Spatial Solitons in Photorefractive Media

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(Received 3 October 1991)

We show that photorefractive media can support a new type of spatial soliton, in which the diffraction is balanced by the self-scattering (two-wave mixing) of the beam spatial frequency components. This photorefractive soliton possesses some unique properties, such as independence of the absolute light intensity, and can experience absorption (or gain) with no change in its transverse structure.

PACS numbers: 42.65.−k, 42.50.Qg, 42.70.−a

Light solitons in space (spatial solitons) have been the object of intensive theoretical and experimental research during the last three decades. The solitons evolve from nonlinear changes in the refractive index of the material, induced by the light-intensity distribution. When the confining effect of the refractive index exactly compensates for the effect of diffraction, the beam becomes self-trapped and is called a spatial soliton. The nonlinear effects which are responsible for soliton formation are in general Kerr-like effects, inducing local index changes proportional to the local light power. The index changes needed for spatial solitons require high power densities, and these often exceed 1 MW/cm^2 (see Ref. [1]).

We describe in what follows a new type of spatial soliton, generated by the photorefractive (PR) effect of the medium. The shape of the soliton modulates the refractive index via the PR effect, which results in an exact compensation for the effects of diffraction, and causes the light beam to propagate with an unvarying profile. This index modulation is represented in the formalism as a distribution of index gratings, each one of them induced by the interference between two spatial (frequency) plane-wave components of the light beam. Since the efficiency of this effect is independent of the absolute light intensity, these new solitons can be generated even at very moderate light intensities. Moreover, a given soliton wave form can propagate unchanged in the medium, at very high or very low light intensities (and at all levels in between).

The PR solitons correspond to steady-state solutions of the nonlinear wave equation which describes beam propagation in PR media and accounts for both diffraction and the mutual interaction between each pair of spatial components of the soliton beam. Since the key to this nonlinear scattering process is grating formation by a *continuum* of Fourier (plane-wave) components of the soliton beam, we cannot resort to the two-plane-waves analysis commonly applied to PR materials. Our general formalism accounts for the transverse beam spatial structure.

We start by deriving the nonlinear wave equation which describes the propagation of a monochromatic optical beam of a given frequency (ω) and polarization, traveling in the positive direction of an arbitrary axis z. We assume the absence of nonlinear interaction between orthogonal polarizations (anisotropic scattering [2]), so that our problem can be reduced to a scalar formulation. Light propagation in nonlinear media can be conveniently described using coupled-mode theory [3], applied to the case of unbounded media for which an appropriate set of spatial modes is the continuum of plane waves [4]. The electric field associated with the light beam propagating primarily along the z direction is written as

$$E(\mathbf{r},z,t)=\frac{1}{2}\left[e^{i(kz-\omega t)}\int E(\mathbf{q},\mathbf{r})e^{i(\beta_q-k)z}f(\mathbf{q},z)d\mathbf{q}+\text{c.c.}\right]$$
$$\equiv \tfrac{1}{2}[A(\mathbf{r},z)e^{i(kz-\omega t)}+\text{c.c.}]\,, \qquad (1)$$

where, employing the paraxial approximation, $\beta_q \ll q$, and

$$E(\mathbf{q},\mathbf{r})=(1/2\pi)(\mu_0/\varepsilon_0 n_1)^{1/2}e^{i\mathbf{q}\cdot\mathbf{r}}\,, \qquad (2)$$

$\mathbf{r}\equiv(x,y)$, $k=\omega n_1/c$ is the light wave number, n_1 is the unperturbed index of refraction in the medium, and $f(\mathbf{q},z)$ is the spatial frequency (angular) distribution of the complex amplitude $A(\mathbf{r},z)$. A spatial mode (plane-wave component) is characterized by the projections of its wave vector ($\mathbf{q}$ and β_q) on the transverse ($\mathbf{r}$) and longitudinal (z) directions, respectively, with $\beta_q=(k^2-q^2)^{1/2}$ (where q is restricted to $0\le q\le k$). Assuming negligible absorption, and under the rather general conditions specified in Ref. [4], it is easy to show that $A(\mathbf{r},z)$ obeys, in the presence of a refractive-index distribution $n(\mathbf{r},z)=n_1+\delta n(\mathbf{r},z)$, the differential equation

$$\left(\frac{\partial}{\partial z}-\frac{i}{2k}\nabla_{\mathbf{r}}^2\right)A(\mathbf{r},z)=\frac{ik}{n_1}\delta n(\mathbf{r},z)A(\mathbf{r},z)\,. \qquad (3)$$

The nonlinear term $\delta n(\mathbf{r},z)$ is obtained by considering the matrix process between two plane waves. When only one pair of plane waves (spatial modes) $\mathbf{q}_1$ and $\mathbf{q}_2$, of field amplitudes $a_1(z)$ and $a_2(z)$, is present in the medium, it induces an index grating $\delta n(\mathbf{r},z)$ which is proportional to the time-averaged interference pattern between the waves. The proportionality coefficient is a complex factor $\delta\hat{n}(\mathbf{q}_1,\mathbf{q}_2)$, which represents the PR coupling coefficient between the two plane waves, given the material properties (the orientation of the PR crystalline medium, its trap density P_d, n_1, and dc dielectric constant ε_r) and the polarization of the waves. In this simple case, $\delta n(\mathbf{r},z)$ is [5]:

$$\delta n(\mathbf{r},z)=(1/I_0)\{a_1(z)e^{i(\mathbf{q}_1\cdot\mathbf{r}+\beta_{q_1}z)}a_2^*(z)\times e^{-i(\mathbf{q}_2\cdot\mathbf{r}+\beta_{q_2}z)}\delta\hat{n}(\mathbf{q}_1,\mathbf{q}_2)+\text{c.c.}\}, \quad (4)$$

where $I_0=|a_1|^2+|a_2|^2$ is the absolute light intensity. Since $\delta n(\mathbf{r},z)$ is real (no absorption) we get [5] $\delta\hat{n}(\mathbf{q}_1,\mathbf{q}_2)=\delta\hat{n}^*(\mathbf{q}_2,\mathbf{q}_1)$. The term $\delta\hat{n}(\mathbf{q}_1,\mathbf{q}_2)$ is the product of two factors: $r_{\text{eff}}(\mathbf{q}_1,\mathbf{q}_2)$, which is the scalar product of the material electroptic tensor (and depends on the orientation of the PR crystalline medium), and $E_m(\mathbf{q}_1,\mathbf{q}_2)$, which is the coefficient of the induced space-charge field [6] and depends only on the interference grating wave vector ($\mathbf{K}_g$) between $\mathbf{q}_1$ and $\mathbf{q}_2$.

When more than two plane waves are present, $\delta n(\mathbf{r},z)$ involves a summation over all the possible interacting plane-wave pairs. For a given light beam $A(\mathbf{r},z)$, which consists of a continuous spatial-frequency spectrum of plane waves $f(\mathbf{q},z)$, this summation takes the integral form:

$$\delta n(\mathbf{r},z)=\frac{1}{|A(\mathbf{r},z)|^2}\int d\mathbf{q}_1\int d\mathbf{q}_2 f(\mathbf{q}_1,z)f^*(\mathbf{q}_2,z)\times E(\mathbf{q}_1,\mathbf{r})E^*(\mathbf{q}_2,\mathbf{r})e^{i(\beta_{q_1}-\beta_{q_2})z}\delta\hat{n}(\mathbf{q}_1,\mathbf{q}_2). \quad (5)$$

Note that since the PR nonlinearity is independent of the absolute light intensity, δn is normalized by the factor $|A(\mathbf{r},z)|^2$. In addition, a constant factor representing the dark irradiance [7] may be added to the light intensity in the denominator of Eq. (5), to avoid unrealistic divergence of δn in dark regions. In our analysis we neglect this constant. We note that this model of beam propagation in photorefractive media has proven effective in the interpretation of a variety of wave-mixing processes [6], and in particular, was used for predicting a number of new phenomena (such as incoherent backscattering [8]).

Here we are interested in the simplest way of generating a spatial soliton. Since diffraction can be viewed as due to a linear phase accumulation in each plane-wave component of the light beam, the simplest way to compensate for it is through equal and opposite nonlinear phase delays, as in the Kerr-like solitons. Accordingly, we assume the PR coupling coefficient $\delta\hat{n}(\mathbf{q}_1,\mathbf{q}_2)$ (and hence E_m) to be real, and hence introduce nonlinear phases only (i.e., no "energy transfer" between the pairs of spatial components). Furthermore, since diffraction is essentially a symmetric process, the simplest solution is obtained by requiring a symmetric nonlinear process, i.e., $\delta n(\mathbf{r},z)=\delta n(-\mathbf{r},z)$ [and hence $\delta\hat{n}(\mathbf{q}_1,\mathbf{q}_2)=\delta\hat{n}(-\mathbf{q}_1,-\mathbf{q}_2)$], which implies a symmetric soliton wave form, $A(\mathbf{r},z)=A(-\mathbf{r},z)$.

In the most general case, $\delta\hat{n}(\mathbf{q}_1,\mathbf{q}_2)$ can be expressed as

$$\delta\hat{n}(\mathbf{q}_1,\mathbf{q}_2)=\int\int g(\rho,\rho')e^{-i(\mathbf{q}_1\cdot\rho+\mathbf{q}_2\cdot\rho')}d\rho\, d\rho', \quad (6)$$

so that, from Eqs. (1) and (5), we get

$$\delta n(\mathbf{r},z)=\frac{1}{|A(\mathbf{r},z)|^2}\int\int A(\mathbf{r}-\rho,z)A^*(\mathbf{r}+\rho',z)g(\rho,\rho')d\rho\, d\rho'. \quad (7)$$

Note the explicit nonlocal nature of the PR effect, which is brought out by Eq. (7). By inserting Eq. (7) into Eq. (3), the equation of evolution of the electromagnetic field reads

$$\left(\frac{\partial}{\partial z}-\frac{i}{2k}\nabla_r^2\right)A(\mathbf{r},z)=\frac{ik}{n_1}\frac{1}{A^*(\mathbf{r},z)}\int\int A(\mathbf{r}-\rho,z)A^*(\mathbf{r}+\rho',z)g(\rho,\rho')d\rho\, d\rho', \quad (8)$$

where the integral on the right-hand side accounts for the nonlocal nature of the photorefractive effect. In particular, if we look for soliton solutions, we require

$$A(\mathbf{r},z)=U(\mathbf{r})e^{i\gamma z}, \quad (9)$$

where $U(\mathbf{r})$ and γ are real, γ being the characteristic soliton propagation constant. Equation (8) becomes then

$$\left(\gamma-\frac{1}{2k}\nabla_r^2\right)U(\mathbf{r})=\frac{k}{n_1}\frac{1}{U(\mathbf{r})}\int\int U(\mathbf{r}-\rho)U(\mathbf{r}+\rho')g(\rho,\rho')d\rho\, d\rho'. \quad (10)$$

The integrodifferential Eq. (10) can be transformed into an ordinary differential equation by using the Taylor expansion of $U(\mathbf{r}-\rho)$ around $\rho=0$:

$$U(\mathbf{r},\rho)=U(\mathbf{r})-\nabla_r U(\mathbf{r})\cdot\rho+\tfrac{1}{2}[\nabla_r U(\mathbf{r})\nabla_r U(\mathbf{r})]:\rho\rho+\cdots, \quad (11)$$

along with an analogous expansion for $U(\mathbf{r}+\rho')$ around $\rho'=0$, and inserting it into its right-hand side.

Truncating the Taylor expansion to a given order (the second, in our case) requires that the nonlocal influence of the

PR effect is restricted to a limited region of a given linear dimension [say d, where d is dictated by the form of $\delta\hat{n}(\mathbf{q}_1,\mathbf{q}_2)$] around any position $\mathbf{r}$. It is worthwhile to note that due to the invariance of $\delta n(\mathbf{r},z)$ under the exchange $\mathbf{r}\rightarrow -\mathbf{r}$, the odd-order terms of Eq. (11) do not contribute to the right-hand side of Eq. (10).

For simplicity, we restrict our analysis to a two-dimensional case, allowing diffraction in the y direction and looking for self-trapping in the x direction only (we note that our model is valid in the three-dimensional case as well). In this case $U(\mathbf{r})=U(x)$, and we obtain, by truncating the Taylor expansion after the second term,

$$\left(\gamma-\frac{1}{2k}\frac{d^2}{dx^2}\right)U(x)=\frac{km}{n_1}\frac{d^2U(x)}{dx^2}-\frac{kp}{n_1}\frac{1}{U(x)}\left[\frac{dU(x)}{dx}\right]^2+\frac{kh}{n_1}\frac{1}{U(x)}\left[\frac{d^2U(x)}{dx^2}\right]^2, \tag{12}$$

with

$$m=\frac{1}{2}\int\int g(\rho,\rho')(\rho^2+\rho'^2)d\rho\,d\rho' =-\frac{d^2}{dq_1^2}\delta\hat{n}(q_1,q_2)\bigg|_{q_1=0,q_2=0}, \tag{13}$$

$$p=\int\int g(\rho,\rho')\rho\rho'\,d\rho\,d\rho' =-\frac{d}{dq_1}\frac{d}{dq_2}\delta\hat{n}(q_1,q_2)\bigg|_{q_1=0,q_2=0}, \tag{14}$$

$$h=\frac{1}{4}\int\int g(\rho,\rho')\rho^2\rho'^2\,d\rho\,d\rho' =\frac{1}{2}\frac{d^2}{dq_1^2}\frac{d^2}{dq_2^2}\delta\hat{n}(q_1,q_2)\bigg|_{q_1=0,q_2=0}, \tag{15}$$

where now ρ,ρ' and q_1,q_2 stand for ρ_x,ρ_x' and q_{1x},q_{2x}, and we have also taken advantage of the relations $\delta\hat{n}(0,0)=0$ (see Ref. [9]) and $\delta\hat{n}(q_1,q_2)=\delta\hat{n}(q_2,q_1)$. As will be shown further on, h and the higher-order terms are smaller than m and p by powers of d, and therefore neglected. A rearrangement of the terms in Eq. (12) results in

$$aU'^2+bUU''-\gamma U^2=0, \tag{16}$$

where the prime stands for a derivative with respect to x, and we have set $a=-pk/n_1$ and $b=km/n_1+1/2k$. In the special case, where $a=-2b$, Eq. (16) is satisfied by the solution

$$U(x)=U_0\operatorname{sech}(\alpha x), \tag{17}$$

which is consistent with our requirement of symmetry under the exchange $x\rightarrow -x$, and with a decay with $|x|$ within a region of linear dimension d. The soliton propagation constant is $\gamma=-b\alpha^2>0$, where α satisfies the condition $\alpha d\ll 1$ to justify the truncation of the Taylor expansion.

The nonlinear parameters a and b, and the "effective length" (d) of the PR interaction, are determined from $\delta\hat{n}(q_1,q_2)$. The requirement of a real $\delta\hat{n}(q_1,q_2)$ for all q_1 and q_2 is equivalent having a real $E_m(q_1,q_2)$, which in turn implies the application of an external (dc) electric field E_0 to the material. In our case of PR interaction between pairs of plane waves with relatively small angular deviation (spatial components of the same beam, under the paraxial approximation), the limiting space-charge field E_p is relatively large and the diffusion field E_d is small. Application of an appropriate external field, such that $|E_d|\ll|E_0|\ll|E_p|$, allows us to neglect the imaginary part of the coupling coefficient, so that

$$E_m=\frac{E_0E_p^2}{E_0^2+(E_d+E_p)^2}\cong\frac{E_0}{(E_0/E_p)^2+1}, \tag{18}$$

where $E_p=eP_d/\varepsilon_0\varepsilon_rK_g$ and, under the paraxial approximation, $K_g\sim q_1-q_2$. Note that E_m is always symmetric with respect to both $\mathbf{q}_1,\mathbf{q}_2$, i.e., $E_m(\mathbf{q}_1,\mathbf{q}_2)=E_m(-\mathbf{q}_1,-\mathbf{q}_2)$. In order to maintain this property for $\delta\hat{n}(q_1,q_2)$ we apply this symmetry requirement to $r_{\rm eff}(q_1,q_2)$, and choose the direction of propagation (z) and the polarization of the light beam accordingly. As an example, we consider the coupling coefficient for a field polarized in the x-z plane in $BaTiO_3$ crystal, when the x direction has been adjusted to coincide with the crystalline $\mathbf{c}$ axis (we neglected the relatively small Pockel's coefficients, other than r_{42}):

$$\delta\hat{n}(q_1,q_2)\cong\frac{n_1^3}{2}r_{42}E_0\frac{0.5(q_1+q_2)^2}{1+(E_0\varepsilon_0\varepsilon_r/eP_d)^2(q_1-q_2)^2} \equiv B\frac{[(q_1+q_2)/k]^2}{1+d^2(q_1-q_2)^2}. \tag{19}$$

According to the definition of m, p, and h [see Eqs. (13)-(15)] and the expression of $\delta\hat{n}(q_1,q_2)$, it is a straightforward task to find $m=p=-2B/k^2$ and $h=4Bd^2/k^2$, so that $h=2d^2m$. We have evaluated the factors B and d by employing the parameters used in Ref. [10] and the condition $a=-2b$, thus getting $B\sim+1$ (in dimensionless units) and $d\sim 14$ μm. The neglection of the "h" term, along with higher-order terms, in Eq. (16) is therefore justified. It is then possible to show that, the condition $\alpha d<1$ is equivalent to $\gamma=-b\alpha^2=\alpha^2/2k=k(\alpha/k)^2/2<k$, which can be satisfied by choosing α to be a small fraction of k. Note, that B imposes a positive sign to the externally applied electric field E_0, and a requirement for a large nonlinearity.

The soliton solution for $\alpha=0.05$ μm^{-1}, for a given ("frozen") time t_0, is shown in Fig. 1. The vertical axis represents the light-wave field amplitude $E(x,z,t=t_0)$ in arbitrary units (a consequence of the PR effect is independence from the absolute light intensity $|A|^2$), and the other axes are x and z in μm.

Since our solution was obtained assuming a real cou-

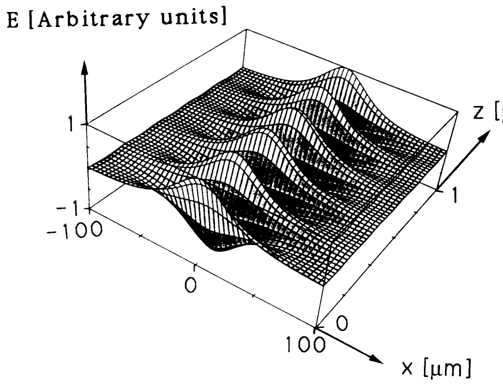

FIG. 1. A three-dimensional plot of the light electric field $E(x,z,t)$ for a "frozen" time $t=t_0$. The vertical axis gives the amplitude in arbitrary units, and the horizontal axes are x and z in μm.

pling coefficient $\delta\hat{n}(q_1,q_2)$, an arbitrary input profile does not evolve into the soliton shape as in the case with temporal Kerr-like solitons. A real $\delta n(\mathbf{q}_1,\mathbf{q}_2)$ does not allow for energy transfer between plane-wave components, so that one must start with the correct wave form which compensates exactly for the diffraction. Once launched, small deviations from the proper solution neither decay or grow, but maintain both the original deviation and the accompanying diffraction, in a "quasistable" situation. A degree of fine tuning of the PR effect (and hence of the soliton width) is allowed by varying the externally applied electric field E_0 (which determines the value of b). Note that for reversal of the polarity of E_0, the linear diffraction and the nonlinear phase are additive and we get "double" the diffraction effect.

The inclusion of absorption (or amplification) in our model results in a soliton which maintains its transverse profile even as the total light intensity increases (gain) or attenuates with propagation. In the absorption case, for example, a linear term $\sigma A(\mathbf{r},z)$ is added to the left-hand side of Eq. (8), and the soliton propagation constant γ is allowed to be complex. If we take the imaginary part of γ equal to $-\sigma$, we still get Eq. (10), with γ replaced by its real part γ_r. The resulting soliton is $U(\mathbf{r})e^{i\gamma_r z-\sigma z}$, and the transverse structure remains unchanged.

Material considerations are of great importance for the practical realization of a PR soliton. We look for a PR media in which the diffusion field E_d [responsible for the imaginary part of $\delta\hat{n}(\mathbf{q}_1,\mathbf{q}_2)$] is as small as possible, but which still presents a strong PR nonlinearity. Another reason for trying to avoid the energy transfer process is the strong noise amplification mechanism ("fanning"), which is present in all the PR materials with an imaginary $\delta\hat{n}(\mathbf{q}_1,\mathbf{q}_2)$. We expect this effect to be very small in our case, both for the above reason (very small E_d) and because of the small cross section of interaction with spontaneously scattered noise (see Ref. [6]). Recently developed quadratic materials, that belong to the KTN group [11], can be excellent candidates since they present a very strong nonlinearity with externally applied electric field, and inherently do not support the energy transfer process.

In conclusion, we have presented a new type of soliton which is based on the photorefractive nonlinearity, discussed the unique properties of this soliton, and considered the conditions necessary for observing it.

The support of the U.S. Air Force Office of Scientific Research and the U.S. Army Research Office is gratefully acknowledged. M.S. gratefully acknowledges the support of the Lester Deutsch Technion Fellowship.

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[1] J. S. Aitchinson, A. M. Winer, Y. Silberberg, M. K. Oliver, J. L. Jackel, D. E. Leaird, E. M. Vogel, and P. W. Smith, Opt. Lett. **15**, 471 (1990).

[2] D. A. Temple and C. Warde, J. Opt. Soc. Am. B **3**, 337 (1986).

[3] S. Solimeno, B. Crosignani, and P. Di Porto, *Guiding, Diffraction and Confinement of Optical Radiation* (Academic, Orlando, 1986), Chap. 8.

[4] B. Crosignani and A. Yariv, J. Opt. Soc. Am. A **1**, 1034 (1984).

[5] J. O. White, S. K. Kwong, M. Cronin-Golomb, B. Fischer, and A. Yariv, in *Photorefractive Materials and their Applications,* edited by P. Gunther and J. P. Huignard (Springer-Verlag, Berlin, 1989), Vol. 2, Chap. 4.

[6] M. Segev, Y. Ophir, and B. Fischer, Opt. Commun. **77**, 265 (1990); Appl. Phys. Lett. **56**, 1085 (1990).

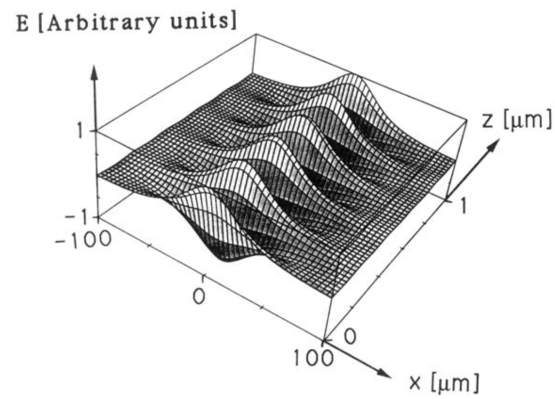
[7] A. V. Mamaev and V. V. Shkunov, Kvantovaya Elektron. (Moscow) **16**, 1863 (1989) [Sov. J. Quantum Electron. **19**, 1199 (1989)].

[8] M. Segev and A. Yariv, Opt. Lett. **16**, 1938 (1991).

[9] If $\delta\hat{n}(0,0)\neq 0$ we need to redefine γ by adding a term proportional to $\delta\hat{n}(0,0)$.

[10] M. B. Klein, in *Photorefractive Materials and their Applications,* edited by P. Gunther and J. P. Huignard (Springer-Verlag, Berlin, 1989), Vol. 1.

[11] A. Agranat, V. Leyva, and A. Yariv, Opt. Commun. **14**, 1017 (1989); A. Agranat and Y. Yacoby, J. Opt. Soc. Am. B **5**, 1792 (1988).



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