Measuring the Probability Distribution of the Relative Velocities in Grid-Generated Turbulence

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Homodyne photon correlation spectroscopy (HCS) and laser Doppler velocimetry (LDV) were used to study the probability density function of velocity differences $\delta v(l)$ between points in the fluid separated by a distance l. The two measuring schemes yield different results for the probability density, $P(\delta v(l))d\delta v(l)$. HCS probes the spatial fluctuations directly, whereas LDV records temporal fluctuations in velocity and relates them to the spatial fluctuations through the Taylor hypothesis. The measurements therefore imply the failure of this hypothesis in the Reynolds number range where we have applied it.

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In turbulent flow the quantity of interest is not so much the velocity itself, $\mathbf{v}(\mathbf{r},t)$, as the instantaneous difference in velocity at two points separated by a distance l. This quantity $\delta\mathbf{v}(\mathbf{l})\equiv\mathbf{v}(\mathbf{r}+\mathbf{l},t)-\mathbf{v}(\mathbf{r},t)$ is characterized by a probability density function, $P(\delta v(l))d\delta v(l)$, where v is the measured component of $\mathbf{v}$ and l is the magnitude of $\mathbf{l}$. Herein we report measurements of $P(\delta v(l))$ in turbulent flow generated by a grid in a water tunnel. The experiments explore a range of l values and span a range of Reynolds numbers under conditions that possibly lie between chaotic behavior and fully developed turbulence.

Our measurements of $P(\delta v(l))$ will be compared with a measured probability density function $P'(\delta v(l))$ deduced from a record of the velocity as a function of time, recorded at a single point in the turbulent fluid. Both types of measurement were made at almost exactly the same position in the turbulent stream. To deduce spatial information from the time record of the velocity, it is necessary to invoke the so-called Taylor hypothesis (frozen turbulence assumption) [1], which simply means that $v(t)$ is replaced by $v(x/U)$, where U is the mean flow velocity and x is a coordinate in the flow direction. Virtually all measurements of the probability density function for velocity differences on scales l have been made in this way [2].

In the work reported here, P and P' are found to be very different, implying a failure of the Taylor hypothesis. As for P, it is well represented by the product of Gaussian and Lorentzian terms. The Lorentzian factor is found to have a much smaller width, $u_L(l)$, than the width $u_G(l)$, of the Gaussian factor. The new measurements reported here substantiate those made in a much smaller water tunnel [3–5]. The function P' is much more Gaussian in form than P, but it also gives a heavier weighting to large velocity fluctuations than does a purely Gaussian function. If one parametrizes P' by the product of Gaussian and Lorentzian factors, the widths of these two factors are comparable in magnitude. For our measurement, the frozen turbulence assumption fails, i.e., P and P' are conveying different information [6].

The schemes for measuring P' and P are laser Doppler velocimetry (LDV) and homodyne photon correlation spectroscopy (HCS), respectively. The former technique is a standard one [7], whereas the HCS method has been used only recently [3,8–11]. Both of these methods require the seeding of the fluid with small particles which follow the local flow and scatter light. The HCS method is sensitive to velocity differences rather than to the velocity itself, because it registers the beating of Doppler shifts coming from pairs of particles moving relative to each other [3]. As will be discussed below, the optical scheme used here records velocity differences for all values of l from the smallest eddy size present out to a size L, which is the width of a slit through which the light passes before it reaches the photodetector. Because one measures a scattered intensity rather than a scattered electric field, the HCS method yields information about the symmetric part of the probability density only. It is this part of the full probability density function that we label as $P(\delta v(l))$. Past measurements of P' in highly turbulent flows show that this function is not always symmetric [12,13]. In the present LDV measurements $P'(\delta v(l))$ is also asymmetric, though the asymmetry is small.

Our HCS measurements of $P(\delta v(l))$ show it to be well approximated by the product of Lorentzian and Gaussian functions only if the Reynolds number of the flow, Re, exceeds some critical value, $\mathrm{Re}_c\simeq 300$. Here we define Re as $\mathrm{Re}=aU/\nu$, where a is the mesh size of the grid ($a=8.5$ mm) and ν is the kinematic viscosity of water (0.01 $\mathrm{cm^2/sec}$). The widths of the Lorentzian and Gaussian factors are explicitly defined by the equation

$$P=\frac{\exp[-u_L(l)^2/2u_G(l)^2]}{\pi\,\mathrm{erfc}[u_L(l)/\sqrt{2}u_G(l)]}\times\exp[-\delta v(l)^2/2u_G(l)^2]\frac{u_L(l)}{\delta v(l)^2+u_L(l)^2}\,. \tag{1}$$

The ratio of these widths, $M \equiv u_G(l)/u_L(l)$, is found to be approximately 3. This ratio is only weakly dependent on l and Re in the range of parameters probed in this work, namely, $0.3 < L < 2.5$ mm, $300 < \mathrm{Re} < 1800$.

As discussed below, the quantity actually measured with the HCS scheme is the intensity correlation function $g(t)$ of light scattered by the seed particles. This function is observed to have a self-similar form, which it would not have unless both widths, $u_L(l)$ and $u_G(l)$, are proportional to l^h, with a common value of the exponent h. In this experiment, as in the earlier work, using a smaller water tunnel, h is observed to increase with increasing Re from $h \simeq 0$ to $\frac{1}{3}$, the Kolmogorov value [14].

The experiments were carried out in the water tunnel that was 10 cm×10 cm in cross section and 1.1 m in length. The measurements were made at an axial point 25 cm downstream from the grid. The spanwise profile of the flow was flat, making the time-averaged velocity gradient at the measuring point for P and P' negligibly small [11].

The water was seeded with polystyrene particles (diameter 0.106 μm), which were small enough to follow the local flow. The light source was the mildly focused beam from an argon-ion laser operating at the wavelength $\lambda = 488$ nm. The incident beam traveled perpendicular to the flow direction. The scattered light was detected in a direction that was also perpendicular to the flow. The incident beam traced out a clearly visible thin line in the flowing water, the diameter of this beam being less than 0.1 mm. The scattered light, at a scattering angle $\theta = 90°$, was imaged at 1:1 magnification on a slit of adjustable width L. This width is a crucial parameter in the experiment because it determines the maximum measurable eddy size, $l = L$, to which $g(t)$ is sensitive. Therefore l is the eddy size in the direction perpendicular to the mean flow. More than 1.5 m behind the slit was a photomultiplier which recorded the scattered intensity $I(t)$. The output of the photomultiplier is a train of identically shaped pulses which are sent to the correlator, whose output is the intensity correlation function $g(t) = \langle I(t'+t)I(t')\rangle/\langle I(t')\rangle^2$.

Under approximations which are well satisfied in these experiments, one can show [3] that $g(t)$ is given by

$$g(t) = 1 + A(t)\int_0^L b(l)dl \int_{-\infty}^{\infty} P(\delta v(l))\cos[k\delta v(l)t]d\delta v(l) = 1 + G(t)\,, \tag{2}$$

where $k = (4\pi n/\lambda)\sin(\theta/2)$, and $\delta v(l)$ is the component of $\delta \mathbf{v}(l)$ along the direction of $\mathbf{k}$. In this experiment $\mathbf{k}$ was perpendicular to the flow direction. The refractive index of the scattering medium (water) is $n = 1.33$. Quite generally, $G(t)$ is a decaying function, with characteristic decay time of the order of $1/ku(L)$, where $u(L)$ is the characteristic velocity difference over a slit width L. The function $b(l) = (2/L)(1 - l/L)$ is the probability that a pair of particles, separated by l, are to be found in the slit [3,11]. This form of $b(l)$ is correct only if L is much greater than the beam diameter [3,8]. The factor $A(t)$ represents the Brownian diffusion of the seed particles and takes account of the fact that even in the absence of turbulent flow, $G(t)$ will decay slowly and is of the form $A(t) \sim \exp(-2Dk^2t)$, where D is the diffusion constant. This parameter is determined from the diameter of the seed particle and the viscosity of water [15]. All of the experimental data have been corrected for this Brownian motion factor [11].

According to Eq. (2), if $P(\delta v(l))$ has the simple scaling form $P(\delta v(l)) = Q(\delta v(l)/u(l))$, and if $u(l) \sim l^h$, then $G(t)$ has the scaling form $G(k,t,L) = G(x)$, where $x = ku(L)t$. Therefore a plot of $\log G(t)$ vs $\log t$, for fixed Re but various values of L, should produce curves all of which should be superimposable when translated along the time axis. This is indeed what was observed in the earlier experiments [3] and in the present ones. Figure 1 shows the normalized $G(t)$ measured at the three slit widths $L = 0.5$, 1.1, and 2 mm, the Reynolds number being $\mathrm{Re} = 1490$. These measurements yield $h = 0.31$.

Figure 2 is a semilog plot of $G(t)$ at $\mathrm{Re} = 1490$, $L = 1.3$ mm. Also shown is the best fit by Eq. (2), assuming that P is purely Gaussian $\{P = [1/\sqrt{2\pi}u_G(l)]\exp[-\delta v(l)^2/2u_G(l)^2]\}$ (dashed line), and purely Lorentzian $\{P = u_L(l)/\pi[u_L(l)^2 + \delta v(l)^2]\}$ (solid line). These two lines correspond to $u_L(L) = u_G(L) = 0.216$ cm/sec. Variation of these parameters could not produce a good fit to these data with either the Gaussian or Lorentzian probability density function, though the Lorentzian fit is slightly superior. We have shown a Gaussian form for $P(\delta v(l))$ because it is sometimes observed for large values of l [16]. A probability density of dominantly Lorentzian form gave a satisfactory fit to the measurements in Ref. [4].

To proceed further we take note of the fact that the cosine factor in Eq. (2) implies that $G(t)$ must approach

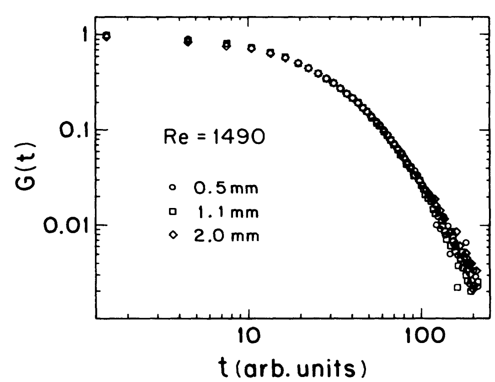

FIG. 1. Superimposed log-log plot of $G(t)$ for the three indicated slit widths L and at the Reynolds number indicated.

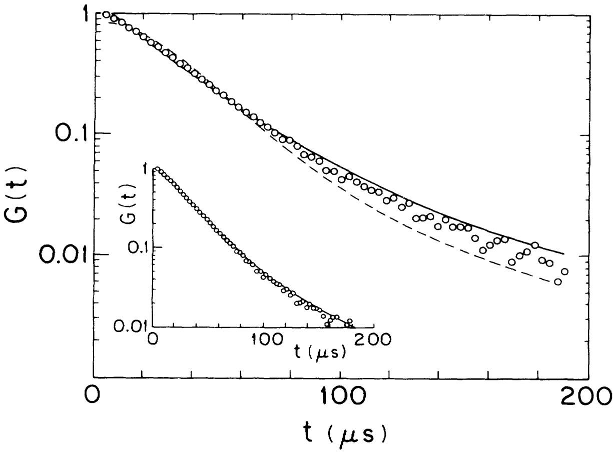

FIG. 2. Semilog plot of $G(t)$ at Re = 1490, L = 1.3 mm. The solid line is a Lorentzian fit to the data, and the dashed line is a Gaussian fit. The characteristic widths $u_L(L)$ and $u_G(L)$ of both factors are 0.216 cm/sec. The inset shows the same data fitted using Eqs. (1) and (2).

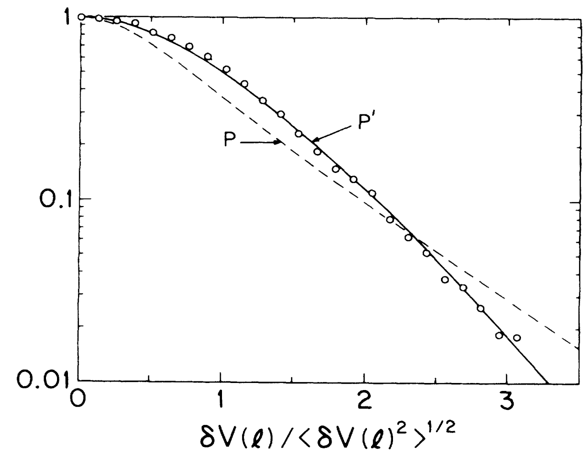

FIG. 3. Semilog plot of the symmetric part of the probability density function as measured by laser Doppler velocimetry, the abscissa being $\delta v(l)/[\langle \delta v(l)^2 \rangle]^{1/2}$. The solid line is a fit to these data using Eq. (1) and the dashed line is the function P extracted from the HCS measurements, using Eqs. (1) and (2). The measurements were at Re = 1490 and L = 1.3 mm.

the origin with zero slope $[G(t) \sim 1 - \text{const} \times t^2]$, even though experimental limitations on the electronics bar an unambiguous observation of this behavior. We therefore attempted the fit to $P(\delta v(l))$ in Eq. (2) with the product of a Lorentzian and a Gaussian function, the widths of these factors being adjustable parameters. As seen in the inset in Fig. 2 (solid line), it was possible to obtain a very good fit to the data with this form of P. A similarly good fit with this form of $G(t)$ was obtained at all values of L and Re. For each of the parameter pairs, $u_G(l)$ and $u_L(l)$, we have evaluated $M(l,\text{Re})$ and find it to be approximately 3. There is no discernible trend with changing Re. Similar values of M were obtained in Ref. [4]. Onuki [17] has suggested that two widths, $u_G(l)$ and $u_L(l)$, characterize velocity fluctuations inside and outside of the active regions. Abundant experiments suggest that the active regions of the turbulent dissipation lie on a fractal or multifractal [18].

Next we turn to the LDV measurements of $P'(\delta v(l))$. With the LDV technique, the seed particles traverse optical interference fringes, modulating the intensity of the scattered light received by the photomultiplier. A commerically available counter-signal conditioner records the frequency of these pulses and hence determines $v(t)$ at the observation point. The Taylor hypothesis is then invoked to convert the $v(t)$, a temporally fluctuating signal, into $v(x)$, where x is a coordinate in the flow direction [7]. The Taylor hypothesis is generally assumed to be valid when the ratio f of the rms fluctuations in the velocity about its mean value, U, is a small fraction of U itself. This condition was well satisfied in the present experiments where f was always less than 0.04. By splitting up the time record $v(t)$ into segments of equal length δt, and replacing δt by l/U, we create an ensemble of velocity differences, $\delta v(l)$, from which $P'(\delta v(l))d\delta v(\delta l)$ can be constructed with a histogram.

Figure 3 is a semilog plot of an LDV measurement of the symmetric part of P' vs $\alpha \equiv \delta v(l)/[\langle \delta v(l)^2 \rangle]^{1/2}$ at Re = 1490 and l = 1.3 mm for $\delta v(l) > 0$. The antisymmetric part of P' was observed to be very small. Over the limited range of α spanned by our measurements, these data agree very well with the measurements of Anselmet *et al.* [12] on the duct flow at a very high Reynolds number (based on the Taylor microscale) of 515. The solid line in the figure is a best fit to these data, assuming that P' is of the Lorentzian-Gaussian form of Eq. (1). The parameters producing this fit are $u_L(l)$ = 0.364 cm/sec and $u_G(l)$ = 0.385 cm/sec, giving M = 1.06. This ratio is much smaller than M obtained from the HCS experiments. Those LDV measurements, which spanned the same range of Re and L as the HCS experiments, could be fitted with a density function containing the product of Lorentzian and Gaussian factors. However, the exponent $h(\text{Re})$ could not be determined without making questionable assumptions [11].

The dashed line in Fig. 3 is the function P extracted from the HCS measurements made using the same values of Re and L as the LDV data in the figure. The parameters associated with this HCS measurement are $u_L(L)$ = 0.256 cm/sec and $u_G(L)$ = 0.695 cm/sec. Note that the Gaussian factor is weighted much more heavily than for the HCS measurements, i.e., M is smaller. The different shapes of the two curves P and P' vs α in Fig. 3 suggest a failure of the frozen turbulence assumption at moderate Re and small scales. This failure would be assured if the turbulence were established to be isotropic. It was not possible to measure P with precision when $\delta\mathbf{v}$ was parallel to the flow, but it was found that, for different directions of $\delta\mathbf{v}$, M was still larger than its value obtained from the LDV measurement of P'. The value of $h(\mathrm{Re})$ in this experiment was the same as that measured in the experiments discussed above, where $\delta\mathbf{v}$ was perpendicular to the flow.

The failure of the frozen turbulence assumption indicates that the small eddies, which are being probed here, do not maintain their geometrical shape as they are convected past the observation point at the mean velocity U. A possible cause is the coupling between velocity fluctuations at large scales and spatial gradients of velocity at small scales, a point that has received much theoretical attention [19–21].

Recent measurements of the probability density function at high Reynolds numbers, using the Taylor hypothesis, show an exponential behavior for large α, and a Gaussian-like form for small α [12,13]. Even though we could fit our data with a Gaussian-Lorentzian form, our HCS measurements of P look similar to theirs (see Fig. 3).

In summary, we have carried out measurements which expose the Taylor hypothesis (frozen turbulence assumption) to an experimental test and have measured the functional form of the probability density $P(\delta v(l))$ over a range of eddy sizes l in the Reynolds number range 300 to 1800. The function P is well represented by the product of Gaussian and Lorentzian factors but not by either factor alone or by an exponential function. These observations indicate the importance of probing turbulence in both the time and space domains.

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