Origin of the Polarization for Inclusive Λ Production in *pp* Collisions

Jacques Soffer and Nils A. Törnqvist [(a)]

Centre de Physique Theorique, CNRS, Luminy, Case 907, F-13288 Marseille, France
(Received 27 June 1991)

We consider the inclusive reaction $pp \to \Lambda X$ in the fragmentation region and present a dynamical mechanism which leads to a fair description of all the basic features of the observed Λ spectrum. We also show in a simple manner why it generates a nonzero Λ polarization whose *magnitude* and *sign* are consistent with existing data.

PACS numbers: 13.85.Ni, 12.40.Gg, 13.88.+e

Naively it seems reasonable to expect no polarization effects in an inclusive reaction $a+b \to c+X$ since one is summing over many different inelastic channels X which have polarizations of random signs such that the sum would average to zero. In reality the experimental situation is not so simple and polarization effects depend strongly on specific reactions and kinematic regions and, in particular, there is a large amount of significant polarization data for inclusive hyperon production [1]. Sizable effects and some regular behavior have been observed which may help to uncover the underlying mechanism for particle production. This was first discovered in 1976 at Fermilab by studying hyperons produced by 300-GeV/c protons on a beryllium target [2]; more specifically the Λ's produced in the fragmentation region have a large polarization perpendicular to the production plane. Over the last fifteen years a fair number of different experiments have accumulated high statistics data on inclusive Λ production which make it the best-known hyperon inclusive reaction [3]. Note that all observable quantities are in general functions of three independent kinematic variables, $\sqrt{s}$, the center of mass (c.m.) energy; x_F, the fraction of incident proton momentum carried by the Λ in the initial direction of the proton (in the c.m. system); and p_T, the transverse momentum of the Λ relative to the initial proton direction. The beam (target) fragmentation region corresponds to $x_F \sim 0.5$ ($x_F \sim -0.5$), and, in this case, when one assumes that together with the Λ a kaon is also produced carrying a large positive (negative) x_F, say, $x_F > 0.2$.

Let us now briefly recall the main features of the data in the beam fragmentation region [1].

(i) The invariant cross section $E\,d^3\sigma/dp^3$ depends, to a good approximation, only on x_F and p_T and *not* on the c.m. energy.

(ii) The transverse polarization P is *negative* with respect to the direction $\mathbf{n} = \mathbf{p}_{\rm inc} \times \mathbf{p}_\Lambda$.

(iii) P is almost energy independent for an incident energy ranging from 12 to 2000 GeV/c.

(iv) For p_T below 1 GeV/c, the magnitude of P is approximately linear with p_T with a slope increasing with x_F.

(v) For p_T above 1 GeV/c, the magnitude of P is independent of p_T up to $p_T \sim 3.5$ GeV/c and approximately linear with x_F.

On the theoretical side we recall that in terms of the constituent quarks, proton fragmentation into a Λ with $p_T \neq 0$ corresponds to the replacement of a valence u quark in the projectile by a strange quark s coming from the sea which must be accelerated and must get a nonzero p_T. By assuming an SU(6) wave function, the (ud) system of the Λ is in a singlet state, so the polarization of the Λ is that of the strange quark.

In the Lund semiclassical fragmentation model [4], the confined linear color field is stretched and the strange quark needed to make the final Λ with $p_T \neq 0$ is produced by an $\bar{s}s$ pair whose orbital angular momentum must be balanced by the spin of the strange quark. From this mechanism results a negative Λ polarization increasing with p_T but whose magnitude is difficult to predict.

The recombination model [5] is another approach based on classical arguments. If $\mathbf{F}$ is the color force which provides the acceleration to the strange quark, this s quark of velocity $\mathbf{v}$ feels the effect of a Thomas precession given by $\mathbf{w}_T \sim \mathbf{F} \times \mathbf{v}$, which has the direction of the normal $\mathbf{n}$ to the hadronic scattering plane. In order to minimize the energy $\mathbf{s} \cdot \mathbf{w}_T$ associated with this effect, the spin $\mathbf{s}$ of the s quark should be opposite to $\mathbf{w}_T$ so one expects a negative polarization in $pp \to \Lambda X$ whose magnitude is not known. In neither of these two approaches does one give a quantitative description of the invariant cross section.

Let us finally mention a third dynamical model suggested for the fragmentation [6]. In this picture high-spin baryon resonances Y_i^*, Σ_i^* are produced incoherently and then decay to give the observed Λ, whose polarization is obtained by means of a final-state interference mechanism. Apparently this attempt leads to a reasonable quantitative understanding of the experimental situation [7].

We now turn to the mechanism which we believe is at work for Λ inclusive production in the fragmentation region. This involves the Reggeized one-pion-exchange model proposed several years ago and gives a successful description of various exclusive and inclusive reactions [8,9]. As is well known, if quantum numbers allow, pion exchange generally dominates hadronic amplitudes especially at small momentum transfers. Therefore following

Ref. [9], we will assume that in the fragmentation region the diagram shown in Fig. 1 dominates such that the multiperipheral chain reduces only to the binary reaction $\pi p \to K\Lambda$ and the total πp cross section connected by the exchange of an off-shell Reggeized pion. Thus the contribution of this diagram to the cross section $pp \to \Lambda K X$ can be expressed in the following form:

$$d\sigma = \frac{1}{2[\lambda(s,m^2,m^2)]^{1/2}} \left\{ 16\pi\lambda(s_1,m^2,\mu^2) \frac{d\sigma_{\pi N \to K\Lambda}}{dt_1}(s_1,t_1) \right\} \times \{2[\lambda(s_2,m^2,\mu^2)]^{1/2} \sigma_{\pi N}^{\rm tot}(s_2)\} F^2(t;s_1,s_2,s) \frac{d^3p_K}{(2\pi)^3 2E_K} \frac{d^3p_\Lambda}{(2\pi)^3 2E_\Lambda}, \quad (1)$$

where p_1, p_2, p_K, and p_Λ are respectively the momenta of the initial protons and of the produced kaon and lambda, and m and μ are the proton and the pion masses. Moreover, the *five* invariants of the reaction are defined as follows: $s=(p_1+p_2)^2$ is the collision energy squared, $s_1=(p_K+p_\Lambda)^2$ is the energy squared of the binary reaction, $s_2=(p_1+p_2-p_K-p_\Lambda)^2$ is the invariant mass squared of X, t_1 is the invariant momentum transfer squared from the proton beam to the Λ, and the mass squared of the off-shell pion is $t=(p_1-p_K-p_\Lambda)^2$. In Eq. (1), we also have the usual kinematical function $\lambda(x,y,z) = (x-y-z)^2-4yz$ and we denote by $d\sigma_{\pi N \to K\Lambda}/dt_1(s_1,t_1)$ the differential cross section for the binary reaction and by $\sigma_{\pi N}^{\rm tot}(s_2)$ the πN total cross section. The function $F(t;s_1,s_2,s)$ which includes the Reggeized pion propagator and describes the off-mass-shell behavior will be given explicitly later. The five invariants of the reaction can be reexpressed in terms of five more convenient variables, namely, x_Λ and x_K, the x_F's of the Λ and of the kaon, $p_{T\Lambda}$ and p_{TK}, the corresponding transverse momenta relative to the initial proton direction (in the c.m. system), and φ, the angle between the two directions $\mathbf{p}_{T\Lambda}$ and $\mathbf{p}_{TK}$.

It is important for our argument that in the fragmentation region, i.e., $x_\Lambda \sim 0.5$ and $x_K \sim 0.2$, the effective energy of the binary s_1 is *always much smaller* than s and, in general except for $x_K=0$, it is reduced to values in the range of 10 GeV^2 or less. In fact we have

$$s_1 \sim (x_\Lambda - x_K)\left(\frac{m_{T\Lambda}^2}{x_\Lambda} + \frac{m_{TK}^2}{x_K}\right) - (\mathbf{p}_{T\Lambda}+\mathbf{p}_{TK})^2,$$

where the m_T's are the lambda and kaon transverse masses $m_T=(m^2+p_T^2)^{1/2}$. On the other hand, s_2 remains of the order of s and approximately $s_2 \sim (1-x_\Lambda-x_K)s$. Therefore from Eq. (1) it results (provided F^2 remains small when s increases) that the cross section obeys *scaling*, in agreement with the observed inclusive Λ spectrum. Another important kinematic observation is that t is generally not very large in the region considered, where $p_{T\Lambda}$ and p_{TK} are small and $x_\Lambda + x_K$ is not far from unity. Thus the pion will not be far off shell. The invariant cross section for Λ production is obtained from Eq. (1) after integration over the invariant phase-space element of the kaon, d^3p_K/E_K, and the final expression reads

$$E_\Lambda \frac{d^3\sigma}{dp_\Lambda^3} = \frac{4\pi}{(2\pi)^6} \int_0^{2\pi} d\varphi \int_{-1}^{+1} dx_K \int_0^{p_{TK}^{\max}} p_{TK}\, dp_{TK} \frac{\lambda(s_1,m^2,\mu^2)}{(x_K^2 + m_{TK}^2/p_{\rm c.m.}^2)^{1/2}} \times \left(\frac{\lambda(s_2,m^2,\mu^2)}{\lambda(s,m^2,m^2)} \right)^{1/2} \frac{d\sigma_{\pi N \to K\Lambda}}{dt_1}(s_1,t_1)\, \sigma_{\pi N}^{\rm tot}(s_2) F^2(t;s_1,s_2,s), \quad (2)$$

where $p_{\rm c.m.}$ is the c.m. momentum of the initial collision. Strictly speaking, since one integrates over x_K there is a region where the diagram shown in Fig. 1 does not dominate the reaction $pp \to \Lambda K X$ because the kaon is no longer in the fragmentation region, and one should consider other exchange diagrams, like kaon exchange, which correspond to direct Λ production. They must be suppressed at high energy compared to pion exchange and moreover since they do not contribute to the Λ polarization one can ignore them.

It now remains to discuss the form of $F(t;s_1,s_2,s)$. Following Refs. [8,9] we will replace the pion propagator by the function

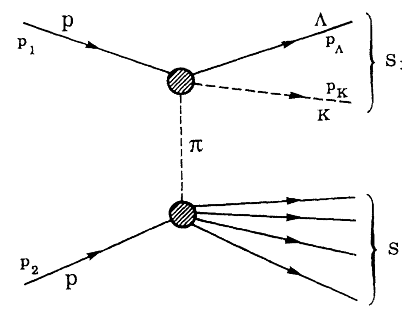

FIG. 1. Single-pion exchange diagram for the process $pp \to \Lambda K X$.

$$F(t;s_1,s_2,s)=\exp\left[\left(R^2+\alpha'_\pi\ln\frac{sm_{TK}^2}{s_1s_2}\right)(t-\mu^2)\right]\times\begin{cases}\frac{1}{2}\pi\alpha'_\pi/\sin[\pi\alpha_\pi(t)/2] & \text{for } |t|<|T_0|\,,\\ \frac{1}{2}\pi\alpha'_\pi\exp[R_\text{I}^2(t-T_0)]/\sin[\pi\alpha_\pi(T_0)/2] & \text{for } |t|>|T_0|\,,\end{cases} \quad (3)$$

and assume a linear parametrization $\alpha_\pi(t)=\alpha'_\pi(t-\mu^2)$ for the pion trajectory. This expression leads to the usual single and double Regge behavior in the corresponding kinematic regions and, due to the signature factor, it reduces for small t to the ordinary pion-pole propagator $1/(t-\mu^2)$. In order to prevent a too slow decrease of F with increasing t, one imposes the bound $R^2+R_\text{I}^2+\alpha'_\pi\ln(sm_{TK}^2/s_1s_2)\geq\lambda_0$. To summarize, $F(t;s_1,s_2,s)$ depends on five parameters, R^2, R_I^2, α'_π, T_0, and λ_0, whose values were obtained in Refs. [8,9] from the analysis of exclusive reactions. The calculation of the invariant cross section [Eq. (2)] requires the knowledge of the binary cross section $\pi p\to K\Lambda$ at low and moderate energies for which we use an interpolation of the low-energy data [10,11] for $\pi^-p\to K^0\Lambda$, for $p_\text{lab}<4$ GeV/c, in terms of an expansion of Legendre polynomials

$$\frac{d\sigma}{dt_1}(s_1,t_1)=\sum_i a_i(s_1)P_i(\cos\theta)\,, \quad (4)$$

where for the calculation of the scattering angle θ in terms of s_1 and t_1, the pion is given its off-shell mass, in order to avoid unphysical values. For $p_\text{lab}>4$ GeV/c we use a Regge parametrization of the data [12]. The contribution of the unobservable reaction $\pi^0p\to K^+\Lambda$ is obtained from the isospin relation

$$\frac{d\sigma}{dt_1}(\pi^0p\to K^+\Lambda)=\frac{1}{2}\frac{d\sigma}{dt_1}(\pi^-p\to K^0\Lambda)\,.$$

We must emphasize that this approach allows us to predict the *absolute normalization* of the invariant inclusive cross section and we now turn to the comparison with experimental data. In Fig. 2 we show, by means of a best fit, the representation of the measured Λ production by 400-GeV/c protons on hydrogen with some typical errors from Ref. [3] together with our theoretical predictions. They correspond to the set of parameters

$$R^2=-0.55\text{ GeV}^2,\quad R_\text{I}^2=1.25\text{ GeV}^2,$$
$$\alpha'_\pi=0.85\text{ GeV}^{-2},\quad T_0=-0.25\text{ GeV}^2, \quad (5)$$
$$\lambda_0=0.35\text{ GeV}^{-2},$$

which are slightly different from those used in Ref. [9], since of course our knowledge on the Λ inclusive spectrum has greatly improved since 1974. Note that we take $\sigma_{\pi N}^\text{tot}(s_2)=24$ mb, an appropriate average value for the relevant range of s_2 under consideration. It is remarkable to see that we get the correct absolute normalization, and the right trend of the data both in x_Λ and $p_{T\Lambda}$, with the best agreement occurring for $x_\Lambda\sim0.5$. Therefore, although no doubt the fit and the model can be refined, we are confident in the validity of the fact that our one model gives the dominant contribution to the cross section in the fragmentation region.

A crucial test of our model is, however, whether it can also predict the Λ transverse polarization P. Clearly it must be related to the Λ polarization P_Λ of the binary reaction $\pi p\to K\Lambda$ at low and moderate energies [9]. There, P_Λ is known to be large and positive for $p_\text{lab}<1.5$ GeV/c [10,11]. But at higher energies for momentum transfer ≤ -0.3 GeV^2 it turns large and negative [13]. Notice that now P is measured along $\mathbf{n}=\mathbf{p}_\text{inc}\times\mathbf{p}_\Lambda$ whereas in the binary reaction the Λ polarization P_Λ is observed along the normal $\mathbf{N}=\mathbf{p}_\pi\times\mathbf{p}_K=-\mathbf{p}_K\times\mathbf{p}_\Lambda$ to the (K,Λ) plane, in the proton (beam) rest frame. Therefore one obtains $(E_\Lambda d^3\sigma/dp_\Lambda^3)P$ from Eq. (2) by replacing $d\sigma/dt_1$ in the integrand by $\cos\Phi P_\Lambda d\sigma/dt_1$, where $\cos\Phi=\mathbf{n}\cdot\mathbf{N}/|\mathbf{n}||\mathbf{N}|$, i.e., the inclusive transverse polarization is obtained as a weighted average of the quantity $\cos\Phi P_\Lambda$. We parametrize the experimental polarization of $\pi^-p\to K^0\Lambda^0$ as $P=+0.5$ for $p_\text{lab}<1.5$ GeV/c, for all scattering angles except near forward and backward directions where it should vanish. For $p_\text{lab}>1.5$ GeV/c we use $P=0.4\sin(-\pi t'/t'_0)$, with $t'_0=0.3$ GeV^2, for

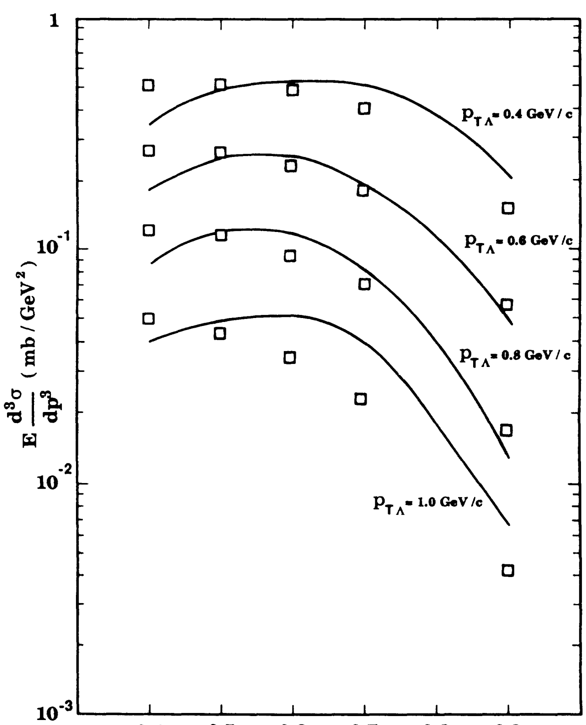

FIG. 2. Inclusive Λ invariant cross section at $p_\text{lab}=400$ GeV/c vs x_Λ for different $p_{T\Lambda}$ values. Solid curves are our theoretical predictions and open squares are the data representation from Ref. [3] with typical errors.

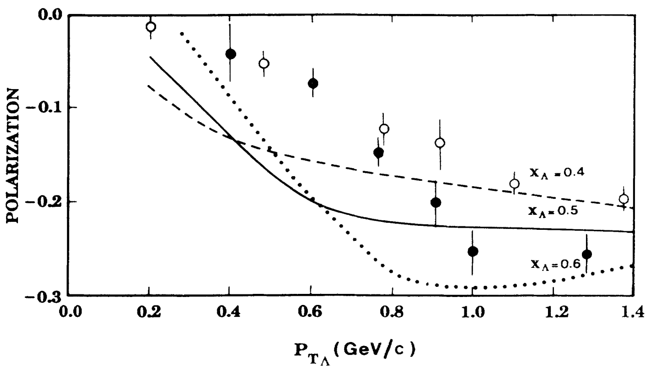

FIG. 3. Comparison of the measured Λ polarization with our theoretical predictions at $x_\Lambda = 0.4$ (dashed curve), $x_\Lambda = 0.5$ (solid curve), and $x_\Lambda = 0.6$ (dotted curve). The open circles are Fermilab data at $p_{lab} = 400$ GeV/c and (average x_Λ) $\bar{x}_\Lambda = 0.44$ from Ref. [3] and the solid circles are CERN ISR data at $\sqrt{s} = 62$ GeV and $x_\Lambda = 0.58$ from Ref. [14].

$t' > -0.45$ GeV^2, while for $t' < -0.45$ GeV^2 we take $P = -0.4$. Here t' is the usual momentum transfer between the pion and the kaon which vanishes for forward scattering. In Fig. 3 we show the results of this calculation at $p_{lab} = 400$ GeV/c for three x_Λ values. Since the cross section is best described for $x_\Lambda = 0.5$, we think the polarization is also most reliable at the same x_Λ. For comparison we show some existing data at two different energies for nearby x_Λ values and again the agreement is remarkably good both in magnitude and sign [15]. We have also checked that the theoretical calculations for P_Λ are energy independent as it should be following (iii) mentioned above. Moreover at large p_T the value of P_Λ flattens out, consistent with a well-established feature of the data [see (v) above]. The sign of P_Λ is directly related to that of P for the binary reaction for $|t'| > 0.3$ GeV^2 and its magnitude increases with x_Λ because the cancellation from the configurations $\cos\Phi > 0$ and $\cos\Phi < 0$ is more effective for small values of x_Λ.

In a rough way, the shape of the inclusive polarization as a function of $p_{T\Lambda}$ is the same as that of the binary reaction as a function of t'. From this argument one can see that the mirror symmetry between the polarizations in $\pi^+ p \to K^+ \Sigma^+$ and $\pi^- p \to K^0 \Lambda^0$ observed in Ref. [13] leads us to predict the inclusive Σ^+ polarization to be mirror symmetric to that of the Λ, in accordance with observation.

Also, regarding the spectacular mirror symmetry effect observed recently at Fermilab in pp collisions with polarized proton beam for π^+ and π^- inclusive production [16], our model can easily account for that due to the mirror symmetry of the polarizations for $\pi^\pm p \to p\pi^\pm$ in the backward direction at low energies. Moreover, we anticipate the correct sign since the binary polarization at $u \sim -0.5$ GeV^2 is opposite to that of the inclusive at $p_T \sim 1$ GeV/c. This opposite sign occurs because for the binary reaction the polarization is measured along $\boldsymbol{\pi}_{in} \times \boldsymbol{\pi}_{out}$ which corresponds for the inclusive to $-\mathbf{p}_{inc} \times \boldsymbol{\pi}_{out}$. Finally in $\bar{p}p$ collisions with polarized $\bar{p}$ it has been observed at Fermilab that the polarizations for π^+ or π^- inclusive production are opposite in sign to those of π^+ or π^- in pp collisions [17]. This fact follows naturally in our model from charge conjugation at the level of the binary reaction.

We would like to thank C. Bourrely, M. Chaichian, A. Efremov, and A. Turbiner for helpful discussions and clarification at various stages of this work. Warm hospitality at the Research Institute for High Energy Physics, Helsinki, is gratefully acknowledged by one of us (J.S.).

(a)Permanent address: Research Institute for High Energy Physics, University of Helsinki, Siltavuorenpenger 20C, SF-00170 Helsinki, Finland.

[1] K. Heller, in *Proceedings of the Seventh International Conference on Polarization Phenomena in Nuclear Physics, Paris, 1990*, edited by A. Boudard and Y. Terrien [J. Phys. (Paris), Colloq. **51**, C6-163 (1990)].
[2] G. Bunce *et al.*, Phys. Rev. Lett. **36**, 1113 (1976).
[3] L. G. Pondrom, Phys. Rep. **122**, 57 (1985); B. Lundberg *et al.*, Phys. Rev. D **40**, 3557 (1989).
[4] B. Andersson, G. Gustafson, G. Ingelman, and T. Sjöstrand, Phys. Rep. **97**, 31 (1983), and references therein.
[5] T. De Grand, J. Markkanen, and H. Miettinen, Phys. Rev. D **32**, 2445 (1985), and references therein.
[6] G. Preparata, in *Proceedings of the 1980 International Symposium on High Energy Physics with Polarized Beams and Targets, Lausanne*, edited by C. Joseph and J. Soffer, Birkhäuser Experientia Supplementum Vol. 38 (Birkhäuser, Boston, 1980) p. 121.
[7] Ph. Ratcliffe, in *Proceedings of the Polarized Collider Workshop, University Park, Pennsylvania, 1990*, edited by J. C. Collins, Steven Heppelmann, and R. W. Robinett, AIP Conference Proceedings No. 223 (American Institute of Physics, New York, 1991), p. 237.
[8] K. G. Boreskov, A. B. Kaĭdalov, and L. A. Ponomarev, Yad. Fiz. **19**, 1103 (1974) [Sov. J. Nucl. Phys. **19**, 565 (1974)]; L. A. Ponomarev, Fiz. Elem. Chastits At. Yadra **7**, 186 (1976) [Sov. J. Part. Nucl. **7**, 70 (1976)], and references therein.
[9] A. V. Turbiner, Yad. Fiz. **22**, 1057 (1976) [Sov. J. Nucl. Phys. **22**, 551 (1976)]; Pis'ma Zh. Eksp. Teor. Fiz. **22**, 386 (1975) [JETP Lett. **22**, 182 (1975)].
[10] R. D. Baker *et al.*, Nucl. Phys. **B141**, 29 (1978).
[11] D. H. Saxon *et al.*, Nucl. Phys. **B162**, 522 (1980).
[12] H. Navelet and P. R. Stevens, Nucl. Phys. **B104**, 171 (1976).
[13] C. E. Ward *et al.*, Phys. Lett. **48B**, 471 (1974); A. Snyder *et al.*, Phys. Rev. D **21**, 599 (1980).
[14] A. M. Smith *et al.*, Phys. Lett. B **185**, 209 (1987).
[15] The effect of the Σ^0 contamination arising from the decay $\Sigma^0 \to \Lambda^0 + \gamma$ suggests that one underestimates by 10%–20% the magnitude of P_Λ for directly produced Λ^0 [see M. Sullivan *et al.*, Phys. Lett. **142B**, 451 (1984)].
[16] Fermilab E704 Collaboration, D. L. Adams *et al.*, Phys. Lett. B **264**, 462 (1991).
[17] A. Yokosawa (private communication).