Identical Bands and the Varieties of Rotational Behavior

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(Received 17 April 1992)

It is shown that identical rotational energy spacings in pairs of even-even nuclei do not set these nuclei qualitatively apart from others. Rather, they represent the terminus or limiting case of a continuous range of spacings whose behavior is controlled by the balance between residual p-n and pairing interactions and whose phenomenology can be simply described.

PACS numbers: 21.10.Re, 21.60.Cs, 27.70.+q

The recent discovery [1] of rotational bands in adjacent nuclei in which transition energies are nearly identical has caused intense interest and excitement. First noted [1] in high spin superdeformed states of adjacent even-even and odd mass nuclei, the phenomenon has since been identified, by a group from Argonne, in low spin states in pairs of even-even nuclei as well [2]. Other studies [3] of low spin states of adjacent even and odd mass nuclei in the deformed rare-earth nuclei confirm the frequent occurrence of similar moments of inertia in neighboring nuclei. Still more recently, it has been noted [4] that identical bands at low spin are not limited to regions of stable deformation or nearby nuclei but are widespread, can occur in pairs of even-even nuclides as separated as ^{156}Dy and ^{180}Os, and are found even in regions in which isotopic sequences display phase transitional behavior. Apparently therefore, the identical band phenomenon of nearly identical moments of inertia is quite general. Presumably, it must result from basic features of nuclear shell structure and interactions.

It is the purpose of this Letter to point out a key new aspect of this phenomenon that sheds light on its character and origins. Specifically, we will show that despite the *apparently* extraordinary character of these identical bands, they do *not* at all represent a "singular" phenomenon. Rather, they are the terminus or limiting case of a smooth and continuous gradation or range of behavior. Moreover, it will be shown that the degree of similarity of rotational band structure depends in a simple way on the competition between the residual valence p-n interaction and the like-nucleon pairing interaction.

It is useful to phrase the discussion in terms of a measure of the changes in various quantities across a range of nuclei. We therefore define, for any quantity X,

$$F(X)=|X_2-X_1|/X_2\,, \tag{1}$$

where the subscripts refer to nuclei of mass A_1 and A_2. $F(X)$ is the fractional change in X between nuclei 1 and 2. Thus, the degree of "equality" of rotational spacings is measured in terms of

$$F_J(E_\gamma)=|E_{\gamma_2}(J)-E_{\gamma_1}(J)|/E_{\gamma_2}(J)\,, \tag{2}$$

where $E_{\gamma_i}(J)=E(J)-E(J-2)$ in nucleus i. The criterion for nearly identical bands is that the fractional change in $E_\gamma(J)$ is much less than that in some standard measure of expected change such as $A^{5/3}$: In practice, this corresponds to $F_J(E_\gamma)\ll 1$.

To address the issue of this Letter, we want to investigate whether identical bands are *qualitatively* different than other pairs of bands or whether bands of virtually any arbitrary degree of similarity or dissimilarity can be found. To do so, we investigate $F_J(E_\gamma)$ values in a deformed mass region (Dy-Os, $N\geq 88$) which is characterized by varying behavior but yet is also one in which extensive sets of identical bands have been found [4]. To assess the behavior of $F_J(E_\gamma)$ values in such a region, it is useful to compare them to changes in some physical variable that is likely to be correlated with rotational energies. One obvious candidate is the deformation which is traditionally related to the moment of inertia in the standard Bohr-Mottelson model [5]. However, it is also well known [5], especially from studies of high spin states, that moments of inertia depend on pairing. Fortunately, both these quantities can be extracted empirically for many nuclei—the quadrupole deformation ε from measured $B(E2)$ values and the pairing Δ from odd-even mass differences. In the analysis below we take $\varepsilon=0.95\beta$ and obtain β from

$$B(E2;0_1^+\rightarrow 2_1^+)=\frac{5}{16\pi}e^2Q_0^2\,, \tag{3}$$

where $Q_0=(3/\sqrt{5\pi})ZR_0^2\beta(1+0.16\beta)$. The $B(E2)$ values are taken from the recent compilation of Raman *et al.* [6]. The pairing Δ is obtained from the standard binding energy difference equation linking four neighboring nuclei as discussed in Ref. [5].

For $^{156\text{-}170}$Er and for pairs of identical-band nuclei identified in Ref. [4] for which ε and Δ can be obtained with good accuracy, Fig. 1 shows $F_4(E_\gamma)$ values plotted against $F(\varepsilon)$ and $F(\Delta)$. Although there are some regularities in the $F(\varepsilon)$ plot, as might be expected, it is evident that neither plot gives a smooth characterization of the behavior of rotational energies. However, the importance of the competition between deformation and pairing suggests that a quantity embodying *both* ε and Δ might be a better parameter. This is in fact the case as is vividly demonstrated in the bottom right panel of Fig. 1 which plots $F(E_\gamma)$ values for the same nuclei against $F(\varepsilon/\Delta)$.

Now the data suddenly collapse into a single, smooth envelope. Even more noteworthy is the fact that this curve *smoothly joins onto the data* for the identical bands. Pairs of nuclei for which the fractional changes in deformation and pairing are identical, giving $F(\varepsilon/\Delta)=0$, show identical bands while, for other pairs, the fractional difference in transition energies grows with changes in the ratio of ε to Δ. This suggests that identical bands are *not* isolated occurrences that are somehow qualitatively different from rotational behavior elsewhere. There is no "gap" in the plot: Rather, one can find pairs of nuclei, with essentially any chosen fractional difference in transition energies [any $F(E_\gamma)$]. Identical bands are merely the limiting case of a continuous gradation of band structures ranging from extremely dissimilar [$F_4(E_\gamma)\gtrsim 1$] to the identical bands themselves. We note that results similar to Fig. 1 (and Fig. 2—see below) apply to other $F_J(E_\gamma)$ values in the range $J=6^+\text{-}14^+$. It is also worth

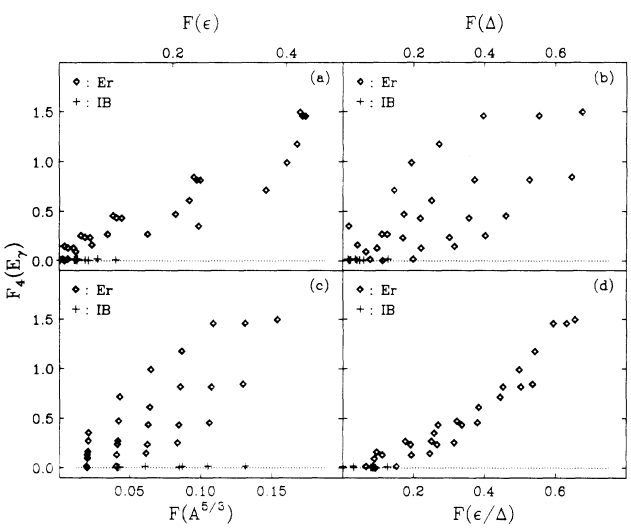

FIG. 1. Plots of $F_4(E_\gamma)$ against $F(\varepsilon)$, $F(\Delta)$, $F(A^{5/3})$, and $F(\varepsilon/\Delta)$ for pairs of Er isotopes and for pairs of identical-band nuclei (IB) discussed in Ref. [4]. Results are shown for nuclei for which the uncertainties in ε and Δ are $\leq 7\%$. (Generally the errors are much smaller than this maximum value.) This eliminates ^{170}Hf and ^{172}W since the errors in Δ are $>7\%$ and ^{180}Os for which the $B(E2)$ value is not known. The data here and in Fig. 2 are based on Refs. [6–8].

noting that the correlation with ε/Δ in Fig. 1, as well as the even better correlation with the P factor to be discussed below, is not simply a reflection that nuclei further apart in N, Z, and A differ more than closer lying pairs. Of course, globally, this is true, but Fig. 1(c) (lower left) shows that $F(E_\gamma)$ is not in fact well correlated with a simple model dependence, $A^{5/3}$, of the moment of inertia. Indeed, as noted in Ref. [4] and seen in Fig. 1(c), nuclei with identical bands can have large $F(A^{5/3})$ values. In passing, we also remark that Fig. 1(d) [or 2(a) below] shows no obvious need to incorporate other deformed degrees of freedom such as ε_4 or γ.

To see if these results are more general, we now inspect a wider ensemble of nuclei. The top part of Fig. 2 shows $F_4(E_\gamma)$ against $F(\varepsilon/\Delta)$ for pairs of isotopes in the Dy, Er, Yb, Hf, W, and Os chains and for identical band pairs of nuclei. Although there is naturally more scatter with this large group of nuclei, the data still show a clear correlation with changes in ε/Δ.

Figures 1 and 2, and the physical content of ε/Δ, suggest an even simpler and more revealing way of studying this phenomenon. It has been shown [9] that the develop-

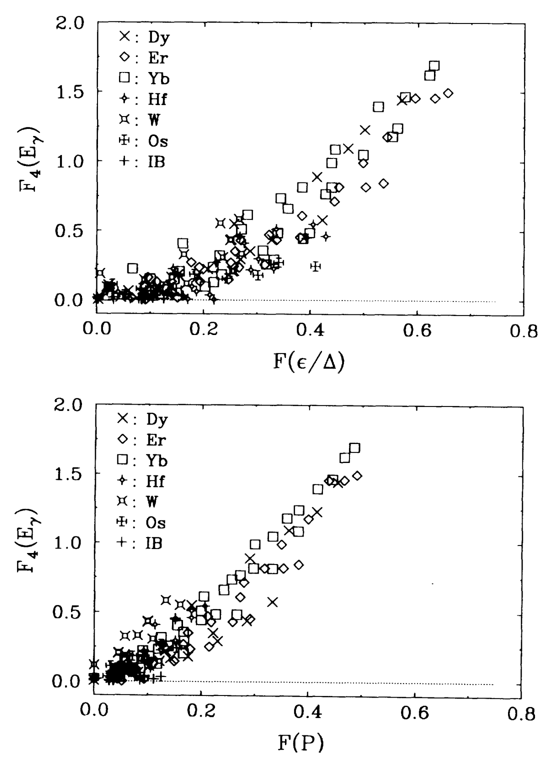

FIG. 2. Plots of $F_4(E_\gamma)$ against $F(\varepsilon/\Delta)$ (top) and $F(P)$ (bottom) for pairs of isotopes of the elements Dy, Er, Yb, Hf, W, and Os, and for pairs of identical-band nuclei (IB) identified in Ref. [4], for which $F(\varepsilon/\Delta)$ can be obtained. Note that, to show a broader spectrum of nuclei, some Δ values used in the upper plot are based on masses estimated from systematics in Ref. [7]. This does not distort the trends as these points fall within the same envelope as those involving only measured values.

ment of collectivity and deformation in medium and heavy nuclei is very smoothly parametrized by the P factor, defined as

$$P=N_pN_n/(N_p+N_n) \tag{4}$$

in terms of the numbers of valence protons (N_p) and neutrons (N_n). P can be viewed as the ratio of the number of valence p-n residual interactions to the number of valence like-nucleon-pairing interactions or, if the p-n and pairing interactions are orbit independent (a rough but pedagogically useful approximation), then P is proportional to the ratio of the integrated p-n interaction strength to the integrated pairing interaction strength. Observables such as $E(4_1^+)/E(2_1^+)$ or $B(E2;0_1^+\rightarrow 2_1^+)$ that are associated with the mean field vary smoothly with P. P is conceptually linked to ε/Δ, and embodies, in a simple formulation, a measure of the balance of competing degrees of freedom that mimics the phenomenological behavior of nuclei. P also has a practical advantage over ε/Δ in that it is trivial to obtain for a given nucleus simply by counting the number of valence nucleons relative to the nearest shell or major subshell closure.

In Fig. 2 we show a plot of $F_4(E_\gamma)$ against P for the same isotopic sequences and identical band nuclei as for the $F(\varepsilon/\Delta)$ plot. The results show an even tighter envelope in which $F_4(E_\gamma)$ values for nuclei with unequal rotational spacings lead asympotically into those for identical bands. Indeed, the fractional change in rotational energy spacings almost scales linearly with fractional changes in P. Thus, both plots in Fig. 2, but especially the $F(P)$ plot, demonstrate, for this wider group of nuclei as well, that identical bands are part of a continuous range of phenomena and, moreover, that the controlling factor is the interplay of p-n and pairing interactions. This confirms and generalizes our previous conclusions. The fact that the P plot is actually a better correlation than ε/Δ has an appealing practical aspect since, empirically, P is available for more nuclei than ε/Δ. Moreover, since P is obtained simply by counting valence nucleons, it is trivial to make predictions of $F_J(E_\gamma)$ for unknown nuclei. A final point worth mentioning is that we have shown results here that compare identical-band nuclei with isotopic sequences of other nuclei. Other "cuts," such as isotonic sequences, can also be studied. Interestingly, the same conclusions are reaffirmed. A plot of $F(E_\gamma)$ vs $F(P)$ is again a compact envelope leading asymptotically to the identical-band nuclei at the origin. Of course, the slopes will be different from those in Fig. 2 since the evolution, or rate of change, of collectivity is not isotropic in the N-Z plane.

To summarize, the identical-band phenomenon has been shown to be part of a smooth range of band structures, from virtually identical to widely different. The identical transition energies observed in pairs of even-even nuclei, reflecting equal moments of inertia, do *not* single out pairs of nuclei that are somehow set apart or qualitatively different from others, but rather pairs that represent the terminus of a range of degrees of diversity. The critical physical concept controlling the diversity of structures is the ratio of residual valence p-n to pairing interactions. In pairs of nuclei with identical bands, changes in one of these are balanced by changes in the other. Both for nuclei with identical bands and for those where the moments of inertia differ, there is a clear correlation, with few exceptions, between changes in rotational spacings and changes in P. Interpretations of identical bands in odd-mass even-mass pairs of nuclei have often involved [10,11] arguments based on symmetry considerations, such as pseudospin or supersymmetry, that focus on the coupling of the odd nucleon to the even-even core. The equal moments of inertia in even-even pairs may also reflect an underlying symmetry and the present results could then be interpreted in terms of a continuous range of symmetry breaking. In any case, the appearance of identical bands is thus not an isolated exception to the normal behavior of nuclei. This in no way, however, diminishes their interest for they are now seen to manifest the extreme limit of a phenomenology that reveals the unity and continuity of collective rotational behavior in heavy nuclei, and its relation to underlying residual interactions.

This work was supported by the U.S. Department of Energy under Contracts No. DE-AC02-76CH00016 and No. DE-FG02-88ER40417, by Contract No. 06OK143 with the BMFT, by NATO Contract No. RG436, by NSF Contract No. INT-9001476, and by Contract No. 62-91-37 with the IAP (Bucharest).

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