Probing the Grand-Unification-Scale Mass Spectrum through Precision Measurements on the Weak-Scale Parameters

J. Hisano, H. Murayama, and T. Yanagida
Department of Physics, Tohoku University, Sendai 980, Japan
(Received 12 May 1992)

We study the renormalization-group evolution of gauge coupling constants including mass threshold effects of both light and heavy particles, in the minimal supersymmetric SU(5) grand unified model assuming supersymmetry breaking at $\lesssim 1$ TeV. We show that a certain combination of superheavy masses $M_V^2 M_\Sigma$ is tightly constrained, while the mass of the color-triplet Higgs boson, M_H, can vary from 2×10^{13} to 2×10^{17} GeV. With a relatively heavy color-triplet mass ($M_H \sim 10^{17}$ GeV), the present nucleon-decay experiments still allow superparticles below 1 TeV. The importance of a precise measurement on α_3 is stressed.

PACS numbers: 12.10.Dm, 11.30.Pb, 12.15.Cc, 14.80.Ly

Supersymmetry (SUSY) provides a beautiful mechanism which protects the large hierarchy between the weak scale and the grand-unification scale against radiative corrections [1,2]. To avoid unnatural fine tunings between bare parameters and radiative corrections, superparticles should be lighter than ~ 1 TeV. Recently, there has been revived interest in the supersymmetric SU(5) grand unified theory (SUSY GUT) [3] after the high-precision measurement on the Weinberg angle $\sin^2\theta_W$ at the CERN e^+e^- collider LEP [4]. Once one regards the SUSY GUT as a realistic theory beyond the standard model, the most important consequence is instability of baryons. The dimension-six operators from gauge-boson exchanges induce nucleon decays, which are not dangerous. They are suppressed by two powers of the GUT scale $\propto 1/M_{\rm GUT}^2$, and the unification scale in the SUSY GUT ($M_{\rm GUT} \simeq 10^{16}$ GeV) is higher than in the non-SUSY GUT, giving the nucleon lifetime $\tau \gtrsim 10^{34}$ yr. However, it was pointed out that the exchanges of a color-triplet Higgs boson give rise to dimension-five operators which are suppressed only by a single power of the GUT scale $\propto 1/M_{\rm GUT}$ [5].

Detailed analyses on the dimension-five operators have been done in Refs. [6–8]. The authors of Ref. [8] claimed that the present limits on the nucleon decay are stringent enough that superparticles lighter than 1 TeV are not allowed unless there is a delicate cancellation between matrix elements of the dimension-five operators. However, these analyses simply assume that the color-triplet Higgs boson lies just on the GUT scale, $M_H \simeq M_{\rm GUT}$. It is extremely important to determine the mass of the color-triplet Higgs boson without any theoretical prejudices.

In this Letter, we show that the mass spectrum at the GUT scale can be determined, as far as one takes the minimal SUSY SU(5) GUT, from the gauge coupling constants and superparticle masses at the weak scale [9]. However, the present data on the strong-coupling constant α_3 do not have sufficient precision to pinpoint the mass of the color-triplet Higgs boson. In fact, we will show that the mass of the color-triplet Higgs supermultiplet should lie in the region

$$2.4\times 10^{13}\,\text{GeV} \le M_H \le 2.3\times 10^{17}\,\text{GeV}\,, \tag{1}$$

whose error is dominated by the uncertainty in the strong-coupling constant α_3. If we adopt a relatively heavy mass of the color-triplet Higgs boson, $M_H \sim 10^{17}$ GeV, then it can be shown that the present nucleon-decay experiments still allow superparticles below 1 TeV. It will also be shown that the masses of superheavy gauge bosons and physical components of the adjoint Higgs multiplet are tightly constrained.

We first discuss the renormalization-group (RG) evolution of gauge coupling constants. It was shown in Refs. [10,11] that the naive step-function approximation is accurate for supersymmetric theories, justified in the "supersymmetric regularization" $\overline{\rm DR}$ (dimensional reduction) scheme. The superheavy particle spectrum in the minimal SUSY SU(5) GUT [2] is characterized by only three masses, M_V of the gauge boson, M_Σ of the adjoint Higgs multiplet [12], and M_H of the color-triplet Higgs boson. Then the running of three gauge coupling constants in minimal SUSY SU(5) GUT can be written down easily at the one-loop level as

$$\alpha_3^{-1}(m_Z) = \alpha_{\rm GUT}^{-1}(\Lambda) + \frac{1}{2\pi}\left\{\left(-2-\tfrac{2}{3}N_g\right)\ln\frac{m_{\rm SUSY}}{m_Z} + (-9+2N_g)\ln\frac{\Lambda}{m_Z} - 4\ln\frac{\Lambda}{M_V} + 3\ln\frac{\Lambda}{M_\Sigma} + \ln\frac{\Lambda}{M_H}\right\}, \tag{2}$$

$$\alpha_2^{-1}(m_Z) = \alpha_{\rm GUT}^{-1}(\Lambda) + \frac{1}{2\pi}\left\{\left(-\tfrac{4}{3}-\tfrac{2}{3}N_g-\tfrac{5}{6}\right)\ln\frac{m_{\rm SUSY}}{m_Z} + (-6+2N_g+1)\ln\frac{\Lambda}{m_Z} - 6\ln\frac{\Lambda}{M_V} + 2\ln\frac{\Lambda}{M_\Sigma}\right\}, \tag{3}$$

$$\alpha_1^{-1}(m_Z) = \alpha_{\rm GUT}^{-1}(\Lambda) + \frac{1}{2\pi}\left\{\left(-\tfrac{2}{3}N_g-\tfrac{1}{2}\right)\ln\frac{m_{\rm SUSY}}{m_Z} + \left(2N_g+\tfrac{3}{5}\right)\ln\frac{\Lambda}{m_Z} - 10\ln\frac{\Lambda}{M_V} + \tfrac{2}{5}\ln\frac{\Lambda}{M_H}\right\}. \tag{4}$$

Here the Λ scale is supposed to be larger than any of the superheavy masses M_V, M_Σ, and M_H. The number of generations N_g is three, and we have assumed a common mass $m_{\rm SUSY}$ for all the superparticles and for one of two SU(2)-doublet Higgs bosons. A mass of the other doublet Higgs boson is taken at m_Z. By eliminating $\alpha_{\rm GUT}^{-1}$ from the above equations, we obtain simple relations:

$$(3\alpha_2^{-1}-2\alpha_3^{-1}-\alpha_1^{-1})(m_Z)=\frac{1}{2\pi}\left\{\frac{12}{5}\ln\frac{M_H}{m_Z}-2\ln\frac{m_{\rm SUSY}}{m_Z}\right\}, \quad (5)$$

$$(5\alpha_1^{-1}-3\alpha_2^{-1}-2\alpha_3^{-1})(m_Z)=\frac{1}{2\pi}\left\{12\ln\frac{M_V^2M_\Sigma}{m_Z^3}+8\ln\frac{m_{\rm SUSY}}{m_Z}\right\}. \quad (6)$$

Equations (5) and (6) imply that we can probe the GUT-scale mass spectrum from the weak-scale parameters (i.e., gauge coupling constants and mass spectrum of the superparticles). Equation (5) determines the color-triplet Higgs boson mass M_H, while Eq. (6), a combination of the vector and adjoint-Higgs-multiplet masses $(M_V^2M_\Sigma)^{1/3}$ which we will call the "GUT scale" $M_{\rm GUT}\equiv(M_V^2M_\Sigma)^{1/3}$ hereafter.

So far we have assumed a common mass $m_{\rm SUSY}$ for the superparticles, but the mass splitting among the superparticles is also important to determine the GUT-scale mass spectrum. Assuming degeneracy among generations [13], the effect of the mass splitting can be taken into account by replacing $\ln(m_{\rm SUSY}/m_Z)$ in Eqs. (5) and (6) as

$$-2\ln\frac{m_{\rm SUSY}}{m_Z}\rightarrow 4\ln\frac{m_{\tilde g}}{m_{\tilde w}}+\frac{N_g}{5}\ln\frac{m_{\tilde u}^3m_{\tilde d}^2m_{\tilde e}}{m_{\tilde q}^4m_{\tilde l}^2}-\frac{8}{5}\ln\frac{m_{\tilde h}}{m_Z}-\frac{2}{5}\ln\frac{m_H}{m_Z} \quad (7)$$

in Eq. (5), and

$$8\ln\frac{m_{\rm SUSY}}{m_Z}\rightarrow 4\ln\frac{m_{\tilde g}}{m_Z}+4\ln\frac{m_{\tilde w}}{m_Z}+N_g\ln\frac{m_{\tilde q}^2}{m_{\tilde e}m_{\tilde u}} \quad (8)$$

in Eq. (6). Here we have neglected the mixings among gauginos and Higgsinos. Two doublet Higgs bosons are assumed to have masses at m_H and m_Z, respectively. The symbols $\tilde q$, $\tilde l$ represent the squark and slepton doublets; $\tilde u$, $\tilde d$, $\tilde e$ the right-handed up, down squarks and charged slepton; and $\tilde g$, $\tilde w$, $\tilde h$ the gluino, W-ino, and Higgsino. In our analysis we adopt the minimum supergravity model, where the SUSY-breaking mass parameters at the weak scale can be determined from a small number of parameters at the Planck scale, by using the renormalization-group equations [14]. We restrict ourselves to the case where the universal scalar mass dominates the SUSY breaking. Then the terms $\ln(m_{\tilde u}^3m_{\tilde d}^2m_{\tilde e}/m_{\tilde q}^4m_{\tilde l}^2)$ in Eq. (7) and $\ln(m_{\tilde q}^2/m_{\tilde e}m_{\tilde u})$ in Eq. (8) are negligibly small. The term $\ln(m_{\tilde g}/m_{\tilde w})$ stays constant, since $m_{\tilde g}/m_{\tilde w}=\alpha_3/\alpha_2\simeq 3.5$. The dependence on m_H in Eq. (7) is weak due to its small coefficient, and we set $m_H=1$ TeV. Therefore, we find that M_H depends mainly on the Higgsino mass $m_{\tilde h}$, and $M_{\rm GUT}$ on the product of gaugino masses $m_{\tilde g}m_{\tilde w}$.

Now we are at the stage to derive the GUT-scale mass spectrum from the above RG analysis. In our numerical calculation, we include two-loop corrections to the RG equations [10]. In Fig. 1, we show the allowed ranges of M_H and $M_{\rm GUT}$ separately for $m_{\tilde h},m_{\tilde g}=100$ GeV and 1 TeV. We have used the $\overline{\rm MS}$ (modified minimal subtraction) gauge coupling constants on the Z pole given in Ref. [15], $\alpha=127.9\pm0.2$, $\sin^2\theta_W=0.2326\pm0.0008$, and $\alpha_3=0.118\pm0.007$. We find that the "GUT scale" is tightly constrained as

$$0.90\times10^{16}\,{\rm GeV}\le(M_V^2M_\Sigma)^{1/3}\le3.1\times10^{16}\,{\rm GeV}, \quad (9)$$

for 100 GeV $\le m_{\tilde g}\le$ 1 TeV. On the other hand, the color-triplet Higgs boson mass M_H is much less constrained, as shown in Eq. (1), because of the small gauge-group representation for the Higgs supermultiplets [16]. Also shown in Fig. 1 is the case of an improved measurement on α_3, with the same central value but with the smaller error bar by a factor of 2. The figure clearly demonstrates the importance of precise measurements on α_3, as well as the experimental observations of the superparticle masses, to determine the color-triplet Higgs boson mass M_H.

Although we have concentrated on a purely phenomenological analysis on the GUT-scale mass spectrum, there

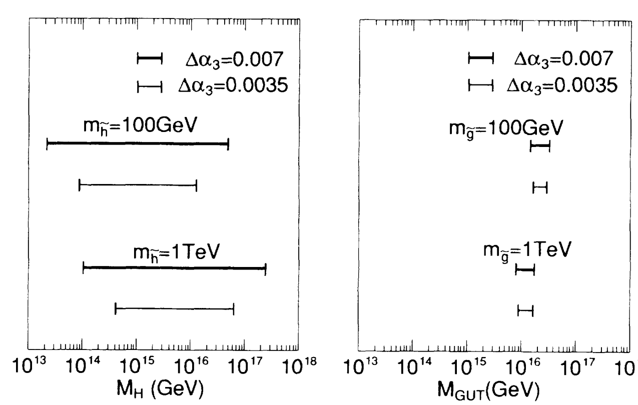

FIG. 1. The allowed ranges of the color-triplet Higgs boson mass M_H and the "GUT scale" $M_{\rm GUT}\equiv(M_V^2M_\Sigma)^{1/3}$ obtained from the renormalization-group analysis (thick lines) by varying $m_{\tilde h}$ and $m_{\tilde g}$ between 100 GeV and 1 TeV. We also utilize the GUT relation $m_{\tilde g}/m_{\tilde w}=\alpha_3/\alpha_2$. The limit on M_H depends only on $m_{\tilde h}$, and similarly $M_{\rm GUT}$ on $m_{\tilde g}$. Also shown are the ranges with an improved measurement on the strong-coupling constant, $\alpha_3=0.118\pm0.0035$ (thin lines).

is an important theoretical constraint on it. The mass M_H comes from an unknown Yukawa coupling between H, $\bar{H}$, and Σ. On the other hand, the mass M_V comes from the gauge coupling of the Σ field, whose strength is obtained by the RG analysis. A large mass splitting $M_V \ll M_H$ requires that the Yukawa coupling constant be very large compared to the SU(5) gauge coupling g_5. Thus the applicability of the perturbation theory restricts the mass splitting to be not large. The same argument applies to the mass M_Σ, which originates in a self-coupling of the adjoint Higgs multiplet.

A constraint arises by requiring that the Yukawa coupling constants do not blow up below the gravitational scale, $M_P/\sqrt{8\pi}=2.4\times10^{18}$ GeV. The running of the Yukawa coupling constants defined in the superpotential

$$W=\tfrac{1}{3}f\,\mathrm{tr}\Sigma^3+\tfrac{1}{2}fV\,\mathrm{tr}\Sigma^2+\lambda\bar{H}(\Sigma+3V)H \tag{10}$$

are described by the RG equations

$$\mu\frac{\partial\lambda}{\partial\mu}=\frac{1}{(4\pi)^2}\left[-\frac{98}{5}g_5^2+\frac{53}{10}\lambda^2+\frac{21}{40}f^2\right]\lambda\,, \tag{11}$$

$$\mu\frac{\partial f}{\partial\mu}=\frac{1}{(4\pi)^2}\left[-30g_5^2+\frac{3}{2}\lambda^2+\frac{63}{40}f^2\right]f\,, \tag{12}$$

$$\mu\frac{\partial g_5}{\partial\mu}=-\frac{3}{(4\pi)^2}g_5^3\,. \tag{13}$$

The conservative limit on λ can be obtained in the case $f=0$. A numerical study shows that the mass M_H is limited from above,

$$M_H=(\lambda/\sqrt{2}g_5)M_V<2.0M_V\,. \tag{14}$$

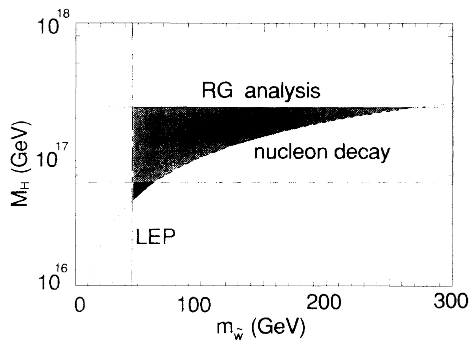

FIG. 2. The lower bound on the color-triplet Higgs boson mass M_H from the nucleon-decay experiments, neglecting the third-generation contribution y^{tK}. We take the following parameters: $\beta=0.01$ GeV^3, $\tan\beta_H=1$, and $m_{\tilde{q}}=1$ TeV. The shaded region is allowed, that is, bounded from below by nucleon-decay experiments (dotted line) and from above by renormalization-group (RG) analysis (solid line). The dashed line corresponds to the upper bound on M_H in the case with an improved α_3 measurement, $\alpha_3=0.118\pm0.0035$. The dot-dashed vertical line represents the limit $m_{\tilde{w}}>45$ GeV from LEP.

A similar limit on M_Σ can be obtained with $\lambda=0$,

$$M_\Sigma=(f/2\sqrt{2}g_5)M_V<1.8M_V\,. \tag{15}$$

It is interesting to see the status of the present nucleon-decay experiments concerning the color-triplet Higgs boson mass M_H. For this purpose, we have reexamined the analyses done in Refs. [6-8], and obtained the partial decay rates as [17]

$$\left.\begin{matrix}\tau(p\to K^+\bar{\nu}_\mu)=0.62\times10^{31}\\ \tau(n\to K^0\bar{\nu}_\mu)=0.35\times10^{31}\end{matrix}\right\}\times\left|\frac{0.01\ \mathrm{GeV}^3}{\beta}\frac{M_H}{10^{16}\ \mathrm{GeV}}\frac{\sin2\beta_H}{1+y^{tK}}\frac{(10\ \mathrm{TeV})^{-1}}{f(q,q)+f(q,l)}\right|^2\ \mathrm{yr}\,. \tag{16}$$

Here $\tan\beta_H$ is the ratio of vacuum expectation values of two Higgs doublets, and the hadron matrix element parameter β ranges from 0.003 to 0.03 GeV^3 [18]. The unknown parameter y^{tK} was introduced in [7], representing the ratio of the third-generation contribution relative to the second-generation one [19]. The function f is defined by

$$f(q,l)\equiv m_{\tilde{w}}\frac{1}{m_{\tilde{q}}^2-m_{\tilde{l}}^2}\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{q}}^2-m_{\tilde{w}}^2}\ln\frac{m_{\tilde{q}}^2}{m_{\tilde{w}}^2}-\frac{m_{\tilde{l}}^2}{m_{\tilde{l}}^2-m_{\tilde{w}}^2}\ln\frac{m_{\tilde{l}}^2}{m_{\tilde{w}}^2}\right)\simeq\frac{m_{\tilde{w}}}{m_{\tilde{q}}^2}\,, \tag{17}$$

where the last relation holds if $m_{\tilde{q}}\sim m_{\tilde{l}}\gg m_{\tilde{w}}$. We have again neglected the mixing between charged W-ino and charged Higgsino.

We can obtain a lower limit on M_H by combining the formulas Eq. (16) with the experimental limits [20], $\tau(p\to K^+\bar{\nu}_\mu)>1.0\times10^{32}$ yr and $\tau(n\to K^0\bar{\nu}_\mu)>8.6\times10^{31}$ yr. The limit is depicted in Fig. 2, taking $\beta=0.01$ GeV^3, $\tan\beta_H=1$, and $m_{\tilde{q}}=1$ TeV, tentatively. The third-generation contribution y^{tK} has been omitted. One can see that the present limits still leave a consistent region with the allowed range of M_H in Eq. (1), even with the superparticles below 1 TeV. One may anticipate that such a large mass ($M_H\gtrsim5\times10^{16}$ GeV) is not possible since the Yukawa coupling $\lambda\bar{H}\Sigma H$ exceeds the bound in Eq. (14). However, if one takes $M_\Sigma\sim10^{15}$ GeV, Eq. (9) suggests $M_V\sim4\times10^{16}$ GeV, and the requirements from the applicability of the perturbation theory Eqs. (14) and (15) are satisfied even with $M_H\sim10^{17}$ GeV. Though this case requires a mass splitting of 2 orders of magnitude among the heavy particles, it is true that this splitting is completely acceptable phenomenologically.

When the error bar of the strong-coupling constant is reduced, then more stringent constraints will be derived. As an example we have also shown the upper bound on M_H in Fig. 2 when the error bar is reduced by a factor of 2. There will be almost no region left with our choice of the parameters. However, one should be warned not to take this figure seriously. The limit on M_H here is only at the level of 1 standard deviation; furthermore, the nucleon-decay rate can be further suppressed if one employs $\beta=0.003$ GeV^3, or allows a cancellation between second- and third-generation contributions. However, still, our results clearly show that higher-precision measurement on the weak-scale parameters is very important to test the minimum SUSY GUT through the nucleon-decay experiments, without any theoretical prejudice on the mass of the color-triplet Higgs boson M_H.

[1] M. Veltman, Acta Phys. Pol. B **12**, 437 (1981); L. Maiani, in *Proceedings of the Eleventh GIF Summer School on Particle Physics, France, 3–7 September 1979*, edited by M. Davier, L. Maiani, C. Savoy, and J. Scherk (Institute for National Physics, Paris, 1980), p. 3; S. Dimopoulos and S. Raby, Nucl. Phys. **B192**, 353 (1981); E. Witten, Nucl. Phys. **B188**, 513 (1981); M. Dine, W. Fischler, and M. Srednicki, Nucl. Phys. **B189**, 575 (1981).

[2] E. Witten, in *Proceedings of the Summer School of Gif-sur-Yvette* (Ref. [1]); S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Rev. D **24**, 1681 (1981); S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**, 150 (1981); N. Sakai, Z. Phys. C **11**, 153 (1981).

[3] P. Langacker and M.-X. Luo, Phys. Rev. D **44**, 817 (1991); U. Amaldi, W. de Boer, and H. Fürstenau, Phys. Lett. B **260**, 447 (1991); W. J. Marciano, Brookhaven National Laboratory Report No. BNL-45999, 1991 (to be published).

[4] LEP Collaborations, Phys. Lett. B **276**, 247 (1992).

[5] N. Sakai and T. Yanagida, Nucl. Phys. **B197**, 533 (1982); S. Weinberg, Phys. Rev. D **26**, 287 (1982).

[6] J. Ellis, D. V. Nanopoulos, and S. Rudaz, Nucl. Phys. **B202**, 43 (1982).

[7] P. Nath, A. H. Chamseddine, and R. Arnowitt, Phys. Rev. D **32**, 2348 (1985).

[8] P. Nath and R. Arnowitt, Phys. Rev. D **38**, 1479 (1988).

[9] It was claimed that the threshold corrections at the GUT scale are so large that one cannot predict the SUSY-breaking scale even if measurements on α_3 become much more precise; R. Barbieri and L. J. Hall, Phys. Rev. Lett. **68**, 752 (1992). This high sensitivity on the GUT-scale mass spectrum implies that one can probe it through precision measurements on the weak-scale parameters, including the superparticle masses. Therefore, our result is consistent with their claim.

[10] M. B. Einhorn and D. R. T. Jones, Nucl. Phys. **B196**, 475 (1982).

[11] I. Antoniadis, C. Kounnas, and K. Tamvakis, Phys. Lett. **119B**, 377 (1982).

[12] The adjoint Higgs multiplet Σ is decomposed into various mass eigenstates after the SU(5) breaking. The real parts of the **(3,2)** and **(3*,2*)** representations will be absorbed by the gauge bosons due to the Higgs mechanism. The imaginary parts in the same representations acquire the mass M_V from the D term, belonging to the vector multiplet of the gauge bosons. The masses of the **(8,1)** and **(1,3)** components have a degenerate mass shown in Eq. (15) which we call M_Σ. The mass of the remaining **(1,1)** representation is $M_\Sigma/5$. However, it does not appear in the RG equations since it is a gauge singlet of SU(3)×SU(2)×U(1).

[13] A high degeneracy is required to suppress flavor-changing neutral currents, especially for first and second generations. See S. Bertolini, F. Borzumati, A. Masiero, and G. Ridolfi, Nucl. Phys. **B353**, 591 (1991), and references therein.

[14] See, for example, M. Drees and M. Nojiri, Nucl. Phys. **B369**, 54 (1992).

[15] P. Langacker, University of Pennsylvania Report No. UPR-0492T, 1992 (to be published).

[16] The limits $2.3\times10^{14}\leq M_H\leq1.4\times10^{18}$ GeV and $1.4\times10^{16}\leq M_{\rm GUT}\leq2.5\times10^{16}$ GeV presented in J. Hisano, H. Murayama, and T. Yanagida, Tohoku University Report No. TU-399 (to be published), were obtained using the older input parameters (given in Langacker and Luo [3]), neglecting the error bars of α and $\sin^2\theta_W$, and setting $m_{\rm SUSY}=m_Z$.

[17] Our analysis is improved in several ways from the previous ones [6–8]. See for details, J. Hisano, H. Murayama, and T. Yanagida, Tohoku University Report No. TU-400 (to be published).

[18] S. J. Brodsky, J. Ellis, J. S. Hagelin, and C. Sachrajda, Nucl. Phys. **B238**, 561 (1984).

[19] This parameter y^{tK} strongly depends on the still-unknown Kobayashi-Maskawa matrix elements and the top-quark mass. It can vary as $|y^{tK}|\sim10^{-2}$–1.

[20] Kamiokande-II Collaboration, K. S. Hirata *et al.*, Phys. Lett. B **220**, 308 (1989); IMB Collaboration, T. J. Haines *et al.*, Phys. Rev. Lett. **57**, 1986 (1986).

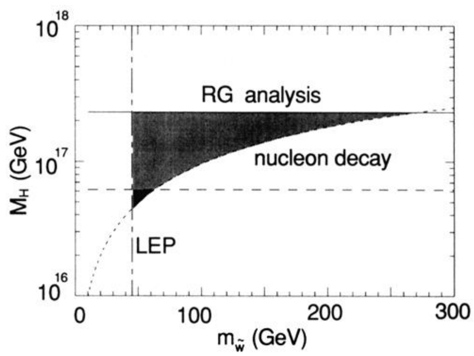

FIG. 2. The lower bound on the color-triplet Higgs boson mass M_H from the nucleon-decay experiments, neglecting the third-generation contribution y^{tK}. We take the following parameters: $\beta = 0.01$ GeV^3, $\tan\beta_H = 1$, and $m_{\tilde{q}} = 1$ TeV. The shaded region is allowed, that is, bounded from below by nucleon-decay experiments (dotted line) and from above by renormalization-group (RG) analysis (solid line). The dashed line corresponds to the upper bound on M_H in the case with an improved α_3 measurement, $\alpha_3 = 0.118 \pm 0.0035$. The dot-dashed vertical line represents the limit $m_{\tilde{w}} > 45$ GeV from LEP.