with low surface to that with high surface. This generates the heaping of powder which maintains convection.

In contrast, our model does not need heaping to cause convection. The effect proposed by Rajchenbach has not yet been observed directly. The viscoelasticity we have introduced is natural, because it corresponds to the

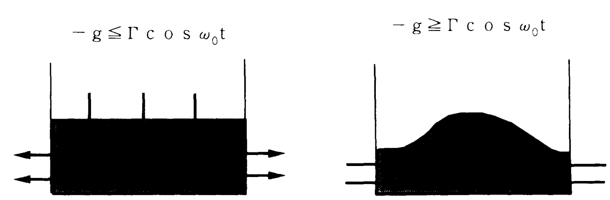


FIG. 6. Illustration of the instability induced by elasticity.

inelasticity of the collisions, as mentioned above. In spite of the simplicity of our assumption, our simulation reproduces some nontrivial phenomena like convection and fluidization.

In order to confirm our results, we propose that experimentalists check the dependence of $\langle \mathcal{J} \rangle$ and $\mathcal{R}$ on e and t_{col}, and compare the results with the numerical simulation [11].

It is also useful to determine the critical value Γ_c by a direct measurement of convective motion. This is because, in our model, convection is not the result of heaping, as assumed by Rajchenbach, but rather the origin of heaping. Since Γ_c should be the same for both convection and heaping, and the critical point is easier to find by observing convection rather than heaping, in the experiment we expect that convection will start apparently before heaping starts. These observations would confirm that our model is much more plausible than Rajchenbach's.

In summary, we have proposed a new origin of convection in powder. Our model has reproduced the convective motion and surface fluidization which are observed in experiments. The mechanism proposed here is new; convection is induced by the elastic interaction between particles.

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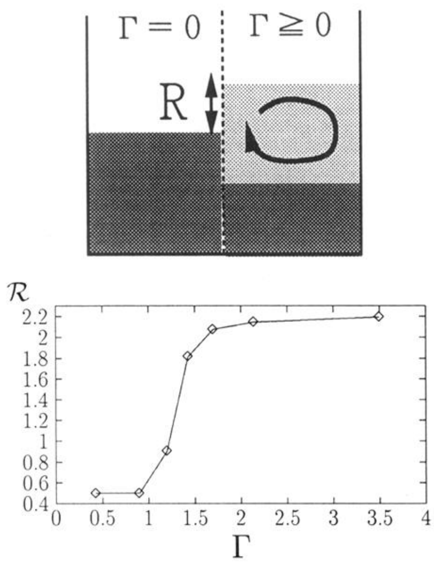
(a)

(b)



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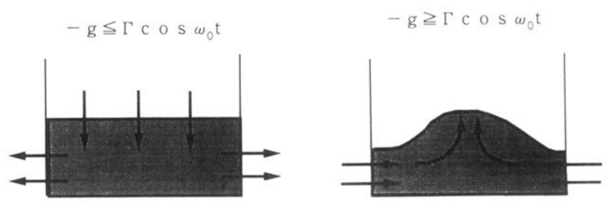

FIG. 6. Illustration of the instability induced by elasticity.