Resonant Transmission and Velocity Renormalization of Third Sound in One-Dimensional Random Lattices

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Localization properties of third sound were studied experimentally in one-dimensional random lattices. Transmission spectra were measured and compared with those in periodic, Fibonacci, and Thue-Morse lattices. Not only the resonant transmission but also the phase-velocity renormalization of third sound was observed in the random lattices.

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Recently, classical-wave propagation has been reinvestigated extensively in one-dimensional (1D) nonperiodic systems. Wave localization in disordered but well-controlled 1D media is one of the most fascinating highlights [1-5]. The wave propagation in 1D quasiperiodic systems (Fibonacci lattice) [6-9] has also been studied, having in mind the close relation to the properties of real quasicrystals [10]. The Thue-Morse lattice is another nonperiodic system studied, an arrangement that is neither periodic, quasiperiodic, nor random [11,12]. These phenomena have the same origin as the Bragg scattering in a periodic system, namely, multiple scattering and interference. The nonperiodic systems show qualitatively different aspects, however, from those of the periodic systems.

When the problem of wave localization in disordered media is discussed, two length scales are important, the wavelength λ and the mean free path l [13]. In the case of weak localization, the relation $\lambda \ll l$ holds. This is the case for electrons in weakly disordered metals. The use of acoustic waves enables us to achieve the opposite condition, $\lambda \geq l$, which is called the strong localization regime. Under such a condition, additional symmetries, for example, that the positions of the scattering centers are restricted to 1D lattice sites, give rise to a resonant transmission of waves [14-16]. That is, the random system becomes transparent at a certain wavelength, but otherwise the system is opaque because of the wave localization. In addition, by using acoustic waves to study the localization problem, we can attain the following advantage: The localization effects can be observed separately from correlation effects or inelastic scattering effects. The aim of this Letter is to show the localization properties of third sound and give experimental evidence for the resonant transmission and the phase-velocity renormalization of third sound in 1D random lattices, both of which have been predicted theoretically [16], but never observed experimentally up to now.

Third sound is a wave in a superfluid helium film which is adsorbed on a solid surface. Because the superfluid is almost free from the viscosity, third sound may propagate more efficiently than surface waves in normal liquids [3,4], as compared on the scale of either the wavelength in question or the depth of the liquid. It is, therefore, advantageous to use third sound for the study of delicate wave interference phenomena such as the localization of classical waves in random media. The experimental method for the study of the third-sound propagation in modulated 1D media has been developed by Smith *et al.* [5,17,18]. We have developed a similar technique independently [9,12,19]. The original ideas of our method are to be credited to Condat and Kirkpatrick (CK) [1,20]. Smith *et al.* employed the scratches of a diamond tip on the surface of a Si wafer as scatterers for third sound, while we used photoetched strips of evaporated aluminum films on a glass substrate. The scatterers of our method are more identical to each other compared to those of the former method. Our technique, therefore, might have the advantage of preserving the coherency of the multiple scattering.

The aluminum strip was 160 nm thick, 80 μm wide, and 8 mm long. We refer to this aluminum strip as A, and to the glass surface of the same width as B. The elements A and B were placed parallel to each other and perpendicular to the direction of the third-sound propagation, and hence they formed a 1D array of scattering centers for third sound. By arranging the elements A and B according to certain rules, one can make 1D binary lattices such as periodic, Fibonacci, Thue-Morse, and random lattices. The periodic lattice is produced by the following inflation rule, ($A \rightarrow AB$, $B \rightarrow AB$). As for the Fibonacci and Thue-Morse lattices, the rules are ($A \rightarrow B$, $B \rightarrow AB$) and ($A \rightarrow AB$, $B \rightarrow BA$), respectively. In the random lattices A and B appear randomly. The Fibonacci, Thue-Morse, and random lattices are nonperiodic. In the present work, each lattice consisted of about 100 elements (total sample size $\sim$8 mm).

Third-sound pulses were propagated on both the patterned and the blank substrate under the same conditions. The transmission spectrum was obtained by dividing the power spectrum of the pulse wave form which was recorded after being propagated on the patterned substrate by that on the blank substrate. This procedure removed the effect of the intrinsic attenuation of third sound. In addition, the measurement was done at 0.8 K to reduce the attenuation effect. The experimental details are described elsewhere [12].

As shown in Fig. 1, the transmission spectrum for the periodic lattice shows (transmission) gaps and bands. In the gaps around $kd/\pi \sim 0.5$, 1.0, and (less pronounced) 1.5, third sound can hardly propagate; k is the wave number of third sound and d is the width of the strip. The other regions are the bands, in which third sound propagates well and the wave function is extended over the lattice. The Fibonacci lattice shows regions in which the transmissivity diminishes (it should be noted that the scale of the ordinate is logarithmic) around $kd/\pi \sim 1/\tau$ and $1/\tau^2$, where τ is the golden ratio $(\sqrt{5}+1)/2 \sim 1.618$. These regions are referred to as (transmission) gaps. The bands are eroded away by the narrower gaps in a nested way. The positions of the gaps were well described by the formula $m+n\tau$ (mod1), where m and n are integers. The Thue-Morse lattice shows several narrow depressions of transmissivity. We call these structures dips rather than narrow gaps, because even the largest one hardly has a well defined width compared to those of the former two lattices. In this sense, the Thue-Morse lattice is closer to the homogeneous medium than the periodic or the Fibonacci lattice. A simple relation holds between the positions of these dips [12]. The detailed properties of these spectra are discussed elsewhere [9,12,19].

The transmission spectrum of third sound in the random lattice (the lowest trace in Fig. 1) is different from any of the three above-mentioned systems. Spiky dips appear randomly and the details of the structure are sample dependent. This fluctuation is essentially the same as that observed by Belzons *et al.* [3,4] in a reflection spectrum of water waves on a rough bottom. They concluded from the direct measurement of the spatial amplitude distribution that it was due to resonance modes of the random bed. The following general and sample-independent features are observed to be intrinsic to the random lattices: (1) At small wave number, $kd/\pi \sim 0$, the random lattice is transparent and the spectrum is flat. (2) With increasing wave number, the transmissivity decreases on the average. (3) At around $kd/\pi \sim 0.5$ and 1.0 there are minima of the transmissivity. (4) At either side of $kd/\pi \sim 1.0$ there are regions where the transmissivity recovers again.

We have averaged the spectra in the random lattices, in order to remove the fluctuations which changed from sample to sample. Five random lattices were prepared, with which the transmission spectra were measured. Those five spectra were overlaid together. Then, intervals were set on the abscissa so that each interval would contain eighty data points. Within each interval the sample mean of $\log_{10} T_n$ was calculated, where T_n is the transmission coefficient. Figure 2 shows the averaged transmission spectra. The amount of adsorbed helium was changed from trace a to c, which changed the scattering power. From Fig. 2 the features (1)-(4) can be seen clearly.

The experimental localization length ξ is obtained from the formula

$$2L_s/\xi = -\langle \ln T_n \rangle \,, \tag{1}$$

where L_s is the sample size. In this experiment, as mentioned in feature (3), the localization effect was most prominent around $kd/\pi \sim 0.5$ and 1.0, in particular, under the condition of Fig. 2, trace c. In these regions the transmission coefficient $\langle \log_{10} T_n \rangle$ reached ~ -2, and

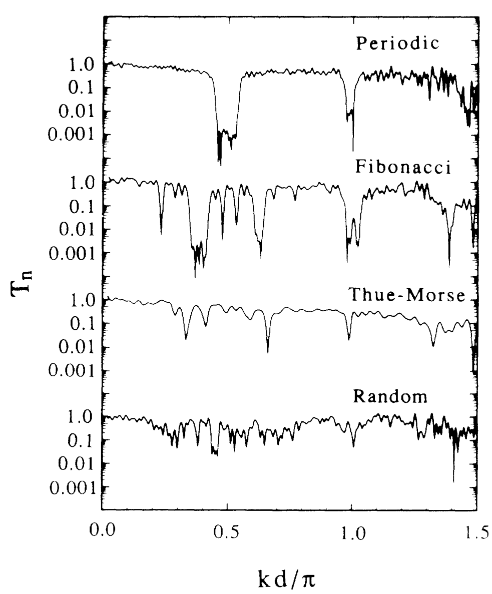

FIG. 1. Transmission spectra of third sound in the periodic, Fibonacci, Thue-Morse, and random lattices.

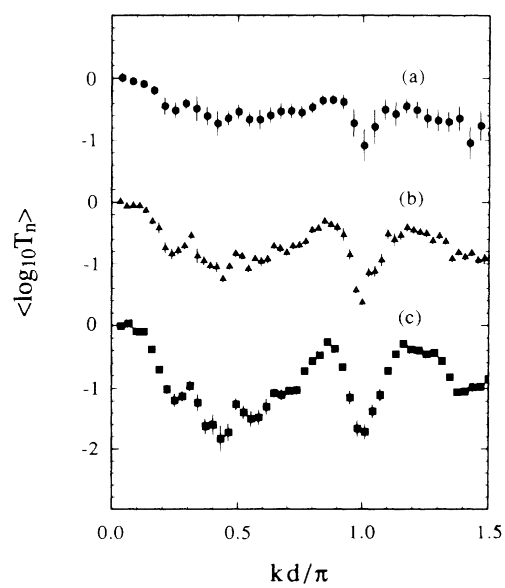

FIG. 2. Averaged transmission spectra of third sound in the random lattices. Traces a-c correspond to different amounts of adsorbed helium, which are given in Table I.

hence from Eq. (1) the localization length ξ was about $0.4L_s$. Therefore, here third sound was effectively localized in the random lattice. On the other hand, in the region where the transmission coefficient becomes close to 1, the localization length can be much longer than the sample size. Under such a condition a quantitative comparison between experiment and theory is difficult, since most theories were developed under the assumption of an infinite sample length. Nevertheless, features (1) and (2) are consistent with the theoretical result of CK [20], namely, $1/\xi \propto -k^2$ for small k.

Feature (4) is to be attributed to the resonant transmission (different from the resonant mode in Refs. [3,4]). When a multiple of the half wavelength of third sound is commensurate with the width d of the scattering center A or B, the scatterer does not contribute to the scattering of third sound. The random lattice becomes transparent in spite of its randomness under this condition [cf. Fig. 3(b) of Ref. [16]]. Figure 2 shows, however, a *splitting* of the resonant transmission. This is due to the fact that the third-sound velocity does depend slightly on substrate A or B. This difference causes there to be two wave numbers, k_A and k_B, according to the substrates A and B, respectively. The wave number k mentioned so far was the average of k_A and k_B, strictly speaking. Thus, the resonant transmission below (above) $kd/\pi \sim 1.0$ corresponds to the condition $k_A d/\pi \sim 1.0$ ($k_B d/\pi \sim 1.0$).

The CK theory [16,20] predicted the renormalization of the third-sound phase velocity. From the phase analysis of the Fourier transform of the pulse wave form, the wave-number dependence of the third-sound phase velocity was obtained experimentally as shown in Fig. 3. As a reference, the phase velocity of third sound in the periodic lattice is indicated by the lines in Fig. 3. Here the drop of the phase velocity at $kd/\pi \sim 0$ is due to the baseline distortion of the pulse wave forms and the bending toward $kd/\pi \sim 0.5$ is due to the resonancelike nature of the transmission gap. But otherwise the phase velocity for the periodic lattice stays constant. In contrast to the periodic lattice, the phase velocity of third sound in the random lattices decreases gradually as the wave number decreases. This fact is thought to be evidence for the phase-velocity renormalization effect in random lattices. The tendency observed in Fig. 3 agrees qualitatively with the CK theory [cf. Fig. 3(a) of Ref. [16]].

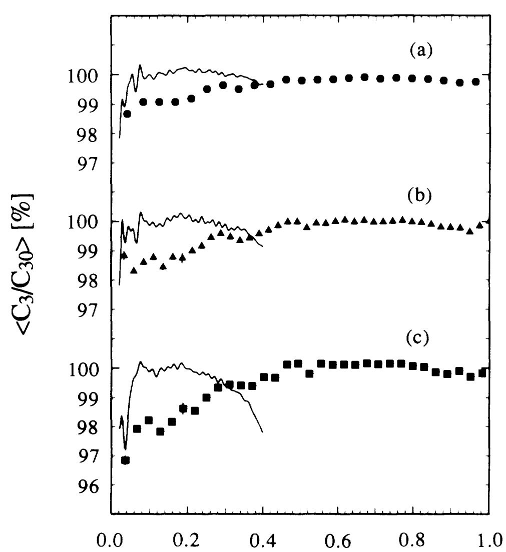

FIG. 3. Relative change of the third-sound phase velocity against wave number. Lines are for the periodic lattice and symbols are for the random lattice, for which the same averaging procedure as that of Fig. 2 was employed. Conditions for a–c are the same as in Fig. 2.

In Table I, we summarize the parameters which are relevant to the evaluation of the formula in the theory, $N = k_A/k_B$ and $r = h_A/h_B$. Here h_A and h_B are the helium film thicknesses on A and B, respectively. These values were obtained from the width of the transmission gaps in the periodic lattice and the time-of-flight data. The scatterer density is $1/2d$ for the present experiment. According to Ref. [16], the renormalization of the third-sound velocity is expressed by the following formula in the long-wavelength limit:

$$\frac{C_3}{C_{30}} \to 1 - \frac{1}{4r}(r^2N^2 - 2r + 1)\,, \tag{2}$$

where C_3 and C_{30} are the renormalized and bare third-sound velocity, respectively. Equation (2) gives the renormalization of the third-sound velocity of about -20% for case c of Table I, while the experimental result is about -3% as can be seen from Fig. 3, trace c. This quantitative disagreement is not surprising, however, partly because the localization length is longer than the sample size in the long-wavelength limit, and partly because the theory was based on the lowest-order perturbation. As mentioned in Ref. [16], if the contribution from the higher-order diagrams is taken into account, the tendency is to remove the discrepancy.

TABLE I. The parameters N and r (defined in the text), which are necessary for the evaluation of Eq. (2). The parameters were obtained from the gap widths in the periodic lattice and the time-of-flight data. In the present cell, 1×10^{-4} mole of helium formed a roughly 1-atomic-layer-thick film on a glass substrate. Trace labels a–c correspond to those in Figs. 2 and 3.

Trace	Amount of He fed into the cell (10^{-4} mole)	N	r	Gap width (kd/π)
a	2.67	1.16	1.10	0.121
b	3.15	1.22	1.15	0.167
c	3.63	1.28	1.20	0.210

In conclusion, we have studied the transmission properties of third sound in the random lattices. The wavenumber dependence of the averaged transmissivity agreed qualitatively with the theoretical prediction of Condat and Kirkpatrick. In particular, the resonant transmission and the phase-velocity renormalization of third sound were observed.

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