Frequency Upconversion of Electromagnetic Radiation upon Transmission into an Ionization Front

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(Received 29 August 1991)

We have demonstrated that an underdense ionization front can significantly upshift the frequency of an impinging electromagnetic wave. Source radiation at 35 GHz was shifted to more than 116 GHz when a relativistically propagating, laser-produced ionization front interacted with microwave radiation that was confined in a resonant cavity. The amount of upshift was proportional to the plasma density in the front, in agreement with theoretical predictions.

PACS numbers: 52.40.Nk, 52.40.Db

When a pulse of electromagnetic radiation reflects from a moving mirror, both its frequency and its duration are altered by the relativistic Doppler effect [1] which depends strongly on the velocity of the mirror. It has been predicted that similar changes would occur if the radiation were to reflect from a moving plasma/neutral-gas boundary, an ionization front [2,3]. The Doppler shift is the same in both cases, but unlike the mirror, the ionization front has no kinetic energy. Therefore, in contrast to the moving mirror, the number of photons is not conserved upon reflection from the front. In order to reflect the impinging radiation, the front must be overdense; that is, the density of the plasma in the front must be above the critical density for electromagnetic radiation *when viewed from the rest frame of the front*. The critical density is defined as that density for which the characteristic plasma frequency ω_p equals the radiation frequency. The plasma frequency is defined by $\omega_p^2=4\pi n_0 e^2/m$, where n_0 is the plasma density, m is the electron's mass, and e is its charge. Because the frequency of the incident radiation is much higher when viewed from this frame and the plasma frequency is Lorentz invariant, creation of an overdense front requires that the plasma be more dense than for a stationary boundary. However, it has recently been predicted that large frequency upshifts and pulse compressions are possible even for *underdense* relativistic fronts, where the incident radiation is transmitted into the plasma [4]. In this case the degree of frequency upshift is proportional to the plasma density, and unlike reflection from an overdense front, is relatively insensitive to the front's velocity.

Developments in laser technology have enabled the generation of relativistically propagating ionization fronts by intense short-pulse lasers via photoionization. Several groups have reported that either a portion of the ionizing radiation or a separate probe pulse was blueshifted as it copropagated with the front [5]. This upshifting results from phase modulation due to the rapid formation of the plasma as the radiation propagates with the density gradient at the plasma/neutral-gas boundary. Novel radiation sources utilizing similar copropagating ionization fronts or relativistic plasma waves have been proposed [6]. To obtain large upshifts, these schemes require that the upshifting radiation and the density gradient copropagate over a considerable distance. Conversely, in a geometry in which radiation impinges upon the front, the radiation and the front need not propagate at the same velocity. The ultimate upshift in this case depends only on the maximum density in the front, and is not subject to dephasing between the upshifted pulse and the front. Furthermore, for large upshifts the incident pulse is significantly reduced in duration [4], as is the case for reflection from overdense fronts [3,7]. Tunable radiation sources derived from this technique have potential applications in many areas, including plasma diagnostics, spectroscopy, and remote sensing.

In this Letter we report the experimental confirmation of frequency upconversion of electromagnetic waves impinging upon relativistically propagating, underdense ionization fronts. Source radiation at 35 GHz was upshifted to more than 116 GHz when an ionizing laser pulse was passed through a resonant microwave cavity operating near its cutoff frequency. The front thus encountered a standing microwave field; a superposition of two counterpropagating waves, one traveling along with (*forward wave*) and the other traveling opposite to (*backward wave*) the front (see Fig. 1). Before the arrival of the laser pulse, the forward and backward radiation have the same frequency, $\omega_f=\omega_b=\omega$. The magnitude of their oppositely directed wave vectors is determined by the dispersion relation for electromagnetic radiation in the uniform, cylindrical waveguide, $\omega^2=\omega_c^2+c^2k_{f,b}^2$, where ω_c (Lorentz invariant in this geometry) is the cutoff frequency for the particular operating mode. They both propagate at the group velocity, $v_g=c(1-\omega_c^2/\omega^2)^{1/2}$, which can be significantly less than c for ω close to ω_c. The front propagates at approximately the velocity of the ionizing laser pulse in the plasma which can be very close to c. This velocity is given by $v_{\text{front}}\simeq c\{1-(\omega_c^2+\omega_p^2)/\omega_l^2\}^{1/2}$, where ω_l is the laser frequency.

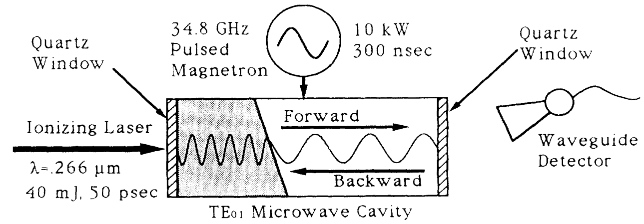

FIG. 1. Schematic of the experimental setup.

The upshifted frequencies can easily be derived in either the laboratory frame or the rest frame of the front [3,8]. Choosing the latter, we first make a Lorentz transformation to the rest frame of the ionization front. In this frame the front is a stationary boundary with plasma streaming away from it at $v_{\rm front}$ on one side and neutral gas streaming toward it on the other. When $v_g < v_{\rm front}$ both the forward and the backward waves propagate toward the front with frequencies in this frame given by $\omega'_{f,b} = \omega\gamma(1 \mp \beta v_g/c)$ ($-$ for f, $+$ for b). Here $\gamma \equiv (1-\beta^2)^{-1/2}$ is the relativistic Lorentz factor for the front, and $\beta \equiv v_{\rm front}/c$. If $(\omega_p^2+\omega_c^2)^{1/2} < \omega'_{f,b}$, then the front is underdense and the radiation is transmitted into it. As with any stationary boundary, the transmitted radiation has the same frequency as the incident radiation, but its wave number adjusts to obey the dispersion relation for electromagnetic radiation in the streaming plasma, namely, $\omega'^2_{{\rm plasma}\,f,b} = \omega_c^2 + \omega_p^2 + c^2 k'^2_{{\rm plasma}\,f,b}$. Thus, inside the plasma in the front's frame we have $\omega'_{{\rm plasma}\,f,b} = \omega'_{f,b}$ and $k'_{{\rm plasma}\,f,b} = (1/c)(\omega'^2_{f,b} - \omega_c^2 - \omega_p^2)^{1/2}$. To evaluate the frequencies of the upshifted radiation in the plasma in the laboratory frame, we now Lorentz transform back and obtain

$$\omega_{{\rm plasma}\,f,b} = \omega'_{{\rm plasma}\,f,b}\,\gamma\left\{1 - \beta\left(1 - \frac{\omega_c^2+\omega_p^2}{\omega'^2_{{\rm plasma}\,f,b}}\right)^{1/2}\right\}. \quad (1)$$

Because all of the cavity boundaries are stationary in the laboratory frame, the frequency of the radiation will not change as it leaves the cavity. For underdense fronts, Eq. (1) can be expanded to give

$$\omega_{{\rm plasma}\,f,b} \approx \frac{\omega}{2}\left(1 \mp \frac{\beta v_g}{c}\right)\left\{1 + \frac{\omega_c^2+\omega_p^2}{\omega^2(1 \mp \beta v_g/c)^2}\right\}. \quad (2)$$

Similarly, the upshifted wave numbers in the laboratory frame are

$$k_{{\rm plasma}\,f,b} \approx \frac{\omega}{2c}\left(1 \mp \frac{\beta v_g}{c}\right)\left\{1 - \frac{\omega_c^2+\omega_p^2}{\omega^2(1 \mp \beta v_g/c)^2}\right\}. \quad (3)$$

It follows from Eq. (2) that the upshifted frequencies increase linearly with plasma density and that $\omega_{{\rm plasma}\,f}$ is always greater than $\omega_{{\rm plasma}\,b}$. The forward wave always propagates in the forward direction; $k_{{\rm plasma}\,f}$ is always negative. On the other hand, $k_{{\rm plasma}\,b}$ changes sign when the term in brackets in Eq. (3) changes sign. Therefore, as the front's density increases, the backward wave's group velocity decreases to zero and then increases in the forward direction. This occurs when the group velocity of the backward wave in the front's frame is less than $v_{\rm front}$ [4]. This type of "reflection" from an underdense front can give rise to large frequency upshifts and temporal compressions that are much different than those predicted by the usual Doppler effect.

The reflection and transmission coefficients for a TE wave polarized perpendicular to the plane of incidence can be calculated in the front's frame by requiring that the tangential components of **E** and **H** be continuous across the boundary, as usual. Also, the transverse current must be continuous at the boundary since at the moment of ionization the electrons have zero net transverse velocity. This third boundary condition gives rise to an additional transmitted mode in the plasma, the free-streaming mode [4], which is a static magnetic field in the laboratory frame. By transforming the fields thus calculated back to the laboratory frame, we obtain the laboratory frame reflected and transmitted wave coefficients:

$$r \equiv \frac{E_r}{E_i} = \frac{(\epsilon'_i)^{1/2} - (\epsilon'_t)^{1/2}}{(\epsilon'_i)^{1/2} + (\epsilon'_t)^{1/2}}, \quad t \equiv \frac{E_t}{E_i} = \frac{2(\epsilon'_i)^{1/2}}{(\epsilon'_i)^{1/2} + (\epsilon'_t)^{1/2}}. \quad (4)$$

Here $\epsilon'_i \equiv 1 - \omega_c^2/\omega'^2_{f,b}$ and $\epsilon'_t \equiv 1 - (\omega_c^2+\omega_p^2)/\omega'^2_{f,b}$, with primed quantities specified in the front's frame.

For the range of plasma densities used in our experiments, $r \approx 0$ and $t \approx 1$. Although the ratio of the transmitted to incident powers at the front is always expected to be nearly unity, for large upshifts most of the incident energy is left in the free-streaming mode's static B field as the transmitted wave's duration shortens. The above model assumes that the ionization front is discontinuous. Similar results are obtained for finite-length underdense fronts, except that the energy in the free-streaming mode is instead converted to thermal energy in the plasma, as is the case for finite-length overdense fronts [3].

The experiments utilize a resonant microwave cavity that consists of a 1.2-cm-diam, 35-cm-long copper cylinder that is closed by 0.3-cm-thick quartz windows at each end (Fig. 1). Microwaves from a pulsed magnetron are fed into the cavity through the side wall at the midplane and excite the TE_{01} mode. The magnetron gives a flattop pulse with an instantaneous power of $\sim$10 kW and a duration of $\sim$300 nsec. The cavity is heated to 100 °C in order to support azulene (which is solid at room temperature) vapor at pressures from a few to several hundred mTorr. Azulene is used because it is easily ionized by 0.266-μm radiation. The ionizing laser pulse is 50 psec long (FWHM) and contains $\sim$40 mJ of energy at 0.266-μm wavelength. The output beam diameter is $\sim$1 cm and thus fills the cavity aperture. Upshifted radiation is monitored by a series of diode detectors, each preceded by a length of rectangular waveguide and a horn, which are mounted at various positions at both ends of the cavity. When the azulene pressure is low, the upshifted frequency of both the forward and backward waves is below the cutoff frequency of the detector waveguide and thus is not detected. Once the plasma inside the cavity is sufficiently dense to shift the waves above the detector cutoff, the signal rises sharply then decreases as the response of the detector rolls off with the higher frequencies that are generated at higher pressures. The measured pulsewidth of the radiation is detection

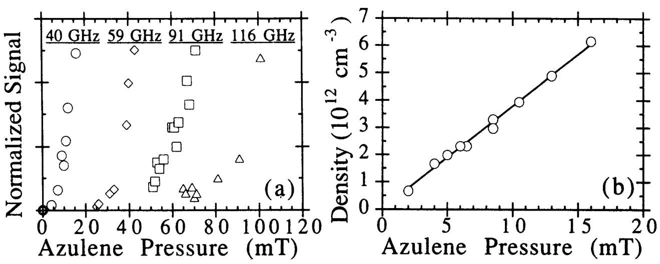

FIG. 2. (a) Onset of the upshifted signals in the forward direction vs azulene pressure; (b) measured line-averaged density vs azulene pressure.

limited to less than 750 psec (FWHM).

In Fig. 2(a) we plot the onset of the upshifted signals measured in the forward direction for four detector channels at 40, 59, 91, and 116 GHz as a function of neutral-gas pressure. In order to compare these frequencies with theory, we must convert the azulene pressures to plasma densities. We have measured the plasma density with a 65-GHz interferometer by counting the interference fringes during the plasma recombination, which occurs on a μsec time scale. The measured line-averaged plasma density is plotted as a function of azulene pressure up to 16 mTorr in Fig. 2(b). A least-squares fit to the data gives a conversion factor of $\sim 3.8 \times 10^{11}$ cm^{-3}/mTorr of azulene. With this information, the onset of the upshifted signals can be plotted versus front density and are shown in Fig. 3 where the horizontal bars represent the range of pressures over which the signals are sharply rising. The solid lines represent the theoretically predicted frequencies given by Eq. (1) for the forward and backward waves. The frequencies and duration of these signals are in reasonable agreement with the theory for the forward wave.

The detection of the backward wave is complicated by several factors. As shown in the inset of Fig. 3, the group velocity of the backward wave, which is initially $0.47c$, is significantly reduced over the whole range of plasma densities obtainable in these experiments. This causes an increase in the pulse length of the backward wave, thus reducing its power, which is what we detect. Also, when the backward wave is near cutoff, its transmission coefficient at the quartz boundary is small and the backward upshifted radiation is trapped in the cavity. The radiation detected in the backward direction has two sources, the backward wave and the forward propagating radiation that has reflected from the quartz output window. Because the upshifted backward wave begins to exit the cavity at the moment the ionizing laser pulse arrives, the reflected forward wave is delayed by the transit time of the laser pulse plus that of the forward radiation, $\sim$2–3 nsec. We use a fast photodiode positioned near the entrance to the cavity to detect the arrival of the laser pulse and provide a time fiducial. We then record the temporal evolution of the radiation emitted in both the forward and backward directions as a function of azulene

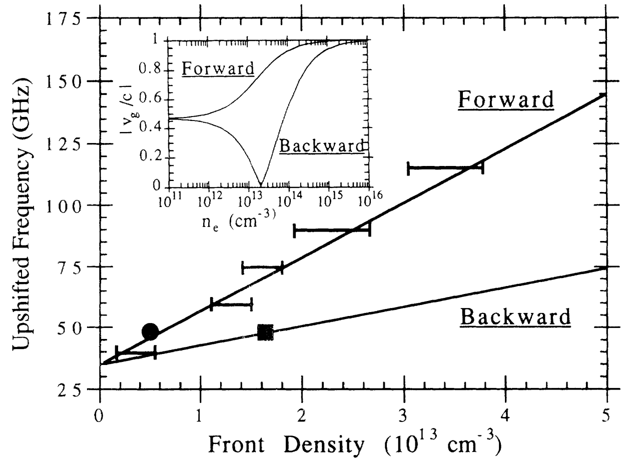

FIG. 3. Experimentally detected upshifted signals vs the measured front density. The solid lines are the predictions of Eq. (1). Inset: The variation of the group velocity of the forward and backward radiation as a function of the front density.

pressure.

Figures 4(a) and 4(b) show the signals detected in the forward and backward directions on the 48-GHz channel at an azulene pressure of 16 mTorr. Both signals exhibit a single subnanosecond peak and onset at approximately the same pressure, which is plotted as the solid circle in Fig. 3. At this pressure, only the forward wave is expected to be upshifted above 48 GHz. We therefore interpret the backward signal as resulting from upshifted forward radiation that has reflected from the output window into the backward direction. Figure 4(c) shows the forward signal at 71-mTorr azulene pressure and Fig. 4(d) shows the backward signal at the same pressure. The forward signal continues to have only one peak, but a second earlier peak appears in the backward direction at higher pressures. The pressure at which this earlier peak onsets

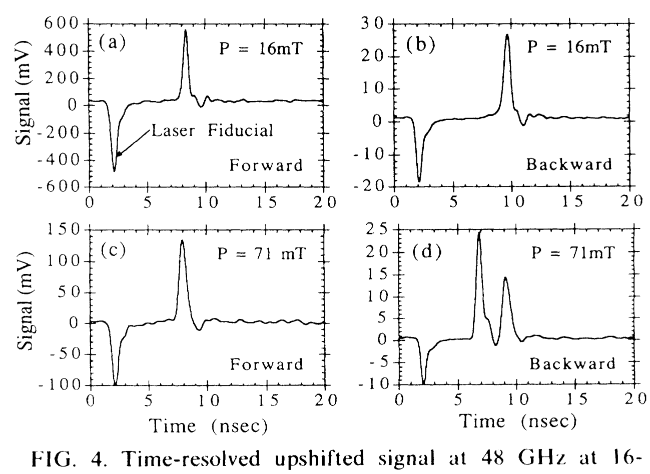

FIG. 4. Time-resolved upshifted signal at 48 GHz at 16-mTorr azulene pressure detected in (a) the forward direction and (b) the backward direction, and at 71 mTorr in (c) the forward direction and (d) the backward direction.

is shown by the solid square in Fig. 3. The onset occurs at approximately the predicted pressure for the backward wave to upshift to more than 48 GHz. This, together with the ~2.4-nsec delay between the arrival of the two peaks, is consistent with identifying the earlier peak as the upshifted backward wave. However, Eq. (3) predicts that the backward wave reverses its propagation direction and exits the cavity in the forward direction when the front density is above 2×10^{13} cm^{-3}. This turn-around density corresponds to an azulene pressure of ~50 mTorr. Above this pressure, the forward and backward radiation are expected to arrive coincidently at the forward detector. Therefore, with our present diagnostics, we are unable to differentiate between the two waves in the forward direction. That the peak associated with the backward wave persists in the backward direction at higher pressures is an issue that requires further investigation.

At 49 GHz, where the backward wave is close to cutoff, we expected that the pulse would be ~10 nsec long. The observed ~1-nsec pulse lengths, however, indicate that the upshifted radiation is being damped on a nanosecond time scale. We obtained an estimate of the damping rate from our interferometry data. At an azulene pressure of 30 mTorr the fringe contrast ratio of the 65-GHz interferometer was reduced to approximately 20% of its value without plasma. This degree of damping for 65-GHz radiation double passing the cavity corresponds to an *e*-folding time of approximately 1.5 nsec at a density of 1×10^{13} cm^{-3}. The backward pulses will be approximately a nanosecond in duration with the remainder of the radiation being damped away at this rate before it can leave the cavity. This should not significantly affect the pulse length of the forward wave, which is already expected to be subnanosecond.

We obtained an estimate of the conversion efficiency of this process by measuring the antenna patterns of the upshifted radiation in the forward direction. The total source power radiated in the forward direction is ~5.8 kW when the laser is blocked. The upshifted power at 40 GHz is ~3.2 kW, giving a conversion efficiency of greater than 50%. The efficiency thus measured is less than 1% at 91 GHz. This lower-than-expected value at higher frequencies may be largely due to a reduction in the detection circuit's response to the progressively shorter subnanosecond pulses.

In conclusion, we have demonstrated that a relativistically propagating, underdense ionization front can significantly upshift the frequency of an impinging electromagnetic wave. The degree of upshift, which is proportional to the plasma density in the front, has been tuned continuously from 35 to more than 116 GHz by varying the neutral-gas pressure in the microwave cavity. The forward-propagating radiation is upshifted by a larger amount than the backward-propagating radiation for a given plasma density, and subnanosecond upshifted pulses were observed. These results are in good agreement with theoretical predictions and herald a new class of tunable radiation sources.

We gratefully acknowledge many useful discussions with G. Shvets, D. Hopkins, G. M. Rebeiz, A. M. Sessler, T. W. Johnston, and C. E. Clayton and the assistance of J. A. Deeds. This work was supported by the U.S. Department of Energy under Contract No. DE-AS03-83-ER40120 and Grant No. DE-FG03-91-ER12114.

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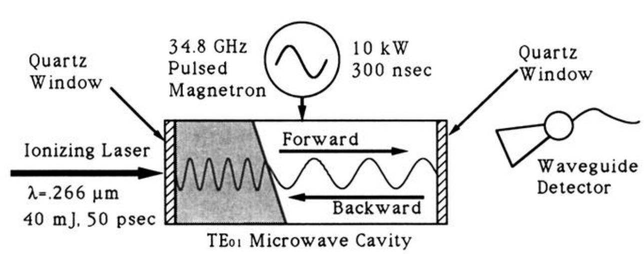

FIG. 1. Schematic of the experimental setup.