Experimental Study of Heaping in a Two-Dimensional "Sandpile"

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In connection with recent vibrationally excited sand-heaping observations we performed a series of experiments dealing with 2D compact triangular packings of monodisperse metallic beads under vibration. We report for the first time the observation of spontaneous heaping and associated convections for this system. We establish the mechanical conditions leading to this phenomenon and the domain of occurrence. We identify the microscopical mechanism to be a block-slip motion generated by the walls and we show that the heaping dynamics obeys a $\ln(t)$ law.

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An increasing number of papers have been devoted recently to the dynamics of granular materials (for a comprehensive review see [1] and references therein). This state of matter turns out to display a number of unusual behaviors such as thixotropy, segregation, fluidization, heaping, arching effects, non-Newtonian shearing, stick-slip motion, etc. This phenomenological complexity prevents any tentative classification among solids or liquids and renders this medium a challenging prototype for the investigation of disordered systems. In addition to this fundamental interest it is noticeable that the domains of application are considerable since numerous industrial processes use all sorts of grains (pharmacology, concrete and building materials, chemical engineering, etc.). Among the open questions is the behavior of a granular material under vibration. Several recent experimental studies have shown the occurrence of a surface instability associated with spontaneous flux of matter when a noncohesive granular material is agitated above a certain threshold of vertical acceleration [2,3]. Theoretical models have also been put forward [4], but up to now, the pertinence of these interpretations is still under question. The sand-heaping experiment has subsequently motivated several computer simulations in 2D geometries [5]. Although these simulations did exhibit convective motions in the bulk of the material, they did not show any heaping process.

In this Letter we report an experimental study on a bidimensional model system made of spherical metal beads submitted to a vertical sinusoidal vibration. A previous study has shown how a 2D model system made of steel beads exhibits surface fluidization and no spontaneous matter flux [6]. This result was recovered by computer simulations [5].

Here we show for the first time that spontaneous matter flow and heaping can occur in an initially ordered 2D bead packing. First we establish clearly the relative importance of internal mechanical parameters such as solid friction and inelasticity as well as external constraints such as vibrational parameters and vertical boundaries. The 2D geometry allows a direct visualization of the microscopic dynamics and, in the case of heap formation, we identify a stochastics block-sliding mechanism initiated by the boundaries to be at the origin of the phenomenon. Then, we investigate the dynamical evolution of the heap formation and measure a $\ln(t)$ law over more than four decades in time.

Our experimental setup consists in a magnetically activated, and vertically guided, piston, supporting the bead container. As we realized in the course of the experiments, the symmetry of the patterns, as well as the reproducibility of the results, is sensitive to very small (less than 1%) spurious oscillations as we checked by optically controlling the motions of the container. We restricted our work to the low-frequency range of excitation (8 to 30 Hz) since it turned out to be very cumbersome to ensure a vertical and sinusoidal motion of the container, free of spurious vibrational modes, at higher frequencies. The container is a rectangular cell; the front and back glass plates are kept apart at a distance of 1.01 times the bead diameter by using two controlled lateral plastic wedges. The aluminum (diameter 1.50 mm) or stainless-steel beads (diameter 1.00 mm) were originally optically polished and perfectly spherical. When filled with metallic monodisperse beads and somewhat shaken, the collection of beads arranges in a perfect 2D (typically 50×50 beads) triangular lattice. For all experiments reported here, we started from a perfectly regular lattice and a flat horizontal upper surface.

We assumed, *a priori*, that the dynamics would be dependent on two mechanical parameters: the shock restitution coefficients (k) and the friction parameters (f). We used a set of four different kinds of beads in order to cover a sufficiently wide range of these parameters which we measured for each situation.

As expected, polished steel beads exhibit a strong energy restitution coefficient ($k=0.9$) and a very low solid friction parameter ($f=0.1$ in the Coulomb sense). In the following, these beads are labeled as (Hk,Lf), standing for high-restitution, low-friction beads. On the other hand, polished and fresh aluminum beads show a rather small restitution coefficient ($k=0.5$) and a low friction

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parameter ($f=0.2$). In order to increase significantly the friction parameters of other sets of beads, we perform a slight surface chemical attack of the steel beads, using a diluted HCl-HNO_3 mixture. It turns out that aluminum beads become oxidized naturally in the presence of wet air, after two or three days of experiments. Neither of these chemical processes perturb the restitution coefficients but they do increase considerably the friction parameters (oxidized Al, $f=0.8$; attacked steel, $f=0.6$). The first two series of experiments are concerned with fresh, steel and aluminum beads. In both cases, we observe a preservation of the triangular network and consequently no flux of matter. The experiments with steel beads (Hk,Lf) exhibit a behavior already described in [6], i.e., a fluidization of the top layers. We do not observe any surface fluidization for aluminum polished beads (Lk,Lf), even at large vibration amplitudes. We propose to link this difference between the two (Hk,Lf) and (Lk,Lf) cases to the different energy transfer coefficients for bead-bead collisions. This is consistent with a mechanism implying a quasi-integral transmission of the impulse from the bottom to the surface in the case of steel beads and an impulse dissipation in the bulk of the packing for aluminum beads. This transition between surface fluidization and packed collective motion has also been identified in experiments and simulation of a 1D column of beads [7].

In a second series of experiments we deal with oxidized steel (Hk,Hf) and oxidized aluminum beads (Lk,Hf) beads. We readily observe that the introduction of an important bead-bead friction parameter leads to a striking change in the macroscopic behavior.

In both cases, we witness the formation of symmetrical rolls generated essentially from the upper left and right corners of the cell. Dealing with the chemically attacked (Hk,Hf) steel beads, and at a moderate vibration amplitude, we observe simultaneously surface fluidization and rolls. At a higher vibration amplitude, we observe a transition to a period-doubling response of the system and, simultaneously, a surface instability. The bottom of the heap no longer shocks the box at a period T but, after a quasi-ballistic flight, it gently touches the piston with zero relative velocity and accompanies the sinusoidal vibration until the whole packing is thrown upwards. Note that this route to chaos by period doubling has also been observed in other granular systems [8]. When reaching these conditions, there is no longer any shock wave and the surface fluidization ceases immediately, being replaced by a heaping process due to the rolls which carry matter upwards from the edges to the center of the heap. This instability is limited by surface avalanches. Oxidized aluminum beads (Lk,Hf) do not exhibit fluidization but a similar surface instability occurs immediately after a certain threshold of vibration is reached. It starts from two symmetrical rolls, and after a while (depending on the excitation parameters) we obtain a "Chinese hat" pattern (see Fig. 1) like in a vibrated 3D sandpile experiment. Note that sand grains also enter in the category (Lk,Hf). From this point on, the dynamics of the heap formation is that investigated on the system made of oxidized aluminum beads.

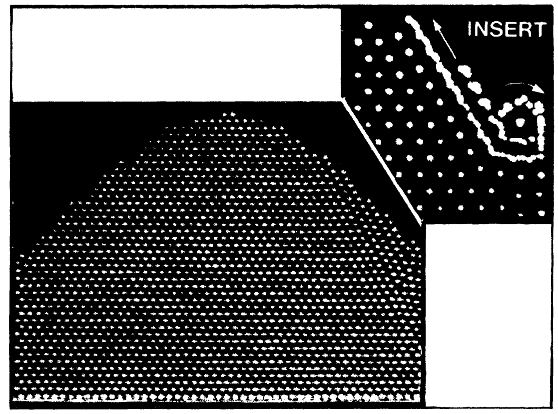

FIG. 1. The quasistationary "Chinese hat" 2D pattern obtained at 15 Hz and at $\Gamma=1.4$ with oxidized aluminum beads. Inset: posed computer photograph of the development of a roll observed at a boundary of the cell at Γ just above the acceleration threshold.

In order to prove unambiguously the decisive role played by the lateral boundaries in the 2D heaping process, we perform an experiment in a 2D cylindrical box (cyclic boundary conditions). For the same excitation parameters, no roll or surface instability is evidenced. In reverse, inserting a vertical 1.5-mm-diam stick between the two front surfaces of the cylindrical container creates a spontaneous heaping, as observed in rectangular-shaped cells. As expected, withdrawing the stick leads to a restoration of the flat surface of the bead packing. The friction of the beads on the boundary walls alone is not sufficient to create rolls and heaping. This process also requires high bead-bead friction. We checked this by operating with fresh steel (Hk,Lf) or fresh aluminum beads (Lk,Lf) and introducing lateral walls that were sawtooth shaped at the scale of the beads size. We did not observe any significant flux of matter or heaping. In this case, there was only a reorganization of the network over a distance of one or two bead sizes from the boundary.

Now, using a rectangular cell, we measure the threshold of occurrence of the instability. For practical reasons, we define this threshold as the possible occurrence of a first roll within a 30-min period of observation. For a range of frequencies from 8 to 30 Hz, and within experimental error bars, we observe, as in the sandpile experiments [2], that the threshold acceleration is independent of frequency. However, we obtain an experimental value $\Gamma_c=(10.8\pm0.5\ \mathrm{m/s^2})/g$, maybe slightly overestimated. It gives a Γ_c value of 1.1 instead of the 1.4 in 3D sandpile experiments.

In the following, we limit ourselves to accelerations

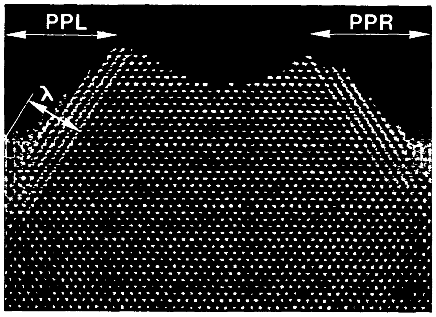

FIG. 2. Computer-processed posed photograph of the bead pile during the heaping process. The white lines on the left and right sides of the pile witness the motion of the beads near the wall during 1 min, 100 s after starting the excitation at $\Gamma = 1.3$ and 15 Hz.

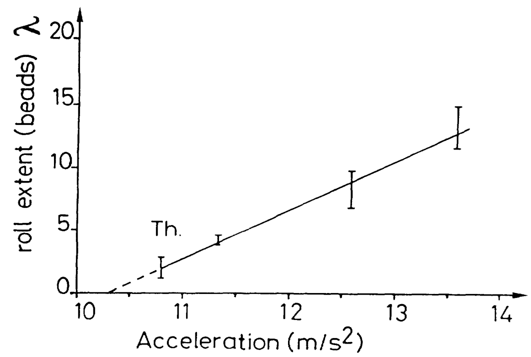

FIG. 3. Extent of the roll, λ, during the heaping process as a function of acceleration in oxidized aluminum beads. "Th" stands for the measured threshold, as reported in the text. Error bars correspond to standard deviations over many computer photographs.

close to the threshold value Γ_c where the heaping process exhibits a simple and clear-cut dynamical behavior.

Starting from a situation where the pile is prepared with a flat horizontal surface and a perfect triangular network, we use the following image processing technique in order to obtain Fig. 2 and the inset of Fig. 1: The excitation is switched on at a time $t = 0$. A stroboscope is flashing at a frequency synchronized with the excitation. The image acquisition process is turned on at a selected time t, for a constant number of images (typically 50 images over 100 s). Every image is digitized and added to the preceding one in the Boolean OR sense. This procedure witnesses the motion of particles active in the heaping process.

Now, we give a brief description of the process as it has been observed all along in our experiments. The heaping process is initiated as a roll occurring at each of the walls (see Fig. 2 and inset of Fig. 1). These rolls are active in creating an upward gliding of particles as blocks following 60° lattice directions, and involving one-lattice-step jumps. These block motions appear stochastically in time *with a duration and a waiting time much larger than the period of excitation.* We attribute the origin of the rolls to the shearing stress generated in the bulk at the vicinity of the walls. This shearing is nonzero only during the lapse of time when the acceleration of the cell is larger than gravity, in a way that rows are constructively pushed always in the same direction, from one period to the next. Note that this implies that the time necessary for a block to return to its previous position between two impulses is much larger than the period of vibration.

We report in Fig. 3 the dependence of the extent of the roll, λ, on the acceleration of the cell. Note that, at a given acceleration, λ turns out to be constant during the heaping process. Within the investigated range of parameters and being conscious of the fact that these results are necessarily discretized by one bead diameter, we observe that the points fall on a straight line which extrapolates to 10.3 m/s^2. This may be considered a better approximation to the threshold value than the one measured before. It is remarkable that the obtained Γ_c value (1.05) suggests that, within the experimental error bars, the acceleration threshold might be exactly the acceleration of gravity g. However, it is still an open question as to whether the threshold for the heaping process is definitely at $\Gamma = 1$ or at a slightly larger acceleration value.

Another important feature may also be derived from the large positive slope of the lines in Fig. 3: One generally observes a tendency for inhomogeneously vibrated granular heaps to escape from zones where vibrations have larger amplitudes and move towards zones of less agitation. We may estimate from the typical slope of the line reported in Fig. 3 that a 2% difference in excitation amplitude from one side of the cell to the other induces a difference between the velocities of both heaps towards the center as large as 15%, thereby pushing the heap towards the less-agitated regions.

In order to characterize the dynamics of the heaping process, we plot in Fig. 4, at two different acceleration

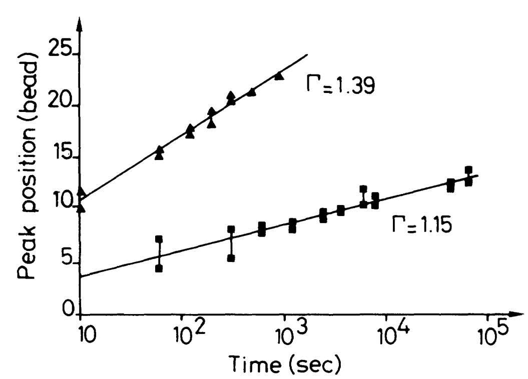

FIG. 4. Plot of the distance of the peaks (as defined in Fig. 2) to the lateral walls as a function of log(t) for two different acceleration values.

amplitudes, the distance of the peaks to the walls (denoted as PPL and PPR in Fig. 2) as a function of time. The data are fitted well by a $\ln(t)$ law over 5 decades in time.

A detailed analysis of this $\ln(t)$ law would require a detailed knowledge of the probability per unit time of the occurrence of a roll as a function of λ. Such an analysis is certainly of interest but far beyond the scope of the present paper. Here, we limit ourselves to a simple phenomenological mapping to the observed experimental features. Starting from the experimental results, we write $x = A\ln(t) + B$, x being the position of the peak relative to the boundary. A and B can be derived from Fig. 4. They both depend on the acceleration Γ. This leads to the following phenomenological differential equation:

$$\frac{dx}{dt} = A \exp\left(\frac{B}{A}\right) \exp\left(-\frac{x}{A}\right) = \alpha \exp\left(-\frac{x}{A}\right),$$

where α is seen to drive the efficiency of the process and depends both on the acceleration and on the bead-wall friction interaction. On the other hand, the argument of the exponential $1/A$ is expected to depend on the row-row or bead-bead interaction in the bulk. At $\Gamma = 1.15$ we find $\alpha = 3.9$ BU/s and $A = 1$ BU. At $\Gamma = 1.39$ we find $\alpha = 16.8$ BU/s and $A = 2.7$ BU, where BU stands for bead units on the vertical scale of Fig. 4.

In this Letter, we have reported, for the first time, the evidence of roll formation in a 2D packing of identical beads where boundaries and solid friction play a capital role. The heaping process is a consequence of this convective motion when the internal dissipation conditions are large enough to forbid surface fluidization. The modes of the dynamics are original because they consist in a series of collective block-sliding motions with a stochastic occurrence. This mechanism has a "motor" different from all the predictions and observations already made in 2D computer experiments [5]. Still, an important question concerns the extension to the 3D sandpile which is not straightforward because, in this last case, the packing structure is amorphous and the range of wall effects as well as their active role in sand heaping is still under question. Nevertheless, we pointed out a striking property of granular materials which may be general: a singular boundary layer effect as a response to an alternative shearing.

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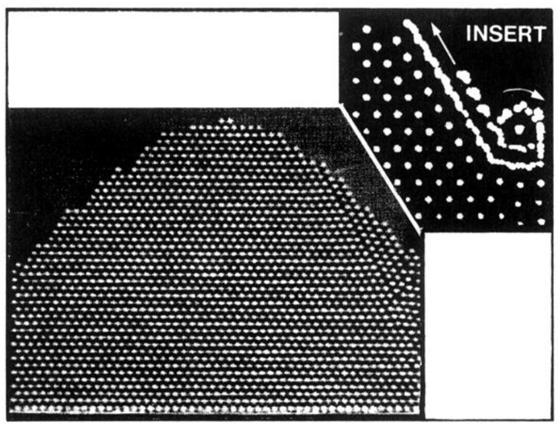

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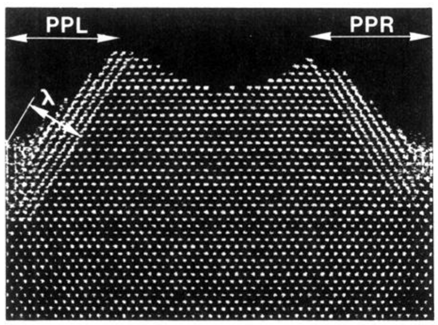

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