Gamow-Teller Strength in the β^+ Decay of ^{37}Ca

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Calculations are presented for the Gamow-Teller strength distribution in the β^+ decay of ^{37}Ca and compared to results extracted from recent beta decay data and from (p,n) data on the analog nucleus ^{37}Cl. The strength distribution is shown to be sensitive to the Hamiltonian, and comparison with experiment indicates a need for further improvement in the $1s0d$ shell Hamiltonian. It also indicates that the quenching of the Gamow-Teller operator in the upper part of the $1s0d$ shell is similar to that deduced from previous analyses.

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Beta decay of hadrons and nuclei has historically played a crucial role in testing strong-interaction models for many-body systems as well as testing the assumptions which go into the standard model of electroweak interactions. Gamow-Teller (GT) beta decay of nuclei is a particularly simple and sensitive test of nuclear structure models. In light nuclei ($A<60$), the GT strength extracted from beta decay is systematically smaller (quenched) relative to that expected from the valence major-oscillator shell model [1-3]. For both light and heavy nuclei, the GT strength summed over low-lying final states (0-15 MeV) inferred from (p,n) reactions is also quenched [4] relative to the model-independent sum rule [5,6] for the GT operator $\lambda\sigma_\mu t_\pm$, which takes into account all of the nucleon degrees of freedom within the nucleus:

$$|B(\mathrm{GT}_-)-B(\mathrm{GT}_+)|\equiv\lambda^2\sum_{\mu,f}|\langle f|\sigma_\mu t_-|i\rangle^2-\langle f|\sigma_\mu t_+|i\rangle^2| =3\lambda^2|N_i-Z_i|\,.$$

In this expression σ is the Pauli spin operator, $t_\pm$ are the isospin raising and lowering operators, i (f) are the initial (final) nuclear wave functions, and $\lambda=g_A/g_V$ is the ratio of the axial-vector to the vector weak-interaction coupling constants for the nucleon beta decay. Sometimes this quenching is referred to as a renormalization of λ relative to its free-nucleon value of $|\lambda|=1.26$ [1]. This quenching is ascribed to nuclear correlations outside of the valence major-oscillator shell and to Δ-isobar admixtures in the nuclear states [7]. Consideration of the experimental results together with related data for nuclear magnetic moments has led to a greatly improved understanding of these terms [7,8]. In a recent Letter [9], it was proposed that the analysis of new ^{37}Ca β^+ decay data called into question the extent to which the weak axial-vector current is renormalized in nuclei. This conclusion was based on the fact that the GT decay strength extracted from the new data was about equal to that obtained from a shell-model calculation with the free-nucleon value for λ. It was also claimed that this result cast some doubt on previous conclusions that the experimental GT strength for nuclei with $A=17$-39 was systematically quenched to only about 60% of that expected from $1s0d$ shell-model calculations [2]. In this Letter, I will show that the quenching extracted from the ^{37}Ca β^+ decay data is more model dependent than most previous analyses of GT data. In particular, I will show that it depends on the shape of the assumed GT strength distribution and that this shape is particularly sensitive to the Hamiltonian chosen for the $1s0d$ shell valence shell-model space. My discussion will also include additional information for the GT strength obtained from (p,n) data [10] for the mirror transitions $^{37}\mathrm{Cl}\to{}^{37}\mathrm{Ar}$. (Based upon calculations with charge symmetry-breaking interactions [11], the mirror symmetry-breaking effects on the GT strengths are found to be not important for the discussion below.)

In Fig. 1 the $B(\mathrm{GT}_+)$ strength extracted from the ^{37}Ca β^+ decay and from the (p,n) data (dashed lines) is compared with $1s0d$ shell-model calculations based upon four different effective Hamiltonians (solid lines). In order to

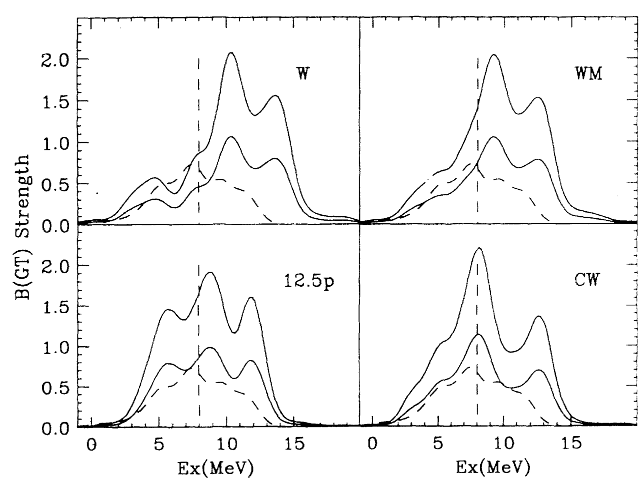

FIG. 1. $B(\mathrm{GT}_+)$ strength distribution for ^{37}Ca. The dashed line is the strength extracted from experiment, and the solid lines correspond to various theoretical calculations (see text for details).

emphasize the qualitative aspects of the comparison, what is shown is the $B(\mathrm{GT}_+)$ strength versus excitation energy averaged over a Gaussian distribution with a FWHM of 2 MeV. The area under each curve is equal to the total $B(\mathrm{GT}_+)$ strength. The experimental data up to about 8 MeV (the vertical dashed line in Fig. 1) are from the ^{37}Ca β^+ decay (Table I of Ref. [12]). Above 8 MeV the $B(\mathrm{GT})$ for states at 9.65 and 11.5 MeV inferred from the (p,n) reaction on the mirror nucleus ^{37}Cl (Table I of Ref. [10]) is also included.

Two solid lines are given for each theoretical calculation in Fig. 1. The upper lines are based upon the free-nucleon value of $|\lambda| = 1.26$ and the areas under these upper lines are equal to the sum-rule value of $3\lambda^2|N_i - Z_i| = 14.1$ [$B(\mathrm{GT}_-) = 0$ for ^{37}Ca in the $1s0d$ model space]. These results will be referred to as the "free-nucleon" calculations. The lower solid lines are based upon the state- and mass-dependent effective GT operator of Ref. [2]. For $A = 37$ this effective operator is within a few percent the same as using a value of $|\lambda| = 0.90$, and the areas under all four of the lower lines are about 7.3. These results will be referred to as the "quenched" calculations. A different effective Hamiltonian was used for each of the four comparisons made in Fig. 1. In order to understand the interpretation of the ^{37}Ca decay data, I will summarize the historical background of these Hamiltonians.

The calculation in the bottom-left part of Fig. 1 is based upon a microscopic G matrix interaction plus core-polarization corrections (the column labeled $12.5p$ in Table I of Ref. [13]). The calculation in the bottom-right part of Fig. 1 is based upon the Chung-Wildenthal (CW) "hole" Hamiltonian [14]. The CW Hamiltonian was obtained from a least-squares fit of binding-energy data for nuclei in the $A = 32$–39 region with the least well-determined linear combinations of two-body matrix elements being kept at the $12.5p$ values. In both cases the single-hole energies are chosen to give excitation energies of 2.50 MeV ($1s_{1/2}$) and 6.12 MeV ($0d_{5/2}$) relative to the $0d_{3/2}$ ground state of $A = 39$.

The calculation shown at the top-left part of Fig. 1 is based upon Wildenthal's (W) Hamiltonian [15]. The W Hamiltonian started with the Chung-Wildenthal "particle" ($A = 17$–24) and "hole" ($A = 32$–39) Hamiltonians and made further adjustments so that 447 binding-energy data across the entire $1s0d$ shell ($A = 17$–39) were reproduced with an rms deviation of 185 keV [15,16]. 47 linear combinations of the 66 Hamiltonian parameters were relatively well determined by these data [15,16]. One additional feature of this interaction was the introduction of a smooth mass dependence to the two-body matrix elements close to that expected from G matrix interactions [15,16]. The goal and achievement was to obtain a universal one- and two-body Hamiltonian for the entire $1s0d$ shell which would reproduce experimental data for binding energies, spectroscopic factors, electromagnetic transitions, beta decay, and electron scattering form factors [15]. With the W Hamiltonian, the $A = 39$ single-hole states come at 2.73 MeV ($1s_{1/2}$) and 7.42 MeV ($0d_{5/2}$). The $0d_{5/2}$ single-hole energy is significantly higher for the W Hamiltonian than for the CW Hamiltonian. The W Hamiltonian was constrained to reproduce a specific value for the energy of the $A = 39$ $0d_{5/2}$ hole state; however, it turns out that the energy obtained with the W Hamiltonian is in good agreement with the centroid of the strength observed in one-nucleon pick-up from ^{40}Ca [17]. The spectrum labeled WM on the top-right part of Fig. 1 was obtained from the W interaction but with the single-particle energies adjusted to give the same single-hole energies as CW.

Now I discuss the implications of these comparisons. The claim of "no quenching" by Adelberger *et al.* [9] is based upon the fact that the areas below 8 MeV under the experimental (dashed line) and the free-nucleon W calculation (upper solid line) in the top-left part of Fig. 1 are about equal to each other. However, it is apparent that only a small part (20%) of the theoretical strength lies below 8 MeV and that the comparison is thus very sensitive to what one assumes for the shape of the strength distribution. If one takes the ^{37}Cl(p,n) data as an indication of the shape of the remaining strength, it is clear that the shape of the total data is much closer to the $12.5p$ or CW calculations than to the W calculation. When compared to the $12.5p$ or CW calculations, the interpretation of the data below 8 MeV from the beta decay is that it is consistent with the effective (quenched) GT operator (lower lines).

Furthermore, it is apparent that the absolute strength obtained from the (p,n) data above 8 MeV is about a factor of 2 smaller than the quenched calculation. One aspect of the nonproportionality between the GT strength extracted from beta decay, $B(\mathrm{GT})_\beta$, and that from (p,n) experiments, $B(\mathrm{GT})_{pn}$, has previously been noted [18]. In particular, the ratio $B(\mathrm{GT})_{pn}/B(\mathrm{GT})_\beta$ for transitions between "jackknife" configurations ($p_{1/2} \rightarrow p_{1/2}$ and $d_{3/2} \rightarrow d_{3/2}$, in particular) was found to be systematically larger than that between "spin-flip" transitions ($0d_{3/2} \rightarrow 0d_{5/2}$ in this case). The implication of this is that if one calibrates the (p,n) reaction to low-lying transitions with a jackknife structure, the strength extracted for the high-lying spin-flip transitions is too small. A recalibration of the old ^{37}Cl$(p,n)^{37}$Ar data and new higher-resolution data would be important for testing this hypothesis. It would also be important to measure the GT strength above 12 MeV in excitation in ^{37}Ar. The proportionality between the GT strength extracted from (p,n) reactions and beta decay must eventually break down for transitions which are weak (a few percent or less) relative to the sum-rule value (transitions to the lowest few $A = 37$, $T = \frac{1}{2}$ levels in this case). There are, of course, additional experimental problems and uncertainties in subtracting the Fermi strength in transitions to analog states (the 5.05-MeV final state in this case).

What one thus learns from these experiments is that the older Hamiltonians used for the upper $1s0d$ shell ($12.5p$ and CW) are in better agreement with the GT distribution than the newer W Hamiltonian. A similar conclusion has been reached previously on the basis of the Ar β^+ decay data [19]. Comparison of W and WM indicates that the difference is partly but not entirely related to the position of the $0d_{5/2}$ single-hole state. The failure of the W interaction to give the correct position and shape of the GT distribution in the upper $1s0d$ shell may imply that it is not possible to describe all binding-energy data in the $1s0d$ shell with a universal smoothly mass-dependent Hamiltonian. However, the CW and W Hamiltonians were determined predominantly from experimental binding energies of low-lying states (up to about 5 MeV in excitation), and consideration of the higher GT strength may be able to better determine the components of the Hamiltonian to which the low-lying data are not very sensitive. [I have been concerned here with the averaged properties of the GT distribution. The detailed comparison with experiment for the very weak states below 4 MeV is about equally poor for all of the Hamiltonians considered here, however, they all are consistent with the fact that there is very little strength, less than 4% of the sum rule, for the sum of the B(GT) to these states. One particular detail, which is important for the calibration of the beta decay and (p,n) experiments, is that the B(GT) value to the analog state is small, 0.13 or less, for all of the Hamiltonians considered here.]

The problem with the position of the GT strength with the W Hamiltonian is primarily in the upper part of the $1s0d$ shell; GT strength distributions observed in (p,n) reactions for nuclei in the lower and middle parts of the $1s0d$ shell ($A=18$–32) are in overall good agreement with the W Hamiltonian (Ref. [15] and references therein). In fact, for $A=18$ and 19 there are several cases where most of the GT strength resides in low-lying levels which are directly populated in mirror beta decay [2] and whose energies were integral in determining the W Hamiltonian. Thus, the previous conclusions concerning the quenching of GT strength [2], which are based primarily on β decay data in the lower and middle parts of the $1s0d$ shell, are still valid. In addition, I point out that the quenching obtained from the ^{39}Ca β^+ decay [2] and the ^{39}K(p,n) data [18] is completely independent of the $1s0d$ Hamiltonian.

Quenching of GT strength is clearly an experimental and model-dependent concept. In the ^{37}Ca case discussed above it the total GT strength inferred from β decay and analog (p,n) data below 12 MeV compared to the sum rule, or the strength observed in β decay below 8 MeV compared to that predicted in the $1s0d$ model space with some effective Hamiltonian. Perturbative calculations of higher-order configuration mixing and Δ-isobar mixing [7] are able to qualitatively account for the quenching observed in the $1s0d$ shell [8]. That is, if one were to compare the experimental strength with calculations which include both the $1s0d$ model space plus these higher-order effects (either explicitly or implicitly in terms of an effective operator), there would be agreement between experiment and theory.

There is not a clear-cut division between the GT strength which resides primarily on the $1s0d$ model space and in the direct contribution due to higher-order configuration mixing in the ground state. In particular, the direct strength ascribed to the lowest-energy two-particle two-hole $2\hbar\omega$ admixture is observed in a few discrete states around 10 MeV in ^{40}Ca. The experimental B(GT) in the strongest of these as deduced from a ^{40}Ca(p,n) experiment is 0.33 ± 0.06 [20], and calculations with the SAS (Sakakura-Arima-Sebe) interaction [21] in the $0d_{3/2}$-$0f_{7/2}$ model space, which reproduces the observed $B(M1)$ for these states [22], predicts a total of $B(\mathrm{GT})=0.8$. The same type of calculation predicts an extra amount of strength $B(\mathrm{GT}_+)=0.8$ for the ^{37}Ca β^+ decay. This is small compared to the $1s0d$ contribution. It also predicts $B(\mathrm{GT}_-)=0.08$ (rather than zero). (Note that the sum rule does not hold within the $0d_{3/2}$-$0f_{7/2}$ model space, and that these results can be interpreted, at best, as an indication of the strength expected up to about 15-MeV excitation in ^{37}K.) A ^{37}Cl(n,p) experiment would be useful to confirm this expectation. One should expect even more GT strength at higher excitation energy within the full $1s0d$-$1p0f$ model space as well as more from higher $\hbar\omega$ correlations. However, this strength is difficult to extract from charge-exchange reactions because of the dominance of higher-multipole excitations at higher excitation energy. I believe that the GT strength observed below 15 MeV in excitation for $A<40$ should be ascribed primarily to the $1s0d$ model space. Finally, I note that the $n\hbar\omega$ excitations greatly increase the level density compared to that expected from the $0\hbar\omega$ $1s0d$ configurations. This most clearly shows up in the spreading of the $0d_{5/2}$ hole strength in $A=39$ [17,18] and in the high level density observed in the ^{37}Ca beta decay (between 5 and 8 MeV, about 3 times that expected in the $1s0d$ model space).

In conclusion, I find that the amount of quenching extracted from the ^{37}Ca β^+ decay data is very sensitive to the shape assumed for the GT strength distribution. If the (p,n) data are taken as an indication of what this shape is, then I find that calculations with the $12.5p$ or CW Hamiltonians are preferred and that the quenching inferred from the β^+ decay is about the same as obtained from the global analysis of all $1s0d$ shell beta decay data. A further evolution of the $1s0d$ shell Hamiltonian which would incorporate both the success of the W Hamiltonian across the shell and the CW interaction for the GT distribution in the upper part of the shell will remain a challenge.

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