Heavy Mesons in Two Dimensions

Benjamín Grinstein[(1),(a)] and Paul F. Mende[(2),(b)]
[(1)]*Superconducting Super Collider Laboratory, Dallas, Texas 75237*
[(2)]*Department of Physics, Brown University, Providence, Rhode Island 02912*
(Received 27 February 1992; revised manuscript received 13 July 1992)

The large-mass limit of QCD uncovers symmetries that are not present in the QCD Lagrangian. These symmetries have been applied to physical (finite mass) systems, such as B and D mesons. We explore the validity of this approximation in the 't Hooft model (two-dimensional QCD) in the large-N approximation). We find that the large-mass approximation is good, even at the charm mass, for form factors, but it breaks down for the pseudoscalar decay constant.

PACS numbers: 12.38.Lg, 11.15.Pg, 13.20.−v, 14.40.Jz

(1) Introduction.— Heavy mesons play a prominent role in our understanding of fundamental processes. From their weak decays we may extract fundamental parameters of the standard model of electroweak interactions. Rare decays are sensitive to the presence of new fundamental forces. And they offer the exciting possibility of observing for the first time violation of CP invariance in a decay process.

Straightforward interpretation of measured lifetimes and branching fractions is marred by the difficulties that strong interactions present for practical calculations. Monte Carlo simulations of lattice QCD may eventually furnish accurate calculations of the matrix elements that are relevant to these processes. An alternative approach is furnished by the heavy-quark effective theory (HQET) formalism. Approximate spin and flavor symmetries [1] of the S matrix in the one-heavy-hadron sector are made explicit. The symmetries become exact in the limiting case of infinitely massive heavy quarks. From these symmetries a number of remarkable results follow, such as the normalization of form factors for semileptonic $B \to D$ and D^* transitions at maximum momentum transfer, $q^2_{\rm max}$, and a set of five relations among the six corresponding form factors which hold at any momentum transfer.

In this regard it is of paramount importance to determine the accuracy of the large-mass approximation since in reality the charm and bottom quark masses are both only factors of a few larger than, say, the ρ-meson mass. Unfortunately, this issue involves nonperturbative matrix elements and is therefore hard to pin down. Moreover, one has to consider each physical quantity separately, as the approach to the asymptotic regime of infinite masses may be faster for some than for others. For example, some form factors in semileptonic decay remain calculable at $q^2_{\rm max}$ even after $1/m$ corrections are included [2], and Monte Carlo simulations of lattice QCD in the quenched approximation [3] indicate that $1/m$ corrections to the pseudoscalar decay constant of a heavy meson are large (of the order of 65% at the charm mass).

In this Letter we report on investigations of this issue using two-dimensional QCD in the $1/N$ expansion as a model of the strong interactions. It is worth emphasizing from the outset that such model calculations are not systematic approximations to four-dimensional QCD, and as such one should refrain from using the quantitative results as estimates for physical observables. Our results are qualitatively similar to those of potential models [4] and QCD sum rules [5]: At the charm mass, $1/m$ corrections to the pseudoscalar decay constant are $\sim 100\%$, while corrections to the normalization of form factors of flavor-changing currents are $\sim 10\%$. By gathering evidence for qualitative features that are common to all physically reasonable (uncontrolled) approximations one can begin to believe that such features are also present in QCD.

In the following sections we review some salient features of the model, compute the pseudoscalar decay constant, compute the form factors for heavy-to-heavy transitions, and close with a discussion of these results.

(2) The 't Hooft model.— We calculate the properties of heavy mesons in 1+1 dimensions. This model of QCD, in the $1/N$ expansion, was solved by 't Hooft [6]. It shares features of the four-dimensional $1/N$ expansion which, in turn, has some common ground with meson phenomenology. The spectrum consists of meson states, with an approximately linear, Regge-type trajectory. Since there is no spin in two dimensions, these are obviously radial excitations— there is no analog of the spin-symmetry relations which appear so fruitful in studies of the real world. Nor are there baryon or glueball states, these being suppressed by the $1/N$ expansion and the lack of transverse dimensions. Yet in spite of the peculiarities of two dimensions, it is nonetheless a nontrivial strong-coupling solvable theory which is ideally suited for testing our dynamical ideas. This model has been extensively studied and we refer the reader for details of the calculations to [7,8] where the formalism for the matrix elements was derived, to [9] where the numerical methods are discussed, and to [10] where they were recently applied to the study of low-energy effective theories.

We take two flavors of quark, one heavy and one light, which we denote as Q and q, respectively, with bare masses M and m. The coupling constant g has dimensions of mass, and we work in units where $g^2N/\pi \equiv 1$.

The lowest heavy-light $Q\bar{q}$ bound state—let us call it the B—is a pseudoscalar of mass μ. We can solve the nonperturbative bound-state equation numerically for the wave functions and the masses [9].

As our interest is in the approach to the asymptotic regime, we hold m fixed (taking the value $m^2=0.3$ throughout the paper), computing the dependence of the universal form factors and pseudoscalar decay constant f_B as functions of M. (It is also possible to take the limit $M\to\infty$ directly in the field theory before solving for the bound states, as have been done in 4D to derive the HQET [11]. We have found the bound-state equation appropriate to this limit, which will be reported elsewhere.)

An important limitation of this model is the lack of spin, and we shall have nothing to say about relations that follow from the spin symmetry of the HQET. On the other hand, because the model is a relativistic field theory exhibiting confinement, it is ideally suited to test the scale of the onset of flavor symmetries. The transition from light- to heavy-quark dynamics was explored in detail in Ref. [10]. Inspection of the spectrum of $Q\bar{Q}$ states and the strength of the singularities in the form factors reveals this "charm" mass to be between 1 and 2 [10].

(3) The pseudoscalar decay constant.—The pseudoscalar decay constant f_B for the meson B is defined by

$$\langle 0|\bar{q}\gamma^\mu\gamma^5 Q|B(p)\rangle = f_B p^\mu . \qquad (3.1)$$

If the states are given the usual relativistic normalization,

$$\langle B(p')|B(p)\rangle = 2E\delta(p-p') , \qquad (3.2)$$

then the large-mass limit gives the scaling behavior $f_B\sqrt{\mu}\sim$const. This follows from the observation that the static properties of a "dressed" heavy source of color are independent of its mass. The factor of $\sqrt{\mu}$ simply reflects the mass-dependent normalization of states, cf. Eq. (3.2). In four dimensions there are logarithmic corrections to this relation [12]. We have omitted these, foreseeing that they are absent in a super-renormalizable theory.

In two dimensions both vector and axial currents are good interpolating fields for the pseudoscalar meson. Since $\gamma^5\gamma^\mu=\epsilon^{\mu\nu}\gamma_\nu$, they are both characterized by the same decay constant. In the 't Hooft model f_B is easily computed [7,8]. It is given by $f_B=\int_0^1 dt\,\phi_B(t)$, where $\phi_B(t)$ is the momentum-space wave function for the B meson and t is the fraction of light-cone momentum carried by the heavy quark [6]. In Fig. 1 we show how the limiting behavior $f_B\sqrt{M}\sim f_B\sqrt{\mu}\sim$const is attained.

Fitting the high-mass portion of the curve ($M\geq 5$) by a quadratic polynomial in $1/M$ gives a description of the $1/M$ corrections:

$$f_B\sqrt{M}=2.0\left[1-\frac{1.4}{M}+\left(\frac{1.4}{M}\right)^2\right] . \qquad (3.3)$$

There are two sources of uncertainty in this calculation: the end-point fit of the wave functions ϕ_B and the few

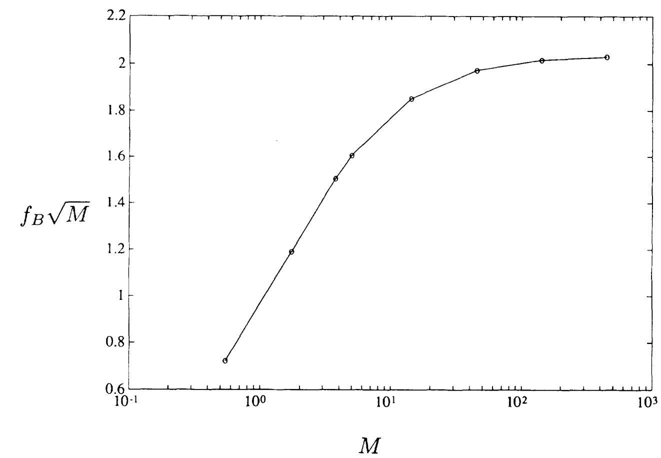

FIG. 1. Plot of the pseudoscalar decay constant $f_B\sqrt{M}$ as a function of the mass of the heavy quark. The mass of the light quark, m, is fixed throughout: $m^2=0.3$.

values of the mass we take for our mass fit. By varying the number of sampling points and successively improving the quality of our wave functions we estimate the error to be less than 2% for the coefficient of $1/M$ and less than 20% for that of $1/M^2$.

(4) Form factors.—Of greatest interest in probing the structure of the theory in the heavy-quark limit is the form factor for a heavy-quark current between heavy mesons:

$$\langle B'(p')|\bar{Q}'\gamma_\mu Q|B(p)\rangle = f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu , \qquad (4.1)$$

where the momentum transfer is given by $q=p-p'$. B and B' have the same light-quark content but different heavy quarks. When $Q'=Q$, conservation of current gives $f_-=0$. The remaining form factor, $f(q^2)$, is normalized, $f(0)=1$.

In the limit of infinite masses, the mesons are more appropriately labeled by their velocities $v=p/\mu$ and $v'=p'/\mu'$, and it is natural to describe the dependence on q^2 through the function $w\equiv v\cdot v'=(\mu^2+\mu'^2-q^2)/2\mu\mu'$. The transition amplitude for a "dressed" heavy source of color with velocity v to a second one with velocity v' is then independent of their masses. Thus all three form factors in this limit are given in terms of a single universal "Isgur-Wise" function [1], $\xi(v\cdot v')$:

$$f_\pm(q^2)=\xi(v\cdot v')\left(\frac{\mu'\pm\mu}{2\sqrt{\mu'\mu}}\right), \quad f(q^2)=\xi(v\cdot v') . \qquad (4.2)$$

As before, we ignore logarithmic corrections [13] which are absent in the super-renormalizable model.

The Isgur-Wise function is normalized—$\xi(1)=1$—by evaluating it for identical heavy mesons so that $v\cdot v'=1$ corresponds to $q^2=0$. This in turn gives a prediction in the case of different heavy mesons at $q^2=q^2_{\max}=(\mu-\mu')^2$:

$$f_{\pm}(q^2_{\max}) = \frac{\mu' \pm \mu}{2\sqrt{\mu'\mu}} . \tag{4.3}$$

Let us now examine the behavior of the form factors for *finite-mass* heavy quarks in two dimensions, where they may be computed exactly. In two and four dimensions the definitions and infinite-mass relations are identical. What are the dominant $1/M$ corrections and where do they set in? How well can the form factors be approximated from the quark-model contributions alone?

It is straightforward to evaluate the current matrix elements of interest. For spacelike momentum transfer the minus component of the current gives the following expression [8] in which the first term dominates our calculation

$$\frac{\langle B'|V_-|B\rangle}{2q_-} = \int_1^\omega dt\, \phi_B\left(\frac{t}{\omega}\right)\phi_{B'}\left(\frac{1-t}{1-\omega}\right) + \frac{1}{1-\omega}\int_0^1 dt\,\phi_B\left(\frac{t}{\omega}\right)\Phi_{B'}\left(\frac{1-t}{1-\omega}\right)G(t;q^2) + \int_1^\omega dt\,\phi_B\left(\frac{t}{\omega}\right)\phi_{B'}\left(\frac{1-t}{1-\omega}\right)\tilde{G}(t;q^2) , \tag{4.4}$$

where $\omega \equiv p_-/q_-$, the full vertex $\Phi(t) \equiv f_0^1 dt'\phi(t')/(t'-t)^2$, the Green function $G(t;q^2) \equiv \sum_n f_n \phi_n^{QQ'}(t)/(q^2-\mu_n^2)$, and $\tilde{G}(t;q^2) \equiv f_0^1 dt' G(t';q^2)/(t-t')^2$.

The first line represents the "quark-model" contribution to the form factor, where the current couples directly to the valence heavy quark. The other terms represent the full set of remaining graphs which arise from gluon exchange in the current channel. These give small contributions: They correct the form factors below by no more than 1.2% or 0.003% for $M=4$ or 45, respectively, provided $v\cdot v' \leq 3$.

First we consider the form factor $f(q^2)$ for identical heavy mesons, $B'=B$. In Fig. 2 we plot the heavy-quark current form factors for $M=5$, 14, and 450.

To better characterize the large-mass behavior we can examine, at fixed $w=v\cdot v'$, the approach to the asymptotic function as the heavy-quark mass is varied away from $M=\infty$. We calculated the form factors for additional masses, $M=45$ and 140. A fit by the quadratic polynomial in $1/M'$,

$$f(v\cdot v') = \xi(v\cdot v')\left[1 - \frac{\kappa_1(v\cdot v')}{M'} - \frac{\kappa_2(v\cdot v')}{M'^2}\right], \tag{4.5}$$

gives $\kappa_1=0.10$, 0.26, and 0.32, and $\kappa_2=0.6$, 1.6, and 3.0, for $v\cdot v'=1.25$, 2.0, and 3.5, respectively. As was the case for the pseudoscalar decay constant, these quantities are good to about 10% accuracy.

Next we consider transitions between different ground-state mesons. At $q^2_{\max}$ the left-hand side of (4.4) is predicted in the HQET, cf. Eqs. (4.1) and (4.3):

$$\langle B'(p')|\bar{Q}'\gamma_- Q|B(p)\rangle|_{q^2_{\max}} = 2\sqrt{\mu\mu'} . \tag{4.6}$$

In Fig. 3 we show

$$d(M') \equiv \frac{\langle B'(p')|\bar{Q}'\gamma_- Q|B(p)\rangle|_{q^2_{\max}}}{2\sqrt{\mu\mu'}} - 1 \tag{4.7}$$

as a function of the heavy-quark mass M' in the lighter B' meson. The heavier B meson has heavy-quark mass held fixed at $M=450$. This plot reveals that the finite-mass corrections are *quadratic* in $1/M'$. This result is expected as a consequence of Luke's theorem [2], which states that there are no corrections of order $1/M'$ to the predicted normalization of form factors at maximum momentum

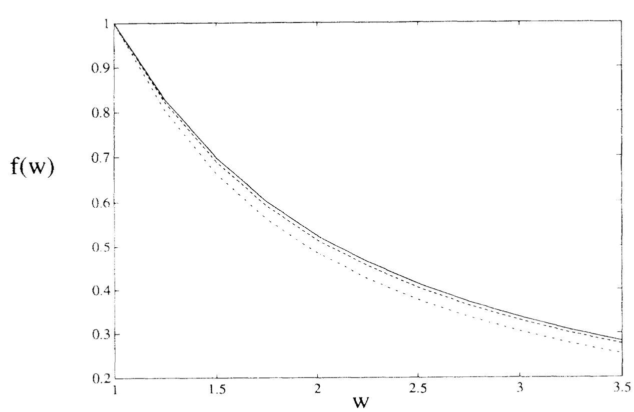

FIG. 2. Heavy-quark current form factors $f(v\cdot v')$ as functions of $v\cdot v'$ for different masses M of the heavy quark. The solid, dashed, and dash-dotted lines correspond to $M=450$ (effectively infinite), 14, and 5, respectively.

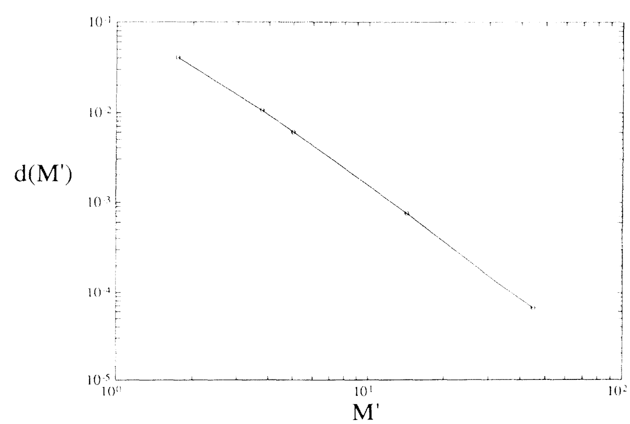

FIG. 3. Difference $d(M')$ from unity of the matrix element of the flavor-changing current between ground-state mesons, at $q^2_{\max}$, normalized to $2\sqrt{\mu\mu'}$, as a function of the heavy-quark mass M' in the lighter meson; see Eq. (4.7). The heavier meson has $M=450$.

transfer; corrections being at order $1/M'^2$. (Luke's theorem, in its original version, ignores corrections of order $\alpha_s(M')/M'$. A stronger version [Cho and Grinstein (to be published)] gives the absence of corrections of order $1/M'$ to all orders in the strong coupling.) Again the result can be fitted by a quadratic polynomial in $1/M'$:

$$d(M') = -\frac{0.002 \pm 0.002}{M'} - \frac{0.14 \pm 0.01}{M'^2}. \qquad (4.8)$$

Alternatively, one can fit by a power A/M'^n, for which we find $A = -0.17$ and $n = 2.1$, which clearly indicates the absence of $1/M'$ corrections.

(5) Discussion and outlook.—The size of $1/M$ corrections is not uniform and cannot be characterized by f_B alone: They are far larger for the pseudoscalar decay constant than they are for the form factors, for which the asymptotic limit is approached rapidly. This is true both for the functional form of the form factors out to moderate $v \cdot v'$ [Eq. (4.5)] as well as for the normalization of the form factors of flavor-changing currents [Eq. (4.8)]. At the "charm" mass the model has corrections of order 100% to the HQET prediction of the pseudoscalar decay constant, while it gives small corrections, (4–14)%, to the normalization of form factors of flavor-changing currents at $v \cdot v' = 1$.

It is apparent that the 't Hooft model is a valuable testing ground for our ideas of the large-mass limit of QCD. One may attempt to investigate the validity of Bjorken's sum rule [14], since one can compute the wave functions of rather high excited states of $Q\bar{q}$ mesons. More interestingly, one can address the question of whether the form factor for the semileptonic decay of a heavy meson into a light one is wave function or pole dominated. The answer remains elusive due to the intrinsically nonperturbative nature of dynamics involved.

The research by B.G. was supported in part by the Alfred P. Sloan Foundation and by the Department of Energy under Contract No. DE-AC35-89ER40486. The research by P.F.M. was supported in part by DOE Grant No. DE-AC02-76-ER03130.

Note added.—In an earlier version the fit shown in Fig. 1 was labeled as $f\sqrt{\mu}$ instead of $f\sqrt{M}$. This does not affect our conclusions but did lead to an apparent discrepancy with the later work of Burkhardt and Swanson [15]. After accounting for a different state normalization, we find both papers are in agreement.

Those authors, however, draw an opposite conclusion—that $1/M$ corrections are as large for form factors as for the decay constant—because they focus on different domains; we believe our choice is more relevant. For flavor-neutral form factors we concentrate on $1 \le v \cdot v' \le 3.5$, while they emphasize behavior for $v \cdot v' \gg 1$, where the HQET is in any case expected to break down. For flavor-changing form factors we emphasize $v \cdot v' = 1$, since the absolute prediction in the heavy-quark limit may allow a determination of CKM angles from the semileptonic decay. Reference [15] focuses instead on $f_-(v \cdot v')$, a form factor whose contribution to $\bar{B} \to De\bar{\nu}$ is suppressed by m_e^2/m_B^2, and which is not protected from $1/M$ corrections by Luke's theorem (as one may readily infer from their Fig. 3.8).

(a)On leave of absence from Harvard University, Cambridge, MA 02138. Electronic addresses: grinstein @sscvx1.bitnet, @sscvx1.ssc.gov.

(b)Electronic address: mende@het.brown.edu.

[1] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).

[2] M. E. Luke, Phys. Lett. B **252**, 447 (1990).

[3] P. Boucaud *et al.*, Phys. Lett. B **220**, 219 (1989); C. R. Allton *et al.*, Nucl. Phys. **B349**, 598 (1991).

[4] W. Wirbel, B. Stech, and M. Bauer, Z. Phys. C **29**, 637 (1985); B. Grinstein, M. B. Wise, and N. Isgur, Phys. Rev. Lett. **56**, 258 (1986); T. Altomari and L. Wolfenstein, Phys. Rev. Lett. **58**, 1583 (1987); J. G. Körner and G. A. Shuler, Z. Phys. C **38**, 511 (1988); N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D **39**, 799 (1989); M. Neubert and V. Rieckert, Report No. HD-THEP-91-6 (unpublished).

[5] C. A. Dominguez and N. Paver, Phys. Lett. B **269**, 169 (1991); A. V. Radyushkin, Phys. Lett. B **271**, 218 (1991); M. Neubert, Phys. Rev. D **45**, 2451 (1992).

[6] G. 't Hooft, Nucl. Phys. **B75**, 461 (1974).

[7] C. G. Callan, N. Coote, and D. J. Gross, Phys. Rev. D **13**, 1649 (1976).

[8] M. B. Einhorn, Phys. Rev. D **14**, 3451 (1976).

[9] A. H. Hanson, R. D. Peccei, and M. K. Prasad, Nucl. Phys. **B121**, 477 (1977); S. Huang, J. W. Negele, and J. Polonyi, Nucl. Phys. **B307**, 669 (1988).

[10] R. L. Jaffe and P. F. Mende, Nucl. Phys. **B369**, 189 (1992).

[11] B. Grinstein, Nucl. Phys. **B339**, 253 (1990); H. Georgi, Phys. Lett. B **240**, 447 (1990); A. Falk, B. Grinstein, and M. Luke, Nucl. Phys. **B357**, 185 (1991).

[12] M. B. Voloshin and M. A. Shifman, Yad. Fiz. **45**, 463 (1987) [Sov. J. Nucl. Phys. **45**, 292 (1987)]; H. D. Politzer and M. B. Wise, Phys. Lett. B **206**, 681 (1988); **208**, 504 (1988).

[13] A. Falk *et al.*, Nucl. Phys. **B343**, 1 (1990).

[14] J. D. Bjorken, Report No. SLAC-PUB-5278, 1990 (unpublished).

[15] M. Burkhardt and E. Swanson, MIT Report No. CTP 2096, 1991 (to be published).