Large Range of Validity of Linear Elasticity of the Vortex Lattice in High-T_c Superconductors

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Linear elasticity theory of the vortex lattice in anisotropic and layered superconductors has a much wider range of applicability than in isotropic superconductors. This finding is used to calculate the energy barrier for thermally activated depinning of vortices from columnar pins along the c axis. The activation energy of these most effective line pins is strongly reduced by anisotropy. For very large anisotropy as in $Bi_2Sr_2CaCu_2O_8$, the length of the depinning vortex segment formally becomes shorter than the layer spacing; this means that the pancake vortices in the CuO layers depin individually.

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The linear theory of elasticity of the lattice of Abrikosov vortices in type-II superconductors has proven highly useful for calculations of, e.g., vortex pinning at temperature $T = 0$ (collective pinning [1]) and at $T > 0$ (collective creep [2]) and of thermal fluctuations of the vortex lattice [3–5]. This is so since the direct calculation of the energy of a distorted vortex lattice from Ginzburg-Landau (GL) theory or from the anisotropic London theory or Lawrence-Doniach (LD) theory [6], more appropriate for high-T_c superconductors (HTSC), would be a formidable task. In general, the condition for linear elasticity to apply is that the strains (i.e., the gradients of the vortex displacements) are small compared to unity. For example, the vortex spacing a_0 should vary little, $\delta a_0/a_0 \ll 1$, and the tilt angle θ away from the equilibrium vortex direction should be small, $|\theta| \ll 1$; otherwise the energy is no longer quadratic in δa_0 or θ.

Surprisingly and much welcome, in anisotropic and layered superconductors with magnetic field B along the crystalline c axis, linear elasticity has a *much larger range of validity*. For example, the elastic energy U_{tilt} of a vortex tilted by an angle θ away from $\mathbf{B}\|\mathbf{c}\|\mathbf{z}$ is proportional to $(\tan\theta)^2$ as long as $|\tan\theta| \ll \Gamma$; this proportionality holds approximately even when $|\tan\theta| \approx \Gamma$. Here $\Gamma = \lambda_c/\lambda_{ab} = \xi_{ab}/\xi_c = (M/m)^{1/2}$ is the anisotropy ratio, $\Gamma \approx 5$ for $YBa_2Cu_3O_7$ (YBCO), and $\Gamma \approx 60$ for $Bi_2Sr_2CaCu_2O_8$ (BSCCO); λ_c, λ_{ab} are the penetration depths for currents along c and in the a-b plane, respectively, and ξ_{ab}, ξ_c are the coherence lengths. Introducing displacements $\mathbf{s}(z) = [x(z);\ y(z);\ 0]$ from the equilibrium vortex position $(0;\ 0;\ z)$, one has $|\tan\theta| = |d\mathbf{s}/dz|$ and thus $U_{\text{tilt}} \propto (d\mathbf{s}/dz)^2$ for $|d\mathbf{s}/dz| \le \Gamma$. This surprising result applies both to *isolated* vortices and to the vortex *lattice*.

In this Letter I will demonstrate this simplifying fact and use it to calculate the activation energy required to depin a vortex trapped by a columnar pin. It turns out that (a) when driven by a current the vortex forms a parabola rather than a circle (which would follow for isotropic line tension); (b) at small currents the depinning nucleus is a steep-sided triangle or trapezoid; (c) the depinning barrier becomes very small for large anisotropy; and (d) without a current a vortex along c does *not* exhibit helical instability.

The elastic energy of the vortex lattice was derived from isotropic GL theory in [7], from microscopic BCS theory in [1], from anisotropic GL theory in [4], and from anisotropic London theory in [8–10]. As a fortunate fact it turns out that the LD theory for layered superconductors yields essentially the same linear elastic energy as the GL and London theories. This is so since the nonlinear term depending on the (gauge invariant) phases $\phi_n(x,y)$ in the LD energy can be linearized, $2-2\cos(\phi_{n+1}-\phi_n) \approx (\phi_{n+1}-\phi_n)^2 \approx d^2(\partial\phi/\partial z)^2$, if the points where the vortex cuts through adjacent planes (superconducting CuO planes of distance $d \ll \lambda_{ab}$) have a distance $\ll \lambda_J$ where $\lambda_J = \Gamma d$ is the Josephson length. This condition means $|\tan\theta| \ll \Gamma$, which coincides with the condition for linearization that will be proven now.

Consider first an isolated vortex limited by planes z and $z + dz$. Tilting this vortex away from $\mathbf{z} \parallel \mathbf{c}$ by an angle θ increases its length to $dl = dz/\cos\theta$. In an isotropic superconductor with vortex self-energy $J = \Phi_0 B_{c1}/\mu_0$ ($\Phi_0 = 2.07 \times 10^{-15}\,\text{T m}^2$ is the quantum of flux, $B_{c1} = \Phi_0[\ln\kappa + 0.5]/(4\pi\lambda^2)$ the lower critical field, and λ the penetration depth) this tilt increases the vortex energy by $dU_{\text{tilt}} = J(dl - dz) = J(1/\cos\theta - 1)dz \approx (J\theta^2/2)dz$ for $\theta^2 \ll 1$. This means J is both line *energy* and line *tension*. In anisotropic superconductors, however, the self-energy $J(\theta)$ depends explicitly on θ. London theory gives "to logarithmic accuracy" (explained below)

$$J(\theta) = J(0)(\cos^2\theta + \Gamma^{-2}\sin^2\theta)^{1/2}. \tag{1}$$

The tilt energy for small θ then becomes $dU_{\text{tilt}} = (J + \partial^2 J/\partial\theta^2)(\theta^2/2)dz$ [8]. Here I have used the equilibrium condition $\partial J/\partial\theta = 0$, which applies for $\theta = 0$ and $\theta = \pi/2$. This argument shows that the line tension in general is $P(\theta) = J(\theta) + \partial^2 J(\theta)/\partial\theta^2$. With (1) this gives $P(0) = J(0)/\Gamma^2$ for vortices along c and $P(\pi/2) = J(0)\Gamma = J(\pi/2)\Gamma^2$ for vortices in the a-b plane tilted out of the a-b plane. For tilt *within* the a-b plane one has $P = J = J(\pi/2)$, see [9] for more details. In this paper only the case $\mathbf{B}\|\mathbf{c}$ will be considered.

For arbitrarily large θ one gets with (1) and $dl = dz/\cos\theta$, $dU_{\text{tilt}} = J(\theta)dl = J(0)(1+\Gamma^{-2}\tan^2\theta)^{1/2}$. With $\tan^2\theta = (\partial\mathbf{s}/\partial z)^2 = s'(z)^2 = x'(z)^2 + y'(z)^2$ one obtains the energy of an arbitrarily curved vortex

$$U_v = J(0)\int [1 + \Gamma^{-2}s'(z)^2]^{1/2}\,dz \tag{2}$$

and the tilt energy $U_{\text{tilt}} = U_v\{s(z)\} - U_v\{0\}$,

$$U_{\text{tilt}} = \frac{P}{2}\int\left[s'^2 - \frac{s'^4}{4\Gamma^2} + \frac{s'^6}{8\Gamma^4} - + \cdots\right]dz\,, \tag{3}$$

where $P = P(0) = J(0)/\Gamma^2$ is the line tension of a vortex along c. Equation (3) shows that the line-tension picture [first term in (3)] is an excellent approximation if $|s'| \le \Gamma$.

Next I extend this picture to the vortex *lattice*. The tilt modulus of the vortex lattice with $\mathbf{B}\|\mathbf{c}$ is from anisotropic London theory [8,9]

$$c_{44}(\mathbf{k}) = \frac{B^2}{\mu_0}\left(\frac{1}{1 + k_z^2\lambda_{ab}^2 + k_\perp^2\lambda_c^2} + \frac{\Phi_0 \ln\tilde{\kappa}}{4\pi\lambda_c^2 B}\right)\,, \tag{4}$$

where $\mathbf{k} = (\mathbf{k}_\perp, k_z)$ is the wave vector of the displacement field $\mathbf{s}(\mathbf{r})$ and $\tilde{\kappa}^2 = (\Gamma^2\kappa^2 + k_z^2\lambda_{ab}^2)/(1 + k_z^2\lambda_{ab}^2)$, $\kappa = \lambda_{ab}/\xi_{ab}$; thus $\tilde{\kappa} \approx \Gamma\kappa$ for $k_z \ll \lambda_{ab}^{-1}$ (nearly straight vortices) and $\tilde{\kappa} \approx 1 + 1/k_z\xi_c$ for $k_z \gg \lambda_{ab}^{-1}$ (strongly curved vortices). In the following we shall always have $k_z \gg \lambda_{ab}^{-1}$, to be confirmed self-consistently. For $k_z \gg \lambda_{ab}^{-1}$ the compressional modulus [8] becomes $c_{11}(\mathbf{k}) \approx \Gamma^2$ times the first term in c_{44}, Eq. (4). The elastic energy per unit length of a vortex then becomes independent of $k_\perp$,

$$\tfrac{1}{2}\langle s^2\rangle[k_\perp^2 c_{11}(\mathbf{k}) + k_z^2 c_{44}(\mathbf{k})]\Phi_0/B$$
$$= \tfrac{1}{2}\langle s^2\rangle B\Phi_0/\mu_0\lambda_{ab}^2 + \tfrac{1}{2}\langle s'^2\rangle P(k_z) \tag{5}$$

with line tension

$$P(k_z) = (\Phi_0^2/4\pi\mu_0\lambda_c^2)\ln\tilde{\kappa}(k_z) \tag{6}$$

($\langle\cdots\rangle$ is the spatial average). The small shear energy is disregarded in (5); its maximum value, reached when only the one vortex is displaced [11], is $\approx \frac{1}{8}$ of the compressional energy, namely, $\frac{1}{2}\langle s^2\rangle\langle k_\perp^2\rangle c_{66}\Phi_0/B$ with $\langle k_\perp^2\rangle = k_{\text{BZ}}^2/2 = 2\pi B/\Phi_0$ and shear modulus $c_{66} = B\Phi_0/16\pi\mu_0\lambda_{ab}^2$. The result (5) has a simple physical interpretation.

The first term means that each vortex sits in a parabolic potential with curvature $B\Phi_0/\mu_0\lambda_{ab}^2$ originating from all other vortices within a range of several λ_{ab}. Only the average density of the neighbors enters but not their precise position (since $k_\perp$ does not enter). This linear restoring force applies if $s \ll \lambda_{ab}$. Since the vortex spacing is typically $a_0 \ll \lambda_{ab}$ this means that even displacements larger than a_0 are allowed provided the vortex core does not come too close to another vortex; but even then the energy does not diverge since the effective interaction potential between vortices is smooth [7,12] and vortices easily cross or cut each other [13].

The second term, originating from the interaction of the vortex with itself, means that for sufficiently short bending wavelengths the vortex in the lattice has the same line tension $P(k_z)$ (6) as an isolated vortex. This line tension has only weak logarithmic dispersion and is strongly reduced by anisotropy. Note that the (approximate) logarithmic factor in P, $\ln\tilde{\kappa} \approx \ln(1/k_z\xi_c)$ does not depend on the penetration depth since $k_z \gg \lambda_{ab}^{-1}$. For completeness I note that when $\Gamma \gg 1$ and $k_z < \lambda_{ab}^{-1}\Gamma/10$, the last term in (4) and the line tension P in (3) have to be replaced by the larger "pancake-stack" tilt energy of the limit $\Gamma \to \infty$.

Thus, sufficiently strongly curved vortices in a vortex lattice *behave like isolated vortices elastically bound to the other vortices*. In the following the binding term will be disregarded since it is typically much smaller than the pinning forces. In a sense, Equation (5) justifies the energy ansatz of Nelson [3] if the correct line tension (6) is used and sufficiently strong (not weak as in [3]) curvature with $k_z \gg \lambda_{ab}^{-1}$ is considered. Both energy terms in (5) apply to much larger displacements $s \ll \lambda_{ab}$ and tilt angles $s' \le \Gamma$ than naive linearity arguments would suggest.

The tilt energy (3) with $P(k_z)$ from (6) inserted is a very general and useful expression if one puts $k_z \approx L^{-1}$ in the logarithm, where L is a typical length of the problem under consideration. For example, consider a flux line rigidly pinned at its ends $z = \pm L$ and deformed by a current density $\mathbf{J} \perp \mathbf{z}$, Fig. 1(a). Minimizing the energy

$$U_J + U_{\text{tilt}} = \int\left[-\Phi_0 J s(z) + \frac{P}{2}s'(z)^2\right]dz, \tag{7}$$

one obtains $s''(z) = -2q = \text{const}$, $q = \Phi_0 J/2P$. This means the vortex "inflates" into a parabola $s(z) = h - qz^2$ with height $h = qL^2$ and maximum slope $s'(z=-L) = 2qL$ which should be $\le \Gamma$. The energy gain is $U_J + U_{\text{tilt}} = \frac{1}{2}U_J = -U_{\text{tilt}} = -\frac{1}{2}\Phi_0 JA$ where $A = \frac{4}{3}Lh = \frac{4}{3}qL^3$ is the parabolic area.

A less trivial example of high relevance for technical application of HTSC is pinning of vortices by long cylindrical defects oriented along $\mathbf{B}\|\mathbf{c}$. Such linear defects were generated in YBCO [14,15] and BSCCO [16,17] by irradiation with heavy Sn or I ions of ≈ 500 MeV energy. It was found that the line pins increase the activation energy U for thermal depinning in YBCO but not in BSCCO, since in YBCO (with $\Gamma \approx 5$) long sections of vortices have to be depinned, whereas in BSCCO (with $\Gamma \approx 60$) single point vortices (pancake vortices in the CuO layers [18]) can easily depin individually [17,19].

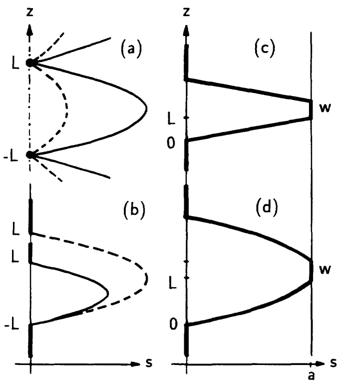

FIG. 1. (a) A vortex line along $\mathbf{z}\|\mathbf{c}$ pinned by a chain of ideal point pins with separation $2L$. A transport current perpendicular to the s-z plane deforms each vortex segment into an ellipse which to a very good approximation coincides with a parabola $s = h - qz^2$. The dashed line is for smaller current. For $J > J_c^{\max}$ neighboring ellipses touch and the vortex depins. (b),(c) Pinning of vortices (bold lines) by line pins (thin lines) along $\mathbf{c}\|\mathbf{z}\|\mathbf{B}$. Thermal fluctuations partly depin the vortex. Shown are the critical deformations with minimum activation energy, i.e., the nuclei which will grow spontaneously if a current is applied. (b) If neighboring pins are far apart, the size of the parabolic nucleus $L = L_c \propto 1/J$ is determined by the current density J. (c) In the limit of zero current the appropriate nucleus is a trapezoid with the short roof pinned by a neighboring line pin and with activation energy U_0 (8). (d) A finite current density J deforms the trapezoid as depicted here. This general nucleus has a reduced activation energy $U_{3\text{D}}(J)$ (9).

The size, shape, and activation energy of the depinning nucleus can be calculated by adding to $U_J + U_{\text{tilt}}$ (7) the pinning energy U_p times the length of the depinned vortex section. One has $U_p = \epsilon\Phi_0^2/4\pi\mu_0\lambda_{ab}^2$ with $\epsilon \le 1$; for a cylindrical hole of radius $R \ge \sqrt{2}\xi_{ab}$ (the vortex core radius [20]) one finds $\epsilon \approx 0.5 + \ln(\lambda_{ab}/\xi_{ab}) - \ln(\lambda_{ab}/R) = 0.5 + \ln(R/\xi_{ab}) \approx 1$. Here the term 0.5 originates from the condensation energy and the $\ln(R/\xi_{ab})$ from the different inner cutoff radii for the magnetic energy of pinned and free vortices.

For large current densities J, or large pin separation a, maximization of $U(L) = U_J + U_{\text{tilt}} + 2LU_p = \frac{1}{2}U_J + 2LU_p$ gives a parabolic nucleus as above, with half-width $L = L_c = (2U_pP)^{1/2}/\Phi_0 J$, height $h_c = qL_c^2 = U_p/\Phi_0 J$, curvature $q = \Phi_0 J/2P$ as above, and energy $U_c = U(L_c) = \frac{4}{3}L_cU_p = \frac{4}{3}(2U_p^3P)^{1/2}/\Phi_0 J$, Fig. 1(b). The maximum slope is $s'(z=-L) = p = (2U_p/P)^{1/2} \approx [2\epsilon/\ln(L_c/\xi_c)]^{1/2}\Gamma < \Gamma$ since $L \gg \xi_c = \xi_{ab}/\Gamma$. A nucleus with $L > L_c$ will grow spontaneously, driven by the current. U_c is thus the activation energy for thermal depinning. Note that both L_c and U_c diverge with decreasing current density as $1/J$.

In the limit of zero current density, the fluctuating vortex has to touch a neighboring line pin (at average distance a) to trigger spontaneous depinning. The nucleus is now a trapezoid with sides of slope $p = (2U_p/P)^{1/2}$ as above and basis width $2L + w = 2a/p + w$, Fig. 1(c). The width w of the trapezoidal top follows from the pinning force per unit length, $\approx U_p/R$, since the pinned top has to balance the tension of the trapezoidal sides, $wU_p/R \ge 2pP$; thus $w \ge pR/U_p = 2R/p \ge 2R/\Gamma \ge 2.8\xi_c$ for slope $p \ge \Gamma$ and pin radius $R \ge 1.4\xi_{ab}$. The top width is thus very small and the nucleus is nearly a triangle. An arbitrarily small current inflates the nucleus by increasing the width w. The energy of the nucleus, the activation energy, is $U_0 = U_c(J=0) = 2a(2U_pP)^{1/2} = 4LU_p$ where $L = 2a/p = a(P/2U_p)^{1/2} = a[2\ln(L/\xi_c)/\epsilon]^{1/2}/\Gamma$. The factor $1/\Gamma$ originates from the small line tension and means that for large anisotropy the activation energy U_0 and the effective depinning length $U_0/U_p = 4L$ are very small. Explicitly one has

$$U_0 = [8\epsilon\ln(L/\xi_c)]^{1/2} a\Phi_0^2/(4\pi\mu_0\lambda_{ab}\lambda_c) \,. \qquad (8)$$

If in layered superconductors U_0 (8) becomes smaller than the pinning energy $U_{2\text{D}} = U_p d$ of a pancake vortex, $U_0 \le U_{2\text{D}}$ equivalent to $2L \le d$ where d is the layer spacing, then the elastic energy of the depinning vortex line can be disregarded and the activation energy for depinning is $U_{2\text{D}} = d\epsilon\Phi_0^2/4\pi\mu_0\lambda_{ab}^2$. This means each point vortex can depin individually.

For arbitrary current density J, the situation is as follows. For $J \ge J_0 = U_p/\Phi_0 a$, the parabolic nucleus determines the depinning since its height is $h \le a$, the pin spacing; in this case $U_c(J) = (2J_0/3J)U_0$. For $0 < J \le J_0$, the triangular nucleus is deformed by the current, Fig. 1(d). This "inflation" reduces the activation energy. The sides of this general nucleus have the shape $s(z) = pz - qz^2$ for $s \le a$ with p and q from above. The half-width is $L = [p - (p^2 - 4qa)^{1/2}]/2q$, and the energy is $U_J + U_{\text{tilt}} + 2LU_p$ as above. After some algebra one obtains the general activation energy

$$U(J \le J_0) = \frac{2J_0}{3J}\left[1 - \left(1 - \frac{J}{J_0}\right)^{3/2}\right] U_0,$$
$$U(J \ge J_0) = \frac{2J_0}{3J}\, U_0, \quad \text{with} \quad J_0 = \frac{U_p}{\Phi_0 a}\,. \qquad (9)$$

This activation energy does *not* depend on the pinning *force* (or pin radius R) but only on the pinning *energy* (depth of the well). Quite generally, pinning energy, pinning force, and activation energy depend on different material parameters.

I will show now that for arbitrary pinning potential $V(s)$ of a line pin, with $V(0) = -U_p$ and $V(\infty) = 0$, the pinning force on a partly depinned vortex is balanced by the line tension P, i.e., the vortex slope $s'(z)$ outside the pin has the value $p = (2U_p/P)^{1/2}$ obtained by energy considerations above. Minimizing the energy $\int[Ps'^2/2 + V(s)]dz$ one obtains $s''(z) = V'(s)/P$. Multiplying this by $s'(z)$ and integrating both sides one obtains $(P/2)[s'(z=\infty)]^2 = \int_{-\infty}^{\infty}(\partial V/\partial s)(\partial s/\partial z)dz = \int_0^{\infty} V'(s)ds = U_p$; thus $s'(\infty) = (2U_p/P)^{1/2}$.

Inside a parabolic pinning well the depinning vortex has exponential shape, $s(z) \propto \exp(-z/l)$ with $l = (P/V'')^{1/2}$. For $V'' \approx 2U_p/R^2$ [e.g., if $V(s) = -U_p \exp(-s^2/R^2)$] the pinning length is $l \approx R(P/2U_p)^{1/2} = R/p$. As a result of the large range of linear elasticity these results apply up to large slopes $p \approx \Gamma$.

How do the nonlinear terms in (3) change these results? From (2) one can show that the *exact* vortex shapes in all the above examples are *ellipses* with axes $Z = \Gamma/2q = \Gamma P/J\Phi_0$ and $X = \Gamma Z$, e.g., $s^2/X^2 + z^2/Z^2 = 1$. The parabola above is an excellent approximation to the ellipse since we always have $|s'(z)| < \Gamma$.

If a vortex is ideally pinned at equidistant points with separation $2L$, depinning is triggered when the current inflates the vortex segments into half ellipses. These touch each other at the pins and merge and depin. Such pinning-force independent depinning occurs when $L = Z = \Gamma P/J\Phi_0$. This process defines a maximum critical current density

$$J_c^{\max} = \frac{\Gamma P}{\Phi_0 L} = \frac{\Phi_0 \ln(L/\xi_c)}{4\pi\mu_0\lambda_{ab}\lambda_c L} \, . \tag{10}$$

Exactly the same result is obtained from the linear approximation and the condition $|s'| \leq \Gamma$.

In conclusion, the linear elastic approximation of the vortex lattice has a much larger range of validity in anisotropic superconductors with $\mathbf{B}\|\mathbf{c}$ than is usually expected. Taking advantage of this fact I obtain the current-dependent activation energy $U(J)$ for depinning of vortices from line pins oriented along the c axis and the critical current density $J_c^{\max}$ of a chain of ideal point pins. Both $U(J)$ [9] and $J_c^{\max}$ [10] *decrease* with increasing anisotropy as $1/\lambda_c$. The values of $U(J)$ and $J_c^{\max}$ are given here for equidistant pins; for randomly positioned pins a more complete statistical treatment is required [21]. If the width of the depinning nucleus for $J \to 0$, $2L = [8\ln(L/\xi_c)/\epsilon]^{1/2} a/\Gamma$ (with $\epsilon \leq 1$), becomes smaller than the distance d of the superconducting layers, $2L \leq d$, the activation energy saturates to the small value $U_{2D} = U_p d \geq U_0 \geq U(J)$. This means that in BSCCO with line pins *the point vortices in the CuO layers depin individually* as found also in [17,19].

Finally, I show that the helical instability predicted by Koyama and Tachiki [22] for a vortex along the c axis does *not* exist. For a helix with $\mathbf{s}(z) = (r\cos k_z z; r \sin k_z z)$, elasticity theory (3) yields a *positive* energy per unit length along z, $U_{\rm hel} = \frac{1}{2}P(k_z)r^2k_z^2$ with line tension $P(k_z)$ from (6). Helical deformations of the vortex will thus not form spontaneously. As shown above, this linear elastic result applies up to very flat helices with slope $|d\mathbf{s}/dz| = k_z r \leq \Gamma$. The exact elastic energy of the helix is

$$U_{\rm hel} = P(k_z)\Gamma^2[(1 + r^2k_z^2/\Gamma^2)^{1/2} - 1]. \tag{11}$$

The linear elastic energy density of a helical deformation of the vortex *lattice* is also positive, $\frac{1}{2}c_{44}(0,0,k_z)r^2k_z^2 > 0$ with $c_{44}(\mathbf{k})$ from (4). The flux-line lattice is thus stable, and helical instability will occur only when a longitudinal current is applied [23].

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