Partial-Wave Analysis of $J/\psi \rightarrow \gamma \eta \pi^+ \pi^-$

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We present a study of the decay $J/\psi \rightarrow \gamma \eta \pi^+ \pi^-$ using the Mark III detector at the SLAC $e^+e^$ storage ring SPEAR. A partial-wave amplitude analysis is performed on the $\eta \pi^+ \pi^-$ system in the mass region from 1.2 to 1.5 GeV. We observe two $\eta \pi^+ \pi^-$ resonances, each decaying via $a_0(980)\pi$. We identify one as the axial-vector meson $f_1(1285)$ and the other as a pseudoscalar resonance at 1.40 GeV.

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Since it was first observed in radiative J/ψ decays by the Mark II [1] and Crystal Ball [2] Collaborations, much effort has been devoted to the study of the $\eta(1440)$ [formerly the i(1440)]. This state has been considered by many [3] to be a prime candidate for gluonium (a bound state of two gluons), since it is copiously produced in radiative J/ψ decays [4] and is not observed in $\gamma\gamma$ collisions [5]. In addition, the spin-parity assignment of $J^{PC} = 0^{-+}$ made it difficult to classify the $\eta(1440)$ into the lowest-lying quark-antiquark nonets [6].

Results from an early amplitude analysis [2] indicated that the $\eta(1440)$ decays through the $J^{PC} = 0^{-+}$ $a_0(980)\pi$ (S wave) intermediate state into $K\bar{K}\pi$, where $a_0(980) \rightarrow K\overline{K}$. Since $\eta\pi$ is the other dominant decay mode of the $a_0(980)$, the $\eta(1440)$ was searched for in the decay $J/\psi \rightarrow \gamma \eta \pi^+ \pi^-$. No signal resembling the broad $K\bar{K}\pi$ enhancement was seen at 1.44 GeV in the $\eta\pi^+\pi^$ invariant mass distribution. Instead, a narrower signal near 1.39 GeV was observed [7].

In order to perform a more sensitive search for states in the 1.2 to 1.5 GeV mass region, the decays $J/\psi \rightarrow \gamma K \overline{K} \pi$ and $J/\psi \rightarrow \gamma \eta \pi^+ \pi^-$ have been studied by the Mark III Collaboration, using an isobar model partial-wave amplitude (PWA) analysis method [8]. The results of the $K\overline{K}\pi$ study [9] indicate that the $\eta(1440)$ is not adequately described as a single resonance, but rather as a superposition of several states. These include a pseudoscalar state near 1.41 GeV decaying through $a_0(980)\pi$, an axialvector state near 1.44 GeV decaying through K^*K , and a second pseudoscalar state near 1.49 GeV decaying through K^*K . We report herein the results of a PWA

analysis of the decay $J/\psi \rightarrow \gamma \eta \pi^+ \pi^-$ and compare the parameters of the states found in this analysis with those obtained in the $K\overline{K}\pi$ channel and in hadron-induced reactions.

This analysis is based upon a sample of 5.8×10^6 produced J/ψ events collected with the Mark III detector [10] at the SLAC e^+e^- storage ring SPEAR. The data presented here contain only photons and charged pions in the final state. The photons are detected in an electromagnetic calorimeter (a barrel and two end caps) with total coverage 94% of 4π . The charged tracks are detected in a multilayer cylindrical drift chamber which covers 85% of 4π .

In our analysis the η is detected in its $\gamma\gamma$ and $\pi^+\pi^-\pi^0$ decays. We select events with no net charge and the required number of charged tracks $(n_{ch}=2 \text{ for } \eta \rightarrow \gamma \gamma \text{ and }$ $n_{\rm ch} = 4$ for $\eta \rightarrow \pi^+ \pi^- \pi^0$) and showers $(n_{\rm sh} \ge 3)$ [11]. We require that the energy of each shower be greater than 100 MeV for $n_{ch}=2$, and greater than 50 MeV for $n_{\rm ch}$ = 4. Four-constraint (4C) kinematic fits are performed to the hypotheses $J/\psi \rightarrow \gamma \gamma \gamma \pi^+ \pi^-$ and J/ψ $\rightarrow \gamma \gamma \gamma \pi^+ \pi^- \pi^+ \pi^-$. If $n_{\rm sh} > 3$, the fit is repeated using all possible combinations of three candidate photons. For events with a good fit, the combination with the largest probability of $\chi^2 [P(\chi^2)]$ is selected. For $n_{\rm ch} = 2$, events with a 5C fit are retained if $P(\chi^2) > 10\%$ (the extra constraint is that of the η mass). For $n_{ch} = 4$, a 6C fit with $P(\chi^2) > 10\%$ is required (π^0 and η mass constraints are imposed). Backgrounds are suppressed by removing asymmetric decays of the π^0 and η . The selected data sample contains feedthrough from other decays (e.g., $J/\psi \rightarrow \rho \pi$ and $\omega \pi$ for $n_{ch} = 2$ and $J/\psi \rightarrow \pi^+ \pi^- \pi^+ \pi^- \pi^0$ for $n_{ch} = 4$). This is compatible with known rates plus Monte Carlo (MC) studies. We find that we can efficiently remove these contaminations by rejecting events satisfying 4C fits to $J/\psi \rightarrow \gamma \gamma \pi^+ \pi^-$, $\gamma \gamma \gamma \gamma \pi^+ \pi^-$, or a 5C fit to $J/\psi \rightarrow \pi^+ \pi^- \pi^+ \pi^- \pi^0$. This rejection does not bias the selection of the genuine signal.

The $\eta \pi^+ \pi^-$ mass distributions from the decay $J/\psi \rightarrow \gamma \eta \pi^+ \pi^-$ are shown in Fig. 1 for both decay modes of the η . The dominant feature is the η' signal (inset), and both channels show significant structures at higher masses. Slight differences between the spectra in the two final states are apparent, reflecting differences in statistics, backgrounds, and above 2.8 GeV the effect of the differing photon energy criteria.

To check that the data selection and reconstruction methods are well understood, we measure the branching ratio $B(J/\psi \rightarrow \gamma \eta')$. Signal strengths in both final states are found by fitting the invariant mass distributions with a nonrelativistic Breit-Wigner function convoluted with a Gaussian mass resolution term (henceforth abbreviated as BWG). The mass resolution ($\sigma = 0.009$ GeV) and the efficiency for finding a signal in an analysis are found by MC simulation [12]. We find

$$B(J/\psi \rightarrow \gamma \eta') = (4.50 \pm 0.14 \pm 0.53) \times 10^{-3}$$

(a)

(b)

100

25

50

25

0

60

40

20

0

1.0

1.5

Events/(0.02 GeV)

150

0.92

40

20

n 92

0.96

0.96

2.5

3.0



2.0

 $\eta \pi^+ \pi^-$ Mass (GeV)

and

$$B(J/\psi \rightarrow \gamma \eta') = (4.30 \pm 0.31 \pm 0.71) \times 10^{-3}$$

for the $\eta \rightarrow \gamma \gamma$ and $\eta \rightarrow \pi^+ \pi^- \pi^0$ modes, respectively. The first error given is statistical, the second systematic. The systematic errors include several terms added in quadrature. These reflect the uncertainties in the J/ψ flux (8.5%), the MC simulation (6.5% for $n_{ch}=2$, 7.1% for $n_{ch}=4$), the fit of the BWG function to the data (0.5% for $n_{ch}=2$, 3.7% for $n_{ch}=4$), and the effect of varying the selection criteria (5.1% for $n_{ch}=2$, 11.9% for $n_{ch}=4$). The results are compatible with previous measurements [13,14].

In the PWA analysis the amplitudes are constructed from Lorentz-invariant combinations of the momenta and polarization 4-vectors [15] in the events. We consider the process as a three-step sequential decay $J/\psi \rightarrow \gamma X$, $X \rightarrow AP$, $A \rightarrow PP$, where X and A are intermediate resonances, and the P's are pseudoscalars (π or η). The relative magnitudes and phases of the amplitudes are found by a maximum likelihood (\mathcal{L}) fit to the data. The binto-bin variation in magnitude and phase of each amplitude reveals the presence of any resonance(s) X.

We consider $J^{PC}(X) = 0^{-+}$ or 1^{++} , and allow X to decay to $a_0(980)\pi$ or $f_0(1400)\eta$. The $a_0(980)$ is included on the basis of studies of $\eta \pi^{\pm}$ invariant masses in our data. To parametrize the $a_0(980)$ line shape we use a relativistic coupled-channel propagator [16]. We include $f_0(1400)$ on the grounds that, due to its large width, it cannot be excluded a priori. The $f_0(1400)$ propagator is parametrized using the formalism of an S-wave $\pi\pi$ scattering amplitude with a pole at 1.4 GeV [17]. To account for events not described by the above amplitudes, we include an additional amplitude which is not allowed to interfere with the others, and corresponds to a phase space decay of X. We fitted the data in 0.02-GeV $\eta \pi^+ \pi^-$ mass bins. Monte Carlo studies indicate that the $\eta \pi^+ \pi^-$ mass resolution (σ) increases from 0.011 to 0.013 GeV in the 1.2 to 1.5 GeV mass range.

To test the ability of the PWA analysis to identify amplitudes correctly, we generate MC signals of various J^{PC} values with statistics similar to that of the data. After analysis we find that the MC signals are correctly identified. The goodness of each fit is checked by comparing the fit predictions of various two-body invariant mass and angular distributions with those occurring in the generated data. Good agreement is found.

Analyzing the data in Fig. 1 up to $\eta \pi^+ \pi^-$ masses of 1.6 GeV, the quantity $\ln \mathcal{L}$ divided by the number of events in each mass bin varies from 0.25 to 0.4. Similar results are obtained in MC studies. The two-body invariant mass and angular distributions are also well reproduced. Above 1.6 GeV the model no longer reproduces these distributions and the goodness of fit decreases. Hence we restrict our analysis to the 1.2 to 1.5 GeV mass region.



FIG. 2. Acceptance corrected intensity distributions using the channel $\eta \rightarrow \gamma\gamma$: (a) 1⁺⁺ $a_0(980)\pi$; (b) 0⁻⁺ $a_0(980)\pi$; (c) phase space; (d)-(f) the same as (a)-(c) but for the $\eta \rightarrow \pi^+\pi^-\pi^0$ channel.

We find that the $1^{++} f_0(1400)\eta$ amplitude contributes negligibly and is thus excluded in subsequent analysis. The $0^{-+} f_0(1400)\eta$ amplitude has a nonzero magnitude in the higher statistics $\eta \rightarrow \gamma\gamma$ sample, but shows no evidence for a $\eta\pi^+\pi^-$ resonance, nor does it improve the likelihood of the fit significantly [18]. Accordingly, we repeat the analysis with no $0^{-+}f_0(1400)\eta$ contribution. The previous $0^{-+}f_0(1400)\eta$ events now appear in the phase-space term, and the contributions to the other amplitudes remain unchanged. These results [i.e., using only the set of $a_0(980)\pi$ amplitudes plus phase-space term] are shown in Fig. 2, and the relative phases in Fig. 3, together with the corresponding data from the η $\rightarrow \pi^+\pi^-\pi^0$ sample.

In the $\eta \rightarrow \gamma \gamma$ final state, we observe two clear features: a resonant structure in $1^{++} a_0(980)\pi$ at ~ 1.3 GeV, and in $0^{-+} a_0(980)\pi$ at 1.4 GeV. This is borne out both in the magnitude of the amplitudes and in the phase motion. A single fit to the whole of the $1^{++} a_0(980)\pi$ spectrum in Fig. 2(a) fails. We therefore extract the component of the spectrum compatible with the well-known resonance $f_1(1285)$. The $f_1(1285)$ line shape (modeled by a BWG function with mass, width, and mass resolution parameters of 1.282 [13], 0.024 [13], and 0.011 GeV, respectively) is shown as a solid curve in Fig. 2(a). The area below the solid curve is normalized



FIG. 3. Phase difference between $a_0(980)\pi$ amplitudes using the $\eta \rightarrow \gamma\gamma$ channel: (a) 1⁺⁺ relative to 0⁻⁺; (b) 0⁻⁺ relative to 1⁺⁺; (c),(d) the same as (a) and (b) but for the $\eta \rightarrow \pi^+\pi^-\pi^0$ channel. An arbitrary zero point is used for each figure.

to that of the data in the mass range 1.24 to 1.32 GeV. In this mass range we find the ratio of the magnitude of the helicity-1 to helicity-0 amplitudes to be 0.64 ± 0.07 . Above 1.32 GeV, there is an excess in $1^{++} a_0(980)\pi$ beyond that expected for $f_1(1285)$. This remains independent of the inclusion or exclusion of the $f_0(1400)\eta$ amplitudes in the PWA analysis, or when the event selection criteria are varied. To better understand this region a much larger data set is needed. We consider the structure in the 1.34 to 1.36 GeV bin in Fig. 2(a) a fluctuation since it appears to be narrower than our mass resolution. In addition we note that we see no associated phase motion in this bin nor a similar structure in the $\eta \rightarrow \pi^+\pi^-\pi^0$ sample.

In the $0^{-+} a_0(980)\pi$ amplitude we see a clear resonant structure [henceforth referred to as $\eta(1400)$]. We fit it with a BWG function (σ =0.013 GeV), obtaining a mass of 1.400 ± 0.006 GeV, and a width Γ of 0.047 ± 0.013 GeV. The fitted line shape is shown as a solid curve in Fig. 2(b).

Having determined the efficiency from MC simulation and studies of the η' signal we find the branching ratios for these two signals:

$$B(J/\psi \to \gamma f_1(1285))B(f_1(1285) \to a_0(980)\pi)B(a_0(980) \to \eta\pi) = (2.60 \pm 0.28 \pm 0.58) \times 10^{-4}$$

$$B(J/\psi \to \gamma n(1400))B(n(1400) \to a_0(980)\pi)B(a_0(980) \to \eta\pi) = (3.38 \pm 0.33 \pm 0.64) \times 10^{-4}.$$

The systematic error is determined in a manner similar to that for $B(J/\psi \rightarrow \gamma \eta')$, with the addition of the following extra terms. To reflect the accuracy of the PWA analysis we add the spread in the results of the PWA analysis of a set of MC generated signals with statistics similar to the real data (15.5%). The change in the efficiency (2%) for various combinations of 1⁺⁺ helicity ratios in included for the $f_1(1285)$ branching fraction. We include a term

reflecting the uncertainty in the magnitude of the non-resonant background [11% for $f_1(1285)$ and 7.7% for $\eta(1400)$].

The $\eta \rightarrow 3\pi$ final state shows features compatible with those seen in the $\eta \rightarrow \gamma\gamma$ channel. The line shapes in Figs. 2(d) and 2(e) use the same masses, widths, and normalizations (including relative η branching fractions) as those in Figs. 2(a) and 2(b), respectively. The agreement between the estimated signal strengths and the data illustrates the high degree of compatibility of the two finalstate samples. The lower statistics and uncertainties in the low-energy γ reconstruction efficiency prevent precise branching fraction determinations in the $\eta \rightarrow 3\pi$ channel. However, due to the more highly constrained η selection this channel has less background [cf. Figs. 2(c) and 2(f)], indicating that the $\eta \pi^+ \pi^-$ data in this mass range are dominated by $a_0(980)\pi$ amplitudes.

In conclusion, we present a PWA analysis of the decay $J/\psi \rightarrow \gamma \eta \pi^+ \pi^-$ in two independent η decay channels, over the 1.2 to 1.5 GeV $\eta \pi^+ \pi^-$ mass range. In each channel we observe two signals, both decaying via $a_0(980)\pi$. We identify the 1⁺⁺ signal at mass 1.29 GeV with the well-known axial vector state $f_1(1285)$. The results show an excess of data at masses slightly above that of the $f_1(1285)$; an improvement in statistics is needed to investigate it definitively. The signal at mass 1.400 $\pm\,0.006$ GeV has a width Γ of 0.047 ± 0.013 GeV and $J^{PC} = 0^{-+}$. We identify the $\eta(1400)$ with a similar signal seen in an analogous PWA analysis of the $K\bar{K}\pi$ system in $J/\psi \rightarrow \gamma K \bar{K} \pi$ [9]. That analysis observes a 0⁻⁺ signal at 1.416 GeV with a width of 0.054 GeV, decaying via $a_0(980)\pi$, the low-mass component of the "iota" (this is also seen elsewhere [19]). There is evidence for a similar $\eta(1400)$ state in other reactions. In the charge exchange reaction $\pi^- p \rightarrow \eta \pi^+ \pi^- n$, a 0⁻⁺ signal has been observed around 1.4 GeV, again decaying via $a_0(980)\pi$ [20,21]. We note, however, that central production, $pp \rightarrow p(\eta \pi \pi)$ at 300-GeV incident p momentum [22], yields no resonant signal around 1.4 GeV, just as the analogous process $p\bar{p} \rightarrow p(K\bar{K}\pi)p$ fails to find a 1.4-GeV signal decaying to $K\overline{K}\pi$ via $a_0(980)\pi$ [23].

We can combine our observation of the $f_1(1285)$ (a well-known axial vector meson) with our measurement in the similar channel $J/\psi \rightarrow \gamma \pi^+ \pi^- \pi^+ \pi^-$ [24] to obtain a total rate for its production in radiative J/ψ decays (a two-gluon-mediated process) which is compatible with predictions from perturbative QCD calculations incorporating longitudinal gluons [25]. Whether the $\eta(1400)$ is a $q\bar{q}$ state or a gluonic meson is not determined.

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FIG. 1. The $\eta \pi^+ \pi^-$ invariant mass distribution above 1 GeV for the (a) $\eta \rightarrow \gamma \gamma$ and (b) $\eta \rightarrow \pi^+ \pi^- \pi^0$ channels. The shaded region is that subjected to the PWA analysis. In both channels, the mass distribution below 1 GeV is shown in the inset with 0.002-GeV bin width.