From Isotropic to Anisotropic Superconductors: A Scaling Approach

G. Blatter,[(1),(2)] V. B. Geshkenbein,[(1),(3)] and A. I. Larkin[(1),(3)]
[(1)]*Theoretische Physik, Eidgenössische Technische Hochschule Hönggerberg, CH-8093 Zürich, Switzerland*
[(2)]*Corporate Research, Asea Brown Boveri, CH-5405 Baden, Switzerland*
[(3)]*L. D. Landau Institute for Theoretical Physics, 117 940 Moscow, U.S.S.R.*
(Received 7 October 1991)

We present a new scaling approach which allows one to map results obtained for isotropic superconductors to anisotropic materials in a simple and direct way. The scaling rules are obtained on the level of Ginzburg-Landau- or London-type equations and applied directly to the desired phenomenological quantity. We illustrate the method by calculating the elasticity moduli, the depinning and melting temperatures, the critical current densities, and the activation barriers for classical and quantum creep in anisotropic superconductors for an arbitrary angle between the magnetic field and the axes of anisotropy.

PACS numbers: 74.60.Ge, 74.20.De, 74.60.Jg

The discovery of high-temperature superconductivity has renewed the interest in the phenomenology of type-II superconductors. Among the recent novel results are the proposed new thermodynamic phases such as the vortex glass [1] or different kinds of vortex liquids [2], the influence of thermal fluctuations leading to thermal depinning [3] and vortex lattice melting [2,4,5], the determination of the elastic properties of a vortex lattice in anisotropic superconductors [4,6,7], and the investigation of pinning and creep, both classical [8,9] and quantum [10,11]. Many of these features have been studied for the case where the magnetic field is aligned with the c axis [2,4] or with the main axes of the anisotropic material [7], others have only been investigated for isotropic superconductors [1,3,5,8,10]. The traditional way to incorporate anisotropy into the phenomenological description of superconductivity is to introduce an anisotropic effective mass tensor into the Ginzburg-Landau or London equations. In the conventional approach one then repeats all the calculations which usually have been done for the isotropic case before. As a result of the appearance of additional parameters and the breaking of spherical symmetry, the corresponding analysis becomes very tedious and thus only few results are known for the general anisotropic case including arbitrary field direction. In this Letter we present a new scaling approach which provides simple and direct access to the most general anisotropic result by rescaling the anisotropic problem to a corresponding isotropic one on the initial level of Ginzburg-Landau (GL) or London equations. The scaling rules extracted out of this mapping are then used to generalize the isotropic results to the anisotropic situation with essentially no effort. The two approaches are schematically illustrated in Fig. 1.

To start with, let us consider the Gibbs free energy for an anisotropic superconductor

$$\mathcal{G}=\int d^3r\left[\alpha|\Psi|^2+\frac{\beta}{2}|\Psi|^4+\sum_{\mu=1}^{3}\frac{1}{2m_\mu}\left|\left(\frac{\hbar}{i}\frac{d}{dx_\mu}-\frac{2e}{c}A_\mu\right)\Psi\right|^2+\frac{B^2}{8\pi}-\frac{\mathbf{H}\cdot\mathbf{B}}{4\pi}\right], \quad (1)$$

where $\Psi(\mathbf{r})$ is the order parameter, $\mathbf{A}$ is the vector potential, and $\mathbf{B}=\nabla\times\mathbf{A}$ is the microscopic magnetic field. The external field $\mathbf{H}$ is chosen to lie in the y-z plane and encloses an angle ϑ with the y axis. For the sake of simplicity and because the oxide superconductors are within high accuracy uniaxial materials, we choose $m_x=m_y=m$, $m_z=M$, and denote the mass anisotropy ratio by $\varepsilon^2=m/M<1$. In order to include effects of pinning [12,13] we introduce scalar disorder in the GL coefficient α, $\alpha=\alpha_0+\delta\alpha(\mathbf{r})$ with $\langle\delta\alpha\rangle=0$, and $\langle\delta\alpha(\mathbf{r})\delta\alpha(\mathbf{r}')\rangle=\gamma\delta(\mathbf{r}-\mathbf{r}')$, describing short-range disorder in the transition temperature T_c.

Several years ago, Klemm and Clem [14] introduced a transformation which mapped Eq. (1) to an isotropic form. Their approach allows one to isotropize all terms in the Gibbs energy; however, the transformation is rather complicated and limited to unidirectional fields. Kogan [15] then pointed out that the magnetic field around a vortex also involves transverse components which the scaling approach fails to take into account. Later, Kogan

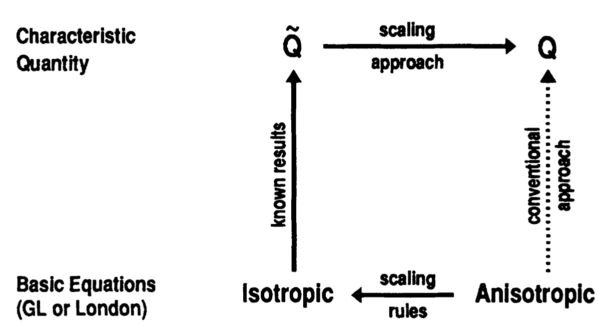

FIG. 1. Schematic illustration of the conventional and of the new scaling approach to obtain physical results for anisotropic superconductors. In the conventional approach one starts from the anisotropic GL or London theory and repeats the calculations done before for the isotropic case. In the new scaling approach we use the scaling rules obtained on the level of GL or London equations and directly transform the isotropic results to anisotropic superconductors.

and Clem [16] and Hao and Clem [17] used a scaling transformation in their calculation of the reversible magnetization and the torque. In particular, they showed that the scaling approach is valid for large $\kappa=\lambda/\xi$ and large magnetic fields (λ and ξ denote the planar London penetration depth and the coherence length). However, it is important to realize that for strong type-II superconductors with $\kappa\gg 1$, fluctuations in the magnetic field can be neglected altogether in most applications. This idea allows us to use the scaling approach in a much wider context than considered before.

In (1) the anisotropy enters only in the gauge-invariant gradient term and a simple rescaling of the coordinate axes

$$x=\tilde{x},\quad y=\tilde{y},\quad z=\varepsilon\tilde{z} \tag{2}$$

together with a scaling of the vector potential, $\mathbf{A}=(\tilde{A}_x,\tilde{A}_y,\tilde{A}_z/\varepsilon)$, will render this term isotropic. The magnetic field is rescaled to $\mathbf{B}=(\tilde{B}_x/\varepsilon,\tilde{B}_y/\varepsilon,\tilde{B}_z)$ and the last two terms in (1) describing the magnetic-field energy are transformed to

$$\mathcal{G}_m=\frac{1}{8\pi}\int d^3r\left[\left(\frac{\tilde{B}_\perp^2}{\varepsilon^2}+\tilde{B}_z^2\right)-2\left(\frac{\tilde{B}_\perp H_\perp}{\varepsilon}+\tilde{B}_zH_z\right)\right]. \tag{3}$$

In short, we have removed the anisotropy from the gradient term but reintroduced it in the magnetic energy term. In general it is not possible to isotropize both terms in the Gibbs energy simultaneously. However, depending on the physical question addressed, we can neglect fluctuations in the magnetic field. For example, the problems of vortex pinning or of vortex lattice melting involve the coherence length ξ or the intervortex distance $a=(\Phi_0/B)^{1/2}$ as their natural length scales. The latter are small compared with the scale of fluctuations of the magnetic field λ, if the superconductor is strongly type II or for large enough magnetic fields with $a<\lambda$, respectively. In such situations the magnetic field is uniform on the natural length scale of the problem and we can adopt a mean-field decoupling scheme, where we first minimize the magnetic-field energy $\mathcal{G}_m$ with respect to $\tilde{\mathbf{B}}$ and then insert the resulting uniform field back into the free energy. More rigorously, let us consider the case $\kappa\to\infty$ or, equivalently, charge $e\to 0$. The coupling between the order parameter Ψ and the gauge field $\mathbf{A}$ is given by the gradient term $|[\nabla/i-(2e/c)\mathbf{A}]\Psi|^2$ and vanishes in the limit $e\to 0$. The external magnetic field then merely fixes the average density of vortices. Hence our approach is exact for the case of an uncharged superfluid.

Minimizing the magnetic-field energy $\tilde{\mathcal{G}}_m$ we obtain $\tilde{\mathbf{B}}=(\varepsilon H_x,\varepsilon H_y,H_z)$, corresponding to $\mathbf{B}=\mathbf{H}$ in the original system. Thus in our rescaled system the magnetic field is reduced to $\tilde{B}=\varepsilon_\vartheta B$, $\varepsilon_\vartheta^2=\varepsilon^2\cos^2\vartheta+\sin^2\vartheta$, as compared with the field in the original system. Next, let us transform energy and temperature: Since the volume scales as $V=\varepsilon\tilde{V}$, the energy scales as $\mathcal{G}=\varepsilon\tilde{\mathcal{G}}$, and for the temperature determining the strength of thermal fluctuations we obtain the rule $T=\varepsilon\tilde{T}$. Finally, we transform the disorder $\delta\alpha(\mathbf{r})$: In the isotropized system the correlator reads $\langle\delta\tilde{\alpha}(\tilde{\mathbf{r}})\delta\tilde{\alpha}(\tilde{\mathbf{r}}')\rangle=(\gamma/\varepsilon)\delta(\tilde{\mathbf{r}}-\tilde{\mathbf{r}}')$, thus the disorder strength γ scales as $\gamma=\varepsilon\tilde{\gamma}$. A second type of disorder is generated by the spatial variation of the mean free path [12,13], which can be described by a variation of the effective masses $m(\mathbf{r})$ and $M(\mathbf{r})$. For a layered superconductor, the disorder in m and M is due to disorder within the conducting plane and between adjacent planes, respectively, and thus in general the two need not be the same after rescaling. The difference between these two kinds of disorder is only relevant in the small angle regime $|\vartheta|<\varepsilon$, since for angles bigger than ε the vortices are redirected mainly along the c axis after rescaling and thus disorder in M can be neglected. Except for this small-angle regime, the disorder in the mean free path can be treated as scalar and therefore transforms in the same manner as the disorder in T_c.

We are now ready to set up a general scaling rule: Consider a uniaxially anisotropic superconductor (axis parallel to z) characterized by the planar coherence length ξ and London penetration depth λ, the anisotropy ε, and the scalar disorder strength γ, in an applied magnetic field $\mathbf{H}$ enclosing an angle ϑ with the x-y plane, at a temperature T. Let Q be the desired quantity for which the isotropic result $\tilde{Q}$ is known. Then we obtain Q for the anisotropic superconductor by the scaling rule

$$Q(\vartheta,H,T,\xi,\lambda,\varepsilon,\gamma)=s_Q\tilde{Q}(\varepsilon_\vartheta H,T/\varepsilon,\xi,\lambda,\gamma/\varepsilon). \tag{4}$$

Typical scaling factors are $s_Q=\varepsilon$ for volume, energy, temperature, and action, and $s_B=1/\varepsilon_\vartheta$ for magnetic field. The scaling rule (4) is the main result of our paper.

In the remainder of the paper we present a few illustrative examples of how to make use of the scaling formalism. Let us start with the elasticity coefficients for a single vortex. The elastic energy of a single vortex can be written as

$$\mathcal{F}_{\rm el}=\int dz'[\tfrac{1}{2}\varepsilon_l^{\parallel}(\vartheta)(\partial_{z'}u_x)^2+\tfrac{1}{2}\varepsilon_l^{\perp}(\vartheta)(\partial_{z'}u_{y'})^2],$$

where we have introduced the rotated system with its z' axis directed along the field and $x'=x$. Transforming the in-plane tilt energy to the isotropic system we obtain the relation $\varepsilon_l^{\parallel}(\vartheta)(\delta u_x)^2/\delta z'=\varepsilon\tilde{\varepsilon}_l(\delta\tilde{u}_x)^2/\delta\tilde{z}'$. Since λ is invariant, $\tilde{\varepsilon}_l=\varepsilon_0=(\Phi_0/4\pi\lambda)^2$. Furthermore, $\delta\tilde{u}_x/\delta u_x=1$ and $\delta z'/\delta\tilde{z}'=\varepsilon/\varepsilon_\vartheta$, since a longitudinal length scales as $l_l=\varepsilon\tilde{l}_l/\varepsilon_\vartheta$. The final expression is $\varepsilon_l^{\parallel}(\vartheta)=\varepsilon_0\varepsilon^2/\varepsilon_\vartheta$, in agreement with the result of the conventional approach [11]. When transforming the out-of-plane tilt energy we should take care about the change of angles due to the scale transformation: The transformed vectors $\delta\tilde{\mathbf{u}}=\delta u_{y'}(0,-\sin\vartheta,\cos\vartheta/\varepsilon)$ and $\delta\tilde{\mathbf{z}}'=\delta z'(0,\cos\vartheta,\sin\vartheta/\varepsilon)$ are no longer orthogonal. Orthogonalizing, we obtain $\varepsilon_l^{\perp}(\vartheta)(\delta u_{y'})^2/\delta z'=\varepsilon\tilde{\varepsilon}_l|\delta\tilde{\mathbf{u}}\times\delta\tilde{\mathbf{z}}'|^2/\delta\tilde{z}'^3$ and the final expression is $\varepsilon_l^{\perp}(\vartheta)=\varepsilon_0\varepsilon^2/\varepsilon_\vartheta^3$, in agreement with Ref. [11].

In addition, we obtain the scaling rule for transverse lengths ($l_t \perp \mathbf{B},\mathbf{x}$ and $\tilde{l}_t \perp \tilde{\mathbf{B}},\mathbf{x}$), $l_t = \varepsilon_\vartheta \tilde{l}_t$.

Next, we discuss the elastic moduli of flux lattices in the limit of strong dispersion ($\tilde{k}\lambda > 1$), where our formalism can be applied. We have to find the two tilt moduli [7] $c_{44}^{\parallel}$ and $c_{44}^{\perp}$, the three compression moduli [18] $c_{11}^{\parallel}$, $c_{11}^{\perp}$, and $c_{11}^{\parallel,\perp}$, and the two shear moduli [6] $c_{66}^{\parallel}$ (easy) and $c_{66}^{\perp}$ (hard). Transforming the tilt energy density we find the relation $c_{44}^{\parallel}(\mathbf{k}) k_z^2 u_x^2 = \tilde{c}_{44}(\tilde{\mathbf{k}}) \tilde{k}_z^2 \tilde{u}_x^2$ and using the scaling rules for longitudinal lengths and magnetic fields we obtain $c_{44}^{\parallel}(\mathbf{k}') = \varepsilon^2 B^2/4\pi\lambda^2 \tilde{k}^2$. Finally, we express $\tilde{\mathbf{k}}$ by $\mathbf{k}'$ and the in-plane tilt modulus becomes

$$c_{44}^{\parallel}(\mathbf{k}') = \frac{B^2}{4\pi}\frac{\varepsilon^2}{\lambda^2} \times \frac{1}{k_{x'}^2 + \varepsilon_\vartheta^2 k_{y'}^2 + \varepsilon_\theta^2 k_{z'}^2 + 2(1-\varepsilon^2)\sin\vartheta\cos\vartheta k_{y'} k_{z'}},$$

with $\varepsilon_\theta = \varepsilon_{\pi/2-\vartheta}$. In the limits $\vartheta = 0$ and $\vartheta = \pi/2$ we can compare our result with Ref. [7] and find agreement. The other elastic moduli are found in a similar way and we quote the results here: $c_{44}^{\perp}(\mathbf{k}') = c_{44}^{\parallel}(\mathbf{k}')/\varepsilon_\vartheta^2$, where the scaling rule for transverse lengths has been used, $c_{11}^{\parallel}(\mathbf{k}') = c_{11}^{\perp}(\mathbf{k}') = c_{11}^{\parallel,\perp}(\mathbf{k}') = c_{44}^{\parallel}(\mathbf{k}')(\varepsilon_\vartheta/\varepsilon)^2$, and $c_{66}^{\parallel} = c_{66}(B)\varepsilon_\vartheta^3$, $c_{66}^{\perp} = c_{66}(B)/\varepsilon_\vartheta$, with $c_{66}(B) = \Phi_0 B/(8\pi\lambda)^2$, in agreement with Ref. [6].

In our next example we generalize the isotropic results for thermal depinning [3] and melting [2,4,5] to anisotropic superconductors and arbitrary angles ϑ of the magnetic field. The thermal depinning temperature T_{dp} is defined by the condition [3] $\langle u^2 \rangle_{\mathrm{th}} = \xi^2$ and given by the expression $T_{\mathrm{dp}} = 4\varepsilon_0 \xi^2 (B/\Phi_0)^{1/2}$. Here $\langle u^2 \rangle_{\mathrm{th}}$ is the mean-squared displacement due to thermal fluctuations. Using our scaling rule (4) with $s_T = \varepsilon$ we obtain the depinning temperature in an anisotropic superconductor,

$$T_{\mathrm{dp}}(\vartheta) = 4\varepsilon\varepsilon_0 \xi^2 (\varepsilon_\vartheta B/\Phi_0)^{1/2}. \tag{5}$$

In order to obtain the melting temperature we may use the Lindemann criterion $\langle u^2 \rangle_{\mathrm{th}} = (c_L a)^2$ with $c_L \sim 0.1$ the Lindemann number. Again using (4) we obtain the generalized melting temperature

$$T_m(\vartheta) = 4\varepsilon\varepsilon_0 c_L^2 (\Phi_0/\varepsilon_\vartheta B)^{1/2}, \tag{6}$$

which agrees with the result of Ref. [4] for the special case $\vartheta = \pi/2$. The results (5) and (6) show that the region where the vortex lattice is thermally depinned but not yet melted grows as the field is tilted away from the c axis. Note that in this region the critical current density is strongly temperature dependent [3].

Let us now turn to the problem of pinning and creep. Here we limit ourselves to the case of single-vortex weak-collective pinning, which seems to describe well the situation in the oxide superconductors at low enough temperatures ($T \lesssim 30$ K for Y-Ba-Cu-O) and magnetic fields ($H \lesssim 1$ T for Y-Ba-Cu-O) [19]. For isotropic superconductors it has been found [8,10,13] that a segment of length $L_c \simeq (\varepsilon_0^2 \xi^2/\gamma)^{1/3}$ is collectively pinned, leading to a pinning potential $U_c \simeq (\gamma L_c)^{1/2}\xi$, an effective action $S_E^{\mathrm{eff}} = (\hbar/e)^2 (\xi/\rho_n)(j_0/j_c)^{1/2}$ (limit of strong damping), and a critical current density $j_c \simeq j_0 (\xi/L_c)^2$, with $j_0 \sim c\Phi_0/\lambda^2\xi$ denoting the depairing current density. Finally, the condition for single-vortex pinning is given by the relation $L_c \le a$ which results from the condition that the interaction between neighboring vortices be smaller than between a vortex and the pinning potential. Using our scaling rule (4) we obtain the following results for the anisotropic superconductor: $L_c(\vartheta) = L_c^c/\varepsilon_\vartheta$, where $L_c^c = (\varepsilon_0^2 \xi^2 \varepsilon^4/\gamma)^{1/3} = \varepsilon^{4/3} L_c^{\mathrm{iso}}$ is the collective pinning length for $\mathbf{H} \parallel \mathbf{c}$. Here we have used the transformation rule for longitudinal lengths and for the disorder coefficient γ. L_c^{iso} denotes the collective pinning length of an equivalent isotropic material with identical parameters λ, ξ, and γ. The pinning potential becomes $U_c^c = \varepsilon^{2/3} U_c^{\mathrm{iso}}$, independent of the angle ϑ. Similarly, the action becomes $S_E^{\mathrm{eff},c} = \varepsilon^{4/3} S_E^{\mathrm{eff,iso}}$, again independent of ϑ. In the anisotropic material we can define two critical current densities, the in-plane critical current density $\mathbf{j}_c^{\parallel} \parallel \mathbf{x}$, and $\mathbf{j}_c^{\perp} \parallel \mathbf{y}'$, the out-of-plane critical current density. The critical current density scales like a length and we obtain $j_c^{\parallel} = j_c^c = \varepsilon^{-2/3} j_c^{\mathrm{iso}}$, independent of ϑ, and $j_c^{\perp}(\vartheta) = \varepsilon_\vartheta j_c^c$. Finally, the condition for single vortex pinning transforms to $L_c^c \le a\varepsilon/\varepsilon_\vartheta^{1/2}$, or using the relation $L_c^c = \varepsilon\xi (j_0/j_c^c)^{1/2}$, $H \le (j_c^c/j_0) H_{c_2}(\vartheta)$. All these results agree with those obtained following the conventional approach [11] but involve essentially no calculations at all. For large fields or small current densities $j \ll j_c^{\parallel,\perp}(\vartheta)$ the response of the system is determined by (small) vortex bundles [3,8,10]. We then can use our scaling approach as long as the transverse size R_c of the bundle is smaller than the London penetration depth λ, or, more precisely, $\tilde{R}_c < \lambda$. The results for this regime will be published elsewhere.

In anisotropic materials an additional degree of freedom is the angle ϑ between the magnetic-field direction and the superconducting planes. Here our scaling rule (4) predicts the angular dependence of physical quantities to be expected due to the anisotropy of the material. For example, the scaling behavior of the in-plane resistivity as measured by Iye, Nakamura, and Tamegai [20] and interpreted by Kes *et al.* [21] finds a natural explanation within our new scaling approach: The scaling factor for the in-plane resistivity is $s_\rho = 1$ and using (4) we obtain $\rho(\vartheta, B) = \rho(\varepsilon_\vartheta B) \simeq \rho(\sin\vartheta B)$, without making any special assumptions about a possible breakdown of the concept of a flux-line lattice in layered superconductors [21]. In addition, for $\vartheta > \varepsilon$, after rescaling, the magnetic field is mainly directed along the c axis and thus the Lorentz force is essentially independent of the direction of the current in the plane. Going back to the original system, one then expects that the in-plane resistivity *should be* independent of the angle between the magnetic field and the current [20] in the regime $\vartheta > \varepsilon$. Similarly, the magnetization experiments by Gyorgy *et al.* [22] find a natural explanation within our scaling approach: Evaluating the critical current densities for in-plane and out-of-plane current flow, they find that $j_c^{\parallel} \simeq j_c^c$ and $j_c^{\perp}(\vartheta) \simeq \sin\vartheta\, j_c^c$, which is the prediction of the scaling theory.

Regarding the regime of applicability we wish to point out that, in spite of starting from a GL-type description, our scaling approach is not limited to the regime near T_c. In fact, our scaling rules can also be obtained by starting from the London equations, which are valid at any temperature. The scaling rules for the disorder will not be changed as long as the anisotropy in the penetration depths and in the core size remain the same. Also, our scaling approach can be used for the case of layered superconductors as long as the discreteness of the structure is not important. The crossover between quasi-2D and 3D anisotropic behavior depends on the physical quantity of interest; however, the regime where the anisotropic description is valid is usually large [11,23].

In summary, we have presented a new scaling approach for obtaining physical results in anisotropic superconductors in a simple and direct way. Our scaling rule (4) shows that the effect of anisotropy is to reduce the field component in the superconducting planes and to enhance the effective strength of the pinning, both favorable effects in view of technological applications of the new materials. On the other hand, the anisotropy increases the temperature of thermal fluctuations, favoring phenomena such as thermal depinning and melting of the vortex lattice, effects which are scientifically very interesting but rather undesired in view of applications.

We wish to thank M. V. Feigel'man, T. M. Rice, and V. M. Vinokur for stimulating discussions. V.B.G. and A.I.L. wish to acknowledge the financial support of the Swiss National Foundation.

[1] M. P. A. Fisher, Phys. Rev. Lett. **62**, 1415 (1989).
[2] D. R. Nelson, Phys. Rev. Lett. **60**, 1973 (1988); D. R. Nelson and H. S. Seung, Phys. Rev. B **39**, 9153 (1989).
[3] M. V. Feigel'man and V. M. Vinokur, Phys. Rev. B **41**, 8986 (1990).
[4] A. Houghton, R. A. Pelcovits, and A. Sudbø, Phys. Rev. B **40**, 6763 (1989).
[5] E. H. Brandt, Phys. Rev. Lett. **63**, 1106 (1989).
[6] V. G. Kogan and L. J. Campbell, Phys. Rev. Lett. **62**, 1552 (1989).
[7] A. Sudbø and E. H. Brandt, Phys. Rev. Lett. **66**, 1781 (1991).
[8] M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Phys. Rev. Lett. **63**, 2303 (1989).
[9] P. H. Kes, J. Aarts, J. van den Berg, C. J. van der Beek, and J. A. Mydosh, Supercond. Sci. Technol. **1**, 242 (1989).
[10] G. Blatter, V. B. Geshkenbein, and V. M. Vinokur, Phys. Rev. Lett. **66**, 3297 (1991).
[11] G. Blatter and V. G. Geshkenbein, Eidgenössische Technische Hochschule Report No. ETH-TH/91-27 (to be published).
[12] A. I. Larkin, Zh. Eksp. Teor. Fiz. **58**, 1466 (1970) [Sov. Phys. JETP **31**, 784 (1970)].
[13] A. I. Larkin and Yu. N. Ovchinnikov, J. Low Temp. Phys. **34**, 409 (1979).
[14] R. A. Klemm and J. R. Clem, Phys. Rev. B **21**, 1868 (1980).
[15] V. G. Kogan, Phys. Rev. B **24**, 1572 (1981).
[16] V. G. Kogan and J. R. Clem, Phys. Rev. B **24**, 2497 (1981).
[17] Z. Hao and J. R. Clem, Phys. Rev. B **43**, 7622 (1991).
[18] A. Sudbø and E. H. Brandt, Phys. Rev. B **43**, 10482 (1991).
[19] A. P. Malozemoff, in Proceedings of the Third International Conference on Materials and Mechanisms of Superconductivity—High-Temperature Superconductors (M^2S-HTSC III), Kanazawa, Japan, 1991 [Physica (Amsterdam) C (to be published)].
[20] Y. Iye, S. Nakamura, and T. Tamegai, Physica (Amsterdam) **159C**, 433 (1989).
[21] P. H. Kes, J. Aarts, V. M. Vinokur, and C. J. van der Beek, Phys. Rev. Lett. **64**, 1063 (1990).
[22] E. M. Gyorgy, R. B. van Dover, K. A. Jackson, L. F. Schneemeyer, and J. V. Waszczak, Appl. Phys. Lett. **55**, 283 (1989).
[23] M. V. Feigel'man, V. B. Geshkenbein, and A. I. Larkin, Physica (Amsterdam) **167C**, 177 (1990).