Eigenstate Assisted Activation

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Transmission through an opaque static barrier is greatly enhanced by an alternating potential. If a particle is activated to its lowest instantaneous eigenenergy, it is "trapped" and will follow the alternating potential to the top of the static barrier ("elevator effect").

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(1) Alternating potential and its implications.—Tunneling and activation in an alternating potential are important in a variety of physical problems [1-4]: field emission, interband breakdown, charge exchange between deep-lying impurity centers in semiconductors, tunneling chemical reactions, Josephson junctions, resonance tunneling [5], Coulomb blockade [6], the destruction of adiabatic invariants [7(a)]. The study of the alternating potential may also be useful for stationary many-body tunneling and evaporation, if the latter is reduced to an approximately single-particle problem. Then some of the degrees of freedom adjust to the progress in particle escape and yield an effective time-dependent potential [2]. Also, the characteristic time T of an alternating potential may be related to an effective temperature $\theta \sim \hbar/T$. Hence, an alternating-potential study may be helpful for the quantum transport problem (e.g., variable-range hopping conductivity).

An extensive and accurate study of particle transmission is also important in view of the experimental rates often being, by dozens of orders of magnitude, above the theoretical values [8,9], and even its upper bound [10]. Tunneling in an alternating potential has been extensively studied [1-3,10,11]. However, this paper for the first time simultaneously considers tunneling and activation of a particle, incoming with energy Ω, through a stationary barrier of height W and length L, in a nonharmonic potential, alternating with the characteristic time T (see Fig. 1). I study the most interesting case of an opaque barrier [7(b)] ($L\sqrt{\Delta W} \gg 1$, $\Delta W = W - \Omega$) and adiabatic potential with a small energy quanta ($1/T \ll \Delta W$); in this paper the particle mass is 0.5 and $\hbar = 1$. I prove that there are *three* types of transmission. When $T \gg t_{\rm BL}$ ($t_{\rm BL} \sim L/\sqrt{\Delta W}$ is the characteristic Buttiker-Landauer tunneling time [2]), the potential is so slow, that the transmission rate is little different from its stationary *tunneling* value. When $T \lesssim t_{\rm BL}$, a particle may rise to the lowest instantaneous resonant level ω_r created by the adiabatic potential below the top of the stationary barrier. Then the particle is trapped in such an instantaneous level. When the level is pushed up by the alternating potential, the trapped particle stays in the "elevating" level and follows it to the top of the stationary barrier ["elevator resonant activation" (ERA)]. The net effect is an exponential enhancement of the total transmission probability. Alternatively, a particle may be activated to a certain lower energy $\tilde{\omega}$, and then tunnel [*activation assisted tunneling* (AAT)]. All the options have exponentially small probabilities of being realized. Their competition is similar to the one in the partition function between different phases. The change (with T or Ω) from one option to another resembles a phase transition.

Thus, a single time-dependent opaque barrier may be a model for exponentially enhanced space-time fluctuations and transmission (and thus diffusion) rates. This may suggest their common origin. The results are very general. They are valid for the penetration of any waves (quantum, electromagnetic, sonic, hydrodynamic, etc.) into a classically forbidden region.

First I demonstrate the approach with a one-dimensional (1D) transmission. Then I sketch it for a general case.

(2) Exact solution.—An opaque barrier is long [7(b)]: $L \gg 1/\Delta W$. In order to have the barrier opaque at all times, one should consider a relatively short alternating potential. I demonstrate the solution and its physics in the extreme case of a point potential $-F(t)\delta(x)$. Then a particle is activated at the point $x=0$ only. Furthermore, it is activated from and into the set of "old" stationary functions. One may study the dependence of the

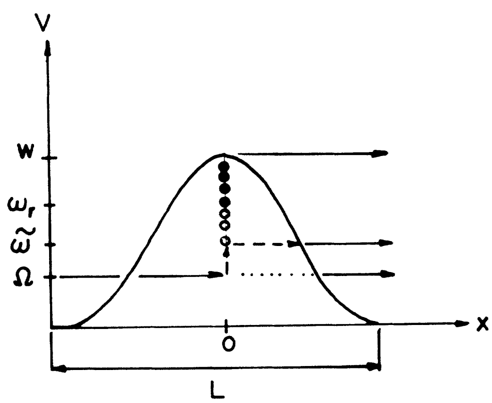

FIG. 1. Types of transmission: Tunneling (dotted line), AAT (dashed line), and ERA (open circles, activation; solid circles, elevation).

transmission on their energy ω and explicitly follow the competition between different options. The Schrödinger equation reads

$$i\dot{\psi} = -\psi'' + [V(x) - F(t)\delta(x)]\psi, \tag{1}$$

where $\dot{\psi} \equiv \partial\psi/\partial t$, $\psi' \equiv \partial\psi/\partial x$. If $V(\pm\infty) = 0$, the plane wave, incoming from $x = -\infty$ with energy Ω, generates reflected plane waves at $x \rightarrow -\infty$ and transmitted plane waves at $x \rightarrow +\infty$. Their energy ω may be arbitrarily due to the alternating potential. Suppose the stationary barrier transforms plane waves $\exp(\pm ix\sqrt{\omega})$ at $x = \pm\infty$ into wave functions $\psi_\omega^\pm(x)$, which satisfy the equations

$$\psi_\omega^{\pm\prime\prime} + (\omega - V)\psi_\omega^\pm = 0. \tag{2}$$

At $x \rightarrow +\infty$ there are only transmitted waves ψ_ω^+. At $x \rightarrow -\infty$ there are the incident wave ψ_Ω^- and reflected waves ψ_ω^-. At $x = 0$, one matches $\psi(t, +0)$ and $\psi(t, -0)$ according to Eq. (1). When, e.g., $x > 0$, one obtains, after simple transformations,

$$\psi(x,t) = \psi_\Omega^+(0) \int b(\omega)[\psi_\omega^+(x)/\psi_\omega^+(0)]\exp(-i\omega t)\,d\omega, \tag{3}$$

where $b(\omega)$ satisfies the integral equation

$$\chi(\omega)b(\omega) - \int_{-\infty}^{\infty} F^*(\omega - \omega')b(\omega')\,d\omega' = \chi(\Omega)\delta(\omega - \Omega), \quad b(\pm\infty) = 0, \tag{4}$$

$$\chi(\omega) = \{\ln[\psi_\omega^-(0)/\psi_\omega^+(0)]\}', \qquad F^*(\omega) = \int F(t)\exp(i\omega t)\,dt/2\pi. \tag{5}$$

The exact Eqs. (3) and (4) reduce the problem in two variables (x,t) to a 1D integral equation. By Eq. (3), the wave function $\psi_\Omega^+(0)$ reaches $x = 0$. There it is activated to the energy ω with the amplitude $b(\omega)$. Then it tunnels at this energy ω to $x > 0$ with the stationary tunneling amplitude $\psi_\omega^+(x)/\psi_\omega^+(0)$.

In a static potential $F(t) = \text{const}$, by Eqs. (4) and (5), $b(\omega) = (1 - F/\chi)^{-1}\delta(\omega - \Omega)$. This implies a tunneling resonance when $F = \chi(\Omega)$. In the WKB case [7(b)] $\chi \simeq 2\sqrt{W - \Omega}$, and the resonance condition is

$$F = \chi(\Omega) \simeq 2\sqrt{W - \Omega}. \tag{6}$$

(3) WKB approximation.—Since [see Sec. (1)] $T\Delta W \gg 1$, Eq. (4) allows for the WKB approach. It must be refined compared to the conventional one [3,4] in virtue of nonlocality and inhomogeneity of Eq. (4). So, first I find the complete set of homogeneous WKB solutions. Then I determine the (non-WKB) inhomogeneous solution in the vicinity of Ω, where $\chi(\omega) \simeq \chi(\Omega)$ allows one to solve Eq. (4) by Fourier transformation. As usual in the WKB approach, the regions where the WKB and $\omega \simeq \Omega$ solutions are valid overlap. By matching the solutions, I finally obtain $b(\omega)$ everywhere:

$$b(\omega) \sim \sum_n \{\chi(\omega)/\dot{F}[t_n(\omega)]\}\exp\left[i\int_\Omega^\omega t_n(\omega_1)\,d\omega_1\right], \tag{7}$$

$$F[t_n(\omega)] = \chi(\omega) \simeq 2\sqrt{W - \omega}. \tag{8}$$

Each term in Eq. (7) is a homogeneous WKB solution; by Eqs. (6) and (8), ω is the instantaneous resonant energy at the moments $t_n(\omega)$.

If $\max_t F(t) = F(0) = \chi(\omega_r)$, then $\omega > \omega_r$ provides a real $t_n(\omega)$. By Eq. (7), this implies that the activation amplitude oscillates with the energy $\omega > \omega_r$ rather than (as usual) exponentially decaying with it. This is the main new point. If a particle is activated to the lowest instantaneous eigenstate $\omega_r = W - F_0^2/4$, $F_0 = F(0)$, it travels from $\omega = \omega_r$ to the barrier top $\omega = W$ with the "elevator" of a time-dependent resonance (ERA). (Of course, this happens due to the long lifetime in the adiabatic level.) Effectively, the particle is lifted to W by the activation to $W - F_0^2/4$. When F_0 exceeds the critical value $F_c = 2\sqrt{\Delta W}$, then $b(\omega)$ in Eq. (7) has a zero activation energy.

If $\omega < \omega_r$, then $t_n(\omega)$ is a complex number. A convenient example to work with is the symmetry $F(-t) = F(t)$. If $F(t) = F_1(t^2/T^2)$, then $t_n(\omega)$ is purely imaginary for $\omega < \omega_r$:

$$t_n(\omega) = iT\tau_n(\omega), \qquad F(i\tau_n T) = F_1(-\tau_n^2) \simeq 2\sqrt{W - \omega}, \quad \tau_n > 0. \tag{9}$$

So, $T\tau_n$ is the "resonance moment" for the energy ω and the effective potential $-F(it)\delta(x) = -F_1^2(-t^2/T^2)\delta(x)$. The activation energy in Eq. (7) is related to $\min \tau_n \equiv \tau$. Substitute $b(\omega)$ from Eqs. (7) and (9) into Eq. (3). Since the barrier is opaque, use the WKB [7(b)] ψ_ω^+. After straightforward (albeit somewhat tedious) technical calculations, one obtains the total transmission amplitude $P(t)$ from $x = 0$:

$$P(t) \sim \int_\Omega^W \exp[\rho(\omega)]\,d\omega, \tag{10}$$

$$\rho(\omega) = -T\int_\Omega^\omega \tau(\omega_1)\,d\omega_1 - \int_0^{x_0(\omega)} |V(x) - \omega|^{1/2}\,dx - i\omega t, \tag{11}$$

where $x_0(\omega) > 0$ is the turning point $V(x_0) = \omega$.

(4) Tunneling AAT and ERA.—In an opaque barrier $|\rho(\omega)|$ is large, and only specific ω significantly contribute to P. The largest $\text{Re}\rho(\omega)$ determines the activation energy $E_A = -\text{Re}(d\ln P/dT)$. Consider $\text{Re}\rho(\omega)$ when $F_1(-\tau^2)$ monotonically increases with τ^2, and $t = 0$. Then $\tau(\omega)$ monotonically decreases with ω to $\tau = 0$ at the branching point ω_r; in its vicinity $\tau \propto (\omega_r - \omega)^{1/2}$. When $\omega > \omega_r$, then, by Eq. (8), $\tau(\omega)$ is imaginary. The change in the second term in Eq. (11) is related to the Buttiker-Landauer time [2]

$$\tau_{\text{BL}}(\omega) = \int_0^{x_0(\omega)} dx/2\sqrt{V(x) - \omega}.$$

Since both the "velocity" $(V-\omega)^{1/2}$ and the tunneling distance $x_0(\omega)$ decrease with ω, $\tau_{BL}(\omega)$ may increase or decrease with ω. In a characteristic general case, $\mathrm{Re}\rho(\omega)$ is schematically presented in Fig. 2. When T is sufficiently large (e.g., $T=T_8$ in Fig. 2), then $\mathrm{Re}\rho(\omega)$ is maximal at $\omega\simeq\Omega$, and "conventional tunneling" dominates in P in Eq. (10). When $T\leq T_7<T_8$, the $\mathrm{Re}\rho(\omega)$ maximum shifts to $\tilde\omega\geq\Omega$, where, by Eqs. (9) and (11),

$$F[i\tau_{BL}(\tilde\omega)T]=F_1^2[-\tau_{BL}^2(\tilde\omega)]=\chi(\tilde\omega)\,, \tag{12}$$

i.e., $\tilde\omega$ is the energy of the instantaneous resonance with the imaginary retardation time $(-i\tau_{BL}T)$. By Eq. (12), it is independent of the incident Ω. When $T=T_6$, the main contribution to transmission comes from particles which are activated to $\tilde\omega_6$ and tunnel with this energy thereafter (AAT). The transition to AAT occurs in a narrow interval $\Delta T/T\sim 1/\Omega T$ of $T=T_7$ and is followed by a quasijump in $dE_A/dT=-d^2\ln P/dT^2$ (see Fig. 3). When $T=T_5<T_6$, the $\rho(\omega)$ maximal value shifts to another AAT energy $\tilde\omega_5'$. The shift is followed by a rapid change in E_A. Finally, at $T=T_2$ AAT changes to ERA (again with the quasijump in dE_A/dT), with the transmitted $\omega\simeq W$ and $E_A\simeq\int_\Omega^{\omega_r}\tau(\omega_1)d\omega_1$. The competition of exponentially small transmission probabilities in P is similar to the competition of different phases in the partition function, with $\ln P$ playing the role of the free energy. This similarity is even more pronounced in a many-body theory [8].

The dependence $E_A(\Omega^{-1})$ is similar to $E_A(T)$. The quasijump in the first (or second) derivative implies a resonance-type shape of the second (or third) derivative. These are the resonances predicted in Ref. [11].

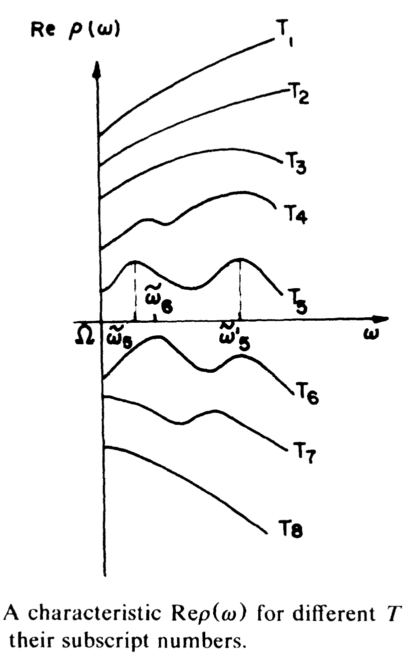

FIG. 2. A characteristic $\mathrm{Re}\rho(\omega)$ for different T's, which increase with their subscript numbers.

Consider a specific example [12] of

$$V(x)=W-a^2x^2,\quad F=\frac{F_0}{\cosh(t/T)} \tag{13}$$

Then AAT exponentially loses to ERA, and

$$P\sim\exp(-\pi\Delta W/4a)+\exp\{-T\Delta W[\eta-0.5\sin(2\eta)]\}\,, \tag{14}$$

$$\eta=\arccos(F_0/2\sqrt{\Delta W})\,.$$

The second term corresponds to ERA. The tunneling-ERA phase boundary is $\eta-0.5\sin(2\eta)=\pi/T$.

(5) Generalization.— Until now I have considered the special case of an alternating potential $-\delta(x)F(t)$ to analyze the accurate solution. However, the approach and the results may be generalized to 2D and 3D cases, as well as to the penetration of classical (electromagnetic, sonic, hydrodynamic, etc.) waves into the region forbidden in geometrical optics. I studied an alternating potential short compared to the length of an opaque barrier. In the leading approximation this led to a δ function. In higher dimensions a δ function may be replaced by a point potential, which has eigenstates; it was introduced in Ref. [13]. Then all of the above approach readily works. It verifies tunneling resonances and elevator activation [14].

In summary, transmission through an opaque stationary barrier and an alternating potential crucially depends on the alternating time T (see a typical Fig. 3). When $T>T_7$, tunneling dominates and the activation energy $E_A\simeq 0$. When T decreases, a quasikink at $T=T_7$ is followed by AAT with T-dependent $E_A(T)$. A quasijump in E_A at $T=T_5$ leads to a new AAT with a different $E_A(T)$. Finally, a quasikink at $T=T_2$ leads to ERA with T-independent E_A. The dependence $E_A(\Omega^{-1})$ is similar to $E_A(T)$.

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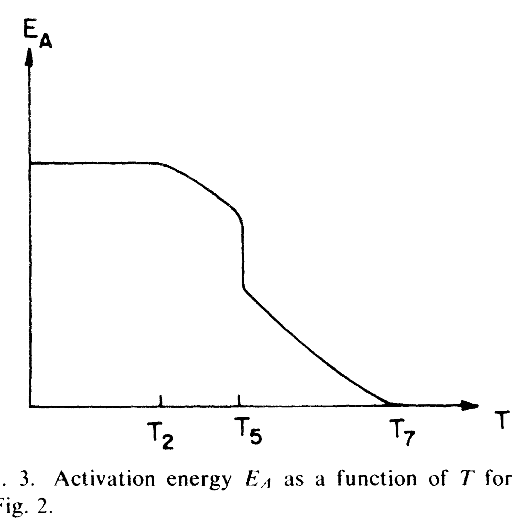

FIG. 3. Activation energy E_A as a function of T for $\rho(\omega)$ from Fig. 2.

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