High-Sensitivity Magnetometer Based on Index-Enhanced Media

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The large dispersion of a phase-coherent medium, at a point of vanishing absorption, is applied to interferometric measurements of detuning between atomic and radiation frequencies. It is shown that, under certain conditions, the interferometer quantum-limited operation is determined by vacuum-fluctuation shot noise while the noise introduced by the interaction of the probe field with the phase-coherent atoms can be made negligible. As a possible application, an optical magnetometer is analyzed whose sensitivity is shown to be potentially superior to the present state-of-the-art devices.

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It is by now generally recognized that phase-sensitive techniques can lead to quantum-noise quenching, e.g., squeezing via a parametric oscillator [1] and spontaneous-emission-noise quenching via the correlated-spontaneous-emission laser [2].

We show here, for the first time, that atomic coherence can be used to enhance the signal in optical interferometry without increasing the noise. Specifically we show that it is possible, using a recently developed technique to tailor the index of refraction [3], to make an ultraprecise measurement of the difference between the frequencies of a monochromatic light field and that of a corresponding atomic transition. In order to make the analysis concrete, and as an interesting example of the present approach, we focus here on the problem of magnetic-field measurements and show that the present considerations lead to a new kind of magnetometer with sensitivity potentially surpassing state-of-the-art devices [4]. However, it is clear that the present techniques are potentially useful in other problems such as optical probes of time-reversal violation and possible new optical frequency standards, as is briefly discussed later on.

The essential physics involved in these considerations is contained in the observation that atomic coherence and interference can be and has been applied to yield electromagnetically induced transparency [5], lasing without inversion [5–8], and under certain conditions enhancement of the index of refraction [3]. In particular we show that electromagnetically induced transparency and the attendant index of refraction can be used to advantage in the context of ultrasensitive interferometry.

It is important to emphasize that enhancing the signal by itself does not ensure a superior measurement sensitivity. In order to assess device sensitivity the appropriate quantum-noise limits must be established. To this end, a quantum Langevin analysis is given herein showing that the process of enhancing the signal need not increase the noise. Obviously, we seek to work in the range of parameter space such that the signal-to-noise ratio is effectively maximized.

The large dispersion of the index of refraction in the vicinity of a sharp atomic resonance provides a mechanism for detecting detunings between atomic and radiation frequencies by interferometric means.

To motivate this let us first consider the case of a simple two-level atom [9]. At resonance of the atomic transition the real part of the susceptibility, χ', is linear in the atom-field detuning with a large slope. Hence a small detuning leads to a substantial change of the index of refraction n, $(n+ik)^2=1+\chi'+i\chi''$, where k is the absorption constant. However, it is well known that the accompanying huge absorption prevents an application of this effect.

With the advent of laser spectroscopy, however, it was realized that it is possible to produce "nonabsorbing" states of matter via quantum interference and atomic coherence. These ideas are the basis of the current research involving lasers which operate without population inversion [5–8], and have been observed for a lower-level doublet (Λ system) in the nonabsorption resonance

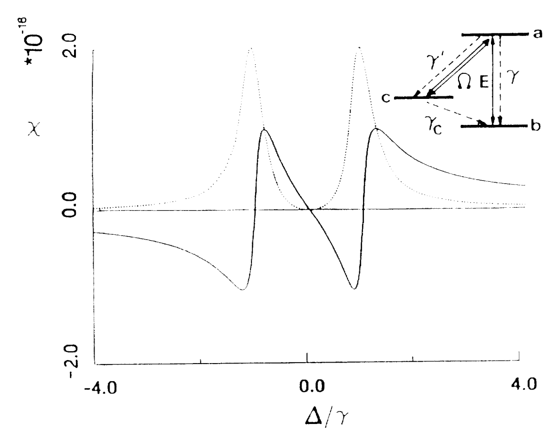

FIG. 1. Real and imaginary parts of the susceptibility χ' (solid line) and χ'' (dotted line) for the Λ scheme indicated in the inset. The strong driving field of Rabi frequency Ω' couples levels a and c. Radiative decays from a to b and a to c go at rates γ and γ' whereas collision depletion of level c occurs at rate γ_c. Electric field of the laser is denoted by E.

quantum coherence experiments [10,11], and the upper-level-doublet (V system) experiments [12] showing markedly reduced absorption due to quantum interference.

An example for the realization of such a situation is the atomic scheme depicted in Fig. 1. A three-level system with one upper and two lower levels is driven by a strong coherent field with Rabi frequency Ω'. The strong driving field leads to an interference of different possible absorption pathways, producing a nonabsorbing resonance. A typical spectrum of the linear susceptibility for this scheme is given in the same figure.

If we place the high-index material in one arm of a Mach-Zehnder interferometer [13] as in Fig. 2, the phase between the arms will be shifted due to the atom-field detuning.

In order to establish the sensitivity of the device, we must compare the interferometer signal $\langle j\rangle$ with the measurement error $\langle \Delta j\rangle$. To accomplish this a fully quantum-field-theoretical analysis is necessary. In particular we investigate the quantum excess noise impressed on the probe laser transmitted through the phase-coherent medium. To this end we carry out a quantum Langevin analysis in terms of the atomic operators for the three-level system of Fig. 1,

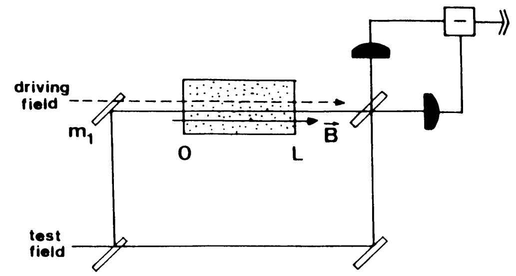

FIG. 2. Mach-Zehnder interferometer. Coherence-producing radiation passes through mirror m_1 which is highly reflecting for laser radiation. Laser radiation acquires a magnetic-field-dependent phase shift and is detected at the two outputs.

$$
\begin{aligned}
&\hat{\sigma}_0=|b\rangle\langle a|, \quad \hat{\sigma}_a=|a\rangle\langle a|,\\
&\hat{\sigma}_1=|b\rangle\langle c|, \quad \hat{\sigma}_b=|b\rangle\langle b|,\\
&\hat{\sigma}_2=|c\rangle\langle a|, \quad \hat{\sigma}_c=|c\rangle\langle c|,
\end{aligned}
\tag{1}
$$

and the positive (negative) frequency part of the probe-field operators, $\hat{E}^{(+)}$ $(\hat{E}^{(-)})$. The essential operator equations in an interaction picture are

$$\dot{\hat{\sigma}}_0=-[i\Delta+\tfrac{1}{2}(\gamma+\gamma')]\hat{\sigma}_0+i\frac{\wp}{\hbar}(\hat{\sigma}_b-\hat{\sigma}_a)\hat{E}^{(+)}+i\Omega'\hat{\sigma}_1+\hat{F}_{\sigma_0}, \tag{2a}$$

$$\dot{\hat{\sigma}}_1=-[i(\Delta-\Delta')+\tfrac{1}{2}\gamma_c]\hat{\sigma}_1-i\frac{\wp}{\hbar}\hat{E}^{(+)}\hat{\sigma}_2^{+}+i\Omega'^{*}\hat{\sigma}_0+\hat{F}_{\sigma_1}, \tag{2b}$$

$$\dot{\hat{\sigma}}_2=-[i\Delta'+\tfrac{1}{2}(\gamma+\gamma'+\gamma_c)]\hat{\sigma}_2+i\frac{\wp}{\hbar}\hat{E}^{(+)}\hat{\sigma}_1^{+}+i\Omega'(\hat{\sigma}_c-\hat{\sigma}_a)+\hat{F}_{\sigma_2}, \tag{2c}$$

where $\Delta=\omega_{ab}-\nu$, and $\Delta'=\omega_{ac}-\nu'$; ν and ν' are the frequencies of the probe and driving field, $\wp$ is the $a\rightarrow b$ transition matrix element, and the decay rates are introduced according to Fig. 1. Note that we have included a decay from level c to b in Eqs. (2).

The quantum-noise operators $\hat{F}_x$ in Eqs. (2) have a zero mean value and are δ correlated,

$$\langle \hat{F}_x(t)\hat{F}_y(t')\rangle=\langle \hat{F}_x\hat{F}_y\rangle\delta(t-t'). \tag{3}$$

The diffusion coefficients, $\langle \hat{F}_x\hat{F}_y\rangle$, are calculated using the generalized fluctuation-dissipation theorem [9].

To obtain the linear response of the phase-coherent medium to a weak probe field, we solve these questions for the mean steady-state values in first order of the probe field. For the polarization of the medium we find

$$
\begin{aligned}
\langle P^{(1)}\rangle=\wp N\langle\sigma_0^{(1)}\rangle=\frac{\wp^2 N}{\hbar}E^{(+)}&\frac{-(\Delta-\Delta')+i\frac{1}{2}\gamma_c}{[|\Omega'|^2+\frac{1}{4}\gamma_c(\gamma+\gamma')-\Delta(\Delta-\Delta')]^2+\frac{1}{4}[(\Delta-\Delta')(\gamma+\gamma')+\Delta\gamma_c]^2}\\
&\times\{|\Omega'|^2+\tfrac{1}{4}\gamma_c(\gamma+\gamma')-\Delta(\Delta-\Delta')-i\tfrac{1}{2}[(\Delta-\Delta')(\gamma+\gamma')+\Delta\gamma_c]\},
\end{aligned}
\tag{4}
$$

where $E^{(+)}=\langle \hat{E}^{(+)}\rangle$.

For sufficiently large $|\Omega'|$, i.e., $|\Omega'|^2\gg\gamma\gamma_c$, the absorption is essentially canceled at $\Delta-\Delta'=0$, while the index of refraction goes like

$$n=n_0-\frac{3}{8\pi^2}\frac{\gamma^2}{|\Omega'|^2}\lambda^3 N\frac{\Delta-\Delta'}{\gamma}, \tag{5}$$

where n_0 is the background index; we have eliminated $\wp$ by γ and the wavelength λ of the optical transition and are working in the limit of small detuning. We thereby neglect collisional dephasing of the a-b polarization as compared to the radiative linewidth. This is a good approximation if the gas pressure is below 10 mtorr, which corresponds to a density of atoms of 2×10^{14} cm^{-3} at room temperature. The phase shift between the interferometer arms associated with the change of n is then

$$\Delta\phi_{\rm sig}=-\frac{3}{4\pi}\lambda^2 NL\frac{\gamma}{|\Omega'|^2}(\Delta-\Delta'), \tag{6}$$

where L is the interaction length in the coherent medium. This phase shift is detected by measuring the difference intensity of the two interferometer outputs. This balancing technique has the advantage, compared to a measurement of one of the outputs, that intensity fluctuations of the input probe field above the shot-noise level are canceled [14]. The mean number of counts is then

$$\langle j\rangle = n_{\rm in}\kappa\cos(\Delta\phi_0+\Delta\phi_{\rm sig}), \tag{7}$$

where $\Delta\phi_0$ is the phase difference between the two interferometer arms in the absence of a magnetic field. At the operating point, determined by $\Delta\phi_0=-\pi/2$, we have

$$\langle j\rangle = n_{\rm in}\kappa\Delta\phi_{\rm sig}. \tag{8}$$

Here $n_{\rm in}$ is the total number of input photons passing through the interferometer during the measurement time t_m, given in terms of the power $P_{\rm in}$ as $n_{\rm in}=(P_{\rm in}t_m)/\hbar\nu$.

$$\kappa=\exp\left\{-\frac{3}{8\pi}\lambda^2LN\frac{\gamma\gamma_c}{|\Omega'|^2}\right\} \tag{9}$$

is the transmittivity of the high-index medium at the resonance point, which approaches unity for $\gamma_c\to 0$. Note that $\gamma_c\ll\gamma,\gamma'$, since the $c\to b$ transition is dipole forbidden.

The principle quantum limit of the measurement error $\langle\Delta j\rangle$ consists of two parts. One is associated with the shot noise of the probe laser and the other originates from the interaction with the phase-coherent atoms. For the shot-noise contribution we have

$$\langle\Delta j^2\rangle_{\rm shot}=\tfrac{1}{2}(1+\kappa^2)n_{\rm in}. \tag{10}$$

From the Langevin equations (2) we obtain for the noise contribution due to the atomic medium

$$\begin{aligned}\langle\Delta j^2\rangle_{\rm atomic}&=\frac{9}{8\pi^2}N\frac{\lambda^4L}{A}\frac{\gamma^2\gamma_c\kappa^2}{t_m|\Omega'|^4}n_{\rm in}^2\\&=\frac{3}{4\pi}N\lambda^2L\frac{\gamma\gamma_c}{|\Omega'|^2}\frac{|\Omega|^2}{|\Omega'|^2},\end{aligned} \tag{11}$$

where A is the effective cross section of the laser beam and $|\Omega|$ is the Rabi frequency of the probe field. In the case of small absorption ($\kappa\approx 1$) expression (11) can be approximated by

$$\langle\Delta j^2\rangle_{\rm atomic}=(1-\kappa^2)\kappa^2\frac{|\Omega|^2}{|\Omega'|^2}n_{\rm in}. \tag{12}$$

It can be recognized from Eq. (12) that for sufficiently small probe-field Rabi frequencies we have $\langle\Delta j^2\rangle_{\rm atomic}\ll\langle\Delta j\rangle_{\rm shot}$, which means that the interaction with the atomic medium contributes only a negligible amount of extra noise.

Besides the quantum-noise contributions to the measurement error, there are additional noise sources, which have to be taken into account. The number of input photons $n_{\rm in}$ from an unstabilized probe laser usually has fluctuations above the shot noise. As mentioned earlier, however, these fluctuations are eliminated by the balancing of the two interferometer outputs [14]. Another error is introduced if the number of atoms in the optical path is fluctuating. The implied signal error $\Delta\phi_{\rm err}\approx(\sqrt{N}/N)\times\Delta\phi_{\rm sig}$ is, however, negligible. Since the atomic-noise contribution, Eq. (12), can be made small by an appropriate choice of system parameters, the sensitivity of the interferometer is essentially shot-noise limited. It is interesting to note that by using squeezed vacuum light instead of ordinary vacuum at the unused input port of the interferometer even sub-shot-noise operation is possible [15].

So far effects of atomic motion have not been taken into account. However, as we can see from Eq. (4), and will be shown in detail elsewhere [16], it is essentially the difference $\Delta-\Delta'$ of the two detunings which determines the polarization and therefore the interferometer operates essentially Doppler free if the $a\to b$ and $a\to c$ transitions are of approximately equal frequencies. In particular if

$$r\Delta_D^2\ll|\Omega'|^2, \tag{13}$$

where Δ_D is the Doppler width and $r=(\omega_{ab}-\omega_{ac})/\omega_{ab}$, Doppler broadening only affects the transmittivity of the high-index medium, which then reads

$$\kappa=\exp\left\{-\frac{3}{8\pi}\lambda^2LN\frac{\gamma}{|\Omega'|^4}[\gamma_c|\Omega'|^2+(\gamma+\gamma')r^2\Delta_D^2]\right\}. \tag{14}$$

Thus equating $\langle j\rangle$, Eq. (8), and $\langle\Delta j\rangle_{\rm shot}$, Eq. (10), and solving for the detuning we obtain for the minimum detectable frequency difference

$$(\Delta-\Delta')_{\rm min}\approx\frac{4\pi}{3}\frac{1}{\lambda^2NL}\frac{|\Omega'|^2}{\gamma}\left[\frac{\hbar\nu}{P_{\rm in}t_m}\right]^{1/2}\left[\frac{1+\kappa^2}{2\kappa^2}\right]^{1/2}. \tag{15}$$

It is useful at this point to consider a numerical example. For $|\Omega'|=\gamma=10^7$ Hz, $P_{\rm in}=1$ mW, $t_m=1$ sec, $L=10$ cm, and an atomic number density of 2×10^{12} cm^{-3} we find a minimum detectable detuning of 10^{-5} Hz. Potential applications of such a precise measurement of frequency shifts are, for instance, optical frequency standards, the detection of small permanent dipole moments, and highly sensitive measurements of magnetic fields.

To illustrate the application to magnetometry let us consider the case where the levels b and c of the scheme in Fig. 1 have different magnetic quantum numbers. For initially resonant conditions a magnetic field shifts level b and c so that we have

$$\Delta-\Delta'=\frac{\mu_B}{\hbar}(g_b+g_c)B=aB, \tag{16}$$

where g_b and g_c are the gyromagnetic factors of the cor-

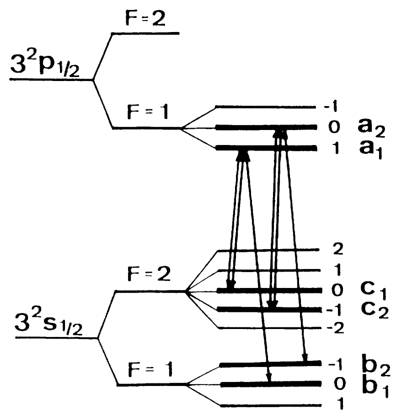

FIG. 3. Realization of the Λ scheme in sodium. A strong right-circular-polarized driving field couples levels $3^2S_{1/2}(F=2)$ and $3^2P_{1/2}(F=1)$. A weak right-circular-polarized field probes the transition from $3^2P_{1/2}(F=1)$ to $3^2S_{1/2}(F=1)$ level.

responding levels, μ_B is the Bohr magneton, and B is the local magnetic field seen by the atoms. $a=\mu_B/\hbar(g_b-g_c)$ is of order 10^7 Hz/G. From Eq. (15) we then find for the minimum detectable field strength

$$B_{\min}\approx\frac{1}{a}\frac{4\pi}{3}\frac{1}{\lambda^2NL}\frac{|\Omega'|^2}{\gamma}\left[\frac{\hbar\nu}{P_{\text{in}}t_m}\right]^{1/2}\left[\frac{1+\kappa^2}{2\kappa^2}\right]^{1/2}. \tag{17}$$

For the parameters used above we obtain a minimum B of 10^{-12} G, which is comparable with state-of-the-art sensitivities without the need of cryogenic cooling.

For an experimental demonstration of the proposed magnetometer consider a low-pressure sodium cell. Two modes of a dye laser around 589 nm with a frequency difference of 1.77 GHz couple the $F=1$ and $F=2$ hyperfine sublevels of the $3^2S_{1/2}$ state to the $3^2P_{1/2}$ $(F=1)$ level as indicated in Fig. 3. The strong driving field between the a and c levels creates nonabsorbing resonances on the $a\to b$ transitions. A magnetic level shift has no influence on transition I, since only level a_1 acquires a Zeeman shift and thus the difference $\Delta-\Delta'$ for this transition remains unaffected. However, transition II is sensitive to a magnetic field in exactly the fashion described above.

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[1] See, for example, special issue on squeezing [J. Opt. Soc. Am. B4 (1987)].
[2] M. O. Scully, Phys. Rev. Lett. **55**, 2802 (1985).
[3] M. O. Scully, Phys. Rev. Lett. **67**, 1855 (1991).
[4] A. Bruno and G. Romani, J. Appl. Phys. **65**, 5 (1989); J. Clarke, Phys. Today **42**, No. 3, 36 (1989); F. A. Franken and F. D. Colegrove, Phys. Rev. Lett. **1**, 316 (1958); L. D. Schearer, F. D. Colegrove, and G. K. Walthers, Rev. Sci. Instrum. **34**, 1363 (1963); C. Cohen-Tannoudjji *et al.*, Phys. Rev. Lett. **22**, 758 (1969).
[5] S. Harris, Phys. Rev. Lett. **62**, 1022 (1989); A. Imamoğlu and S. Harris, Opt. Lett. **14**, 1344 (1989).
[6] O. Kocharovskaya and Ya. I. Khanin, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 581 (1988) [JETP Lett. **48**, 630 (1988)]; O. Kocharovskaya and P. Mandel, Phys. Rev. A **42**, 523 (1990).
[7] M. Scully, S. Zhu, and A. Gavrieliedes, Phys. Rev. Lett. **62**, 2813 (1989); M. Scully, in *Noise and Chaos in Nonlinear Dynamical Systems*, Proceedings of the NATO Advanced Research Workshop, Torino, Italy, 7–11 March 1990 (Plenum, New York, 1990).
[8] A. Lyras, X. Tang, P. Lambropoulos, and J. Zhang, Phys. Rev. A **40**, 4131 (1989); S. Basile and P. Lambropoulos, Opt. Commun. **78**, 163 (1990).
[9] M. Sargent, M. Scully, and W. Lamb, *Laser Physics* (Addison-Wesley, Reading, MA, 1974).
[10] G. Alzetta, A. Gozzini, L. Moi, and G. Orriols: Nuovo Cimento **36B**, 5 (1976); G. Alzetta, in *Coherence in Spectroscopy and Modern Physics*, edited by F. T. Arecchi, R. Bonifacio, and M. O. Scully (Plenum, New York, 1978); G. Alzetta, L. Moi, and G. Orriols, Nuovo Cimento **52B**, 209 (1979).
[11] H. Gray, R. Whitley, and C. Stroud, Opt. Lett. **3**, 218 (1979).
[12] S. Harris *et al.*, Phys. Rev. Lett. **66**, 2593 (1991).
[13] B. Born and E. Wolf, *Principles of Optics* (Pergamon, Oxford, 1980).
[14] C. M. Caves, Phys. Rev. D **23**, 1693 (1981).
[15] For an experimental demonstration of the subshot noise operation of an Mach-Zehnder interferometer see: Min Xiao, Ling-An Wu, and H. J. Kimble, Phys. Rev. Lett. **59**, 278 (1987).
[16] M. Fleischhauer and M. O. Scully (unpublished).