Two-Particle Spectral Properties of Generalized Landau Liquids

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The matrix elements of the up-spin and down-spin electronic density fluctuations taken between the ground state and the gapless elementary excitations of the Hubbard chain in a magnetic field are obtained. The small-ω charge and spin conductivity spectra are derived. Our results clarify the physical character of the pseudoparticles of the new Landau liquid theory, and identify them as the transport carriers. A generalized adiabatic continuity principle concerning the elementary excitations at finite magnetic field in the limit where momentum and frequency tend to zero is established.

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The main purpose of this Letter is to explain the reason for the existence of generalized Landau liquids which, except for the absence of quasiparticle peaks in the single-particle spectral function, have most of the features of a Fermi liquid [1]. We study an exactly soluble model —the Hubbard chain at densities $n<1$ [2]. At zero magnetic field H the ground state of the model is spin rotationally invariant and the low-energy physics can be studied by the method introduced by Haldane [3]. That requires the use of two effective Hamiltonians which correspond to two decoupled harmonic (Gaussian) models, describing charge- and spin-density fluctuations [3,4]. This leads to the holon and spinon excitation picture [5]. Haldane's description is, however, not valid for finite fields because when $H>0$ there is a change in the nature of the gapless excitations and their S matrix [6]. Recently, conformal field theory allowed the calculation of the critical exponents at arbitrary magnetic fields [7]. They were given in terms of the elements of a "dressed charge matrix." Nonetheless, the connection of these relevant parameters to the microscopic electronic representation remained unresolved.

The new Landau liquids include the (single phase) Fermi liquids and the (multiphase) one-dimensional Landau-Luttinger liquids introduced in Ref. [1]. Other generalized Landau liquids are expected to exist in two dimensions [5]. Transitions between the phases considered here can be realized by changes of the particle-particle interactions. In the case of the Hubbard chain we distinguish three phases. Both at $U=0$ and in the ferromagnetic phase at $U>U_c$ the system is a noninteracting Fermi liquid. Otherwise, it is a Luttinger liquid, with a fully incoherent single-particle spectral function. Fixing two of the three quantities U, H, and n, the ferromagnetic phase corresponds either to interactions $U>U_c(H,n)$, fields $H>H_c(U,n)$, or densities $n<n_c(U,H)$, where the critical values are defined by Eq. (2) of Ref. [6] (with H_c replaced by H in the cases of U_c and n_c). The phase diagram of the model for various densities corresponds to Fig. 1(b) of Ref. [6] (with H_c replaced by H). The adiabatic continuity principle, which is a crucial requirement of Fermi-liquid theory, is valid here within each phase. Although it is not valid across the phase boundaries, we show in the following that for electronic Landau liquids it "survives" in the case of excitations of lowest momentum and energy.

Since in the Luttinger phases the single-particle matrix element Z vanishes [5], it is suggestive for a spectral definition of the generalized Landau liquids to consider the two-particle matrix elements, which play a relevant role in the transport properties. Let us consider an electronic interacting system of arbitrary dimension. According to many-body theory the charge-charge and spin-spin response functions can be written as [8]

$$\chi^{(\vartheta)}(\mathbf{k},\omega)=-\sum_j|\langle j|\hat{n}_{\mathbf{k}}^{(\vartheta)}|0\rangle|^2\frac{2\omega_{j0}}{\omega_{j0}^2-(\omega+i\eta)^2}\,, \qquad (1)$$

where $\vartheta=\rho,\sigma_z$ refer to the former and latter functions, respectively, the j summation is over the eigenstates, and ω_{j0} are the corresponding excitation energies. The matrix elements are defined by density and spin fluctuation operators

$$\hat{n}_{\mathbf{k}}^{(\rho)}=\hat{n}_{\mathbf{k}\uparrow}+\hat{n}_{\mathbf{k}\downarrow},\quad \hat{n}_{\mathbf{k}}^{(\sigma_z)}=\hat{n}_{\mathbf{k}\uparrow}-\hat{n}_{\mathbf{k}\downarrow},\quad \hat{n}_{\mathbf{k}\sigma}=\sum_{\mathbf{k}'}c_{\mathbf{k}+\mathbf{k}',\sigma}^{\dagger}c_{\mathbf{k}',\sigma}\,. \qquad (2)$$

We are here particularly interested in the low momentum and frequency limit. In the case of a three-dimensional Fermi liquid there are three kinds of excitations to be considered when $\vartheta=\rho$ in (1): (i) single-pair and (ii) multipair particle-hole processes in the quasiparticle bands, and (iii) zero sound excitations [8]. In the limit $k\rightarrow 0$ only (i) and (iii) contribute to (1). Nonetheless, in that limit the only matrix elements on the right-hand side of (1) which do not vanish are the ones connecting the ground state to the excitations (i) [8] (the same holds true for $\vartheta=\sigma_z$). Let us denote the states (i) by $|\mathbf{k}_0,\mathbf{k};\sigma\rangle$ where $\mathbf{k}_0$ and $\mathbf{k}_0+\mathbf{k}$ are the pseudomomentum of the quasiparticle of spin projection σ ($\sigma=\uparrow,\downarrow$) before and after the particle-hole process, respectively. As $k\rightarrow 0$ we

approach the Fermi surface and the single-pair quasiparticle state becomes a real electron-hole excitation. Thus, in a Fermi liquid,

$$\lim_{\mathbf{k}\to 0}\langle \mathbf{k}_0,\mathbf{k};\sigma|\hat{n}_{\mathbf{k},\sigma'}|0\rangle=\delta_{\sigma,\sigma'}. \tag{3}$$

The equality (3) is an alternative two-particle criterium for a Fermi liquid. In contrast to Z, this type of matrix element does not vanish in the Luttinger phases of the generalized Landau liquids.

In the case of the Hubbard chain at finite fields there are two kinds of single-pair particle-hole states involving the bands for the charge (c) and spin (s) pseudoparticles,

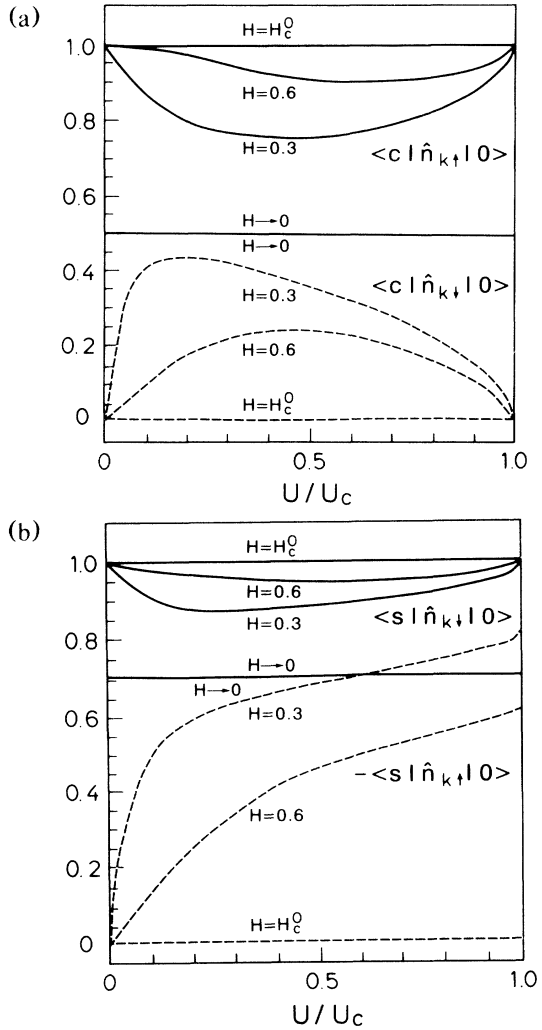

FIG. 1. The matrix elements (a) $\langle c|\hat{n}_{k\uparrow}|0\rangle$ (solid lines), $\langle c|\hat{n}_{k\downarrow}|0\rangle$ (dashed lines) and (b) $\langle s|\hat{n}_{k\downarrow}|0\rangle$ (solid lines), $-\langle s|\hat{n}_{k\uparrow}|0\rangle$ (dashed lines) as a function of U/U_c for $n=0.7$ and various magnetic fields. We note that when $H\to 0$, (5) and (6) change continuously from $1/\sqrt{2}$ for $U\to 0$, to $\frac{1}{2}$ for $U\to\infty$, yet both at $U=0$ and $U=U_c=\infty$ they are given by 1 and 0, respectively. Since $U_c\to\infty$ as $H\to 0$, when $H\to 0$ the U/U_c rescaling hides in (a) their true U dependence for $0<U<U_c$.

respectively [6]. In one dimension the zero sound excitations do not exist. Moreover, as in a Fermi liquid, in the limit $k\to 0$ the multipair excitations do not contribute to (1). We denote the single-pair eigenstates for $0<U<U_c$ by $|k_0,k;\alpha\rangle$, where $\alpha=c,s$, and introduce the notation $\langle\alpha|\hat{n}_{k\sigma}|0\rangle=\lim_{k\to 0}\langle k_0,k;\alpha|\hat{n}_{k\sigma}|0\rangle$. We are then limited to the following four relevant matrix elements

$$\mathbf{N}_0=\begin{bmatrix}\langle c|\hat{n}_{k\uparrow}|0\rangle & \langle c|\hat{n}_{k\downarrow}|0\rangle\\ \langle s|\hat{n}_{k\uparrow}|0\rangle & \langle s|\hat{n}_{k\downarrow}|0\rangle\end{bmatrix}. \tag{4}$$

At $U=0$ the single-pair excitations refer to electrons. If we replace the bands c and s by the bands for up- and down-spin electrons, the corresponding $\mathbf{N}_0$ matrix is the unit matrix, in agreement with (3). Since the one-dimensional pseudo-Fermi surfaces reduce to two points, in the limit of very low momentum and energy the velocities $v_c=v_c(2k_F)$, $v_s=v_s(k_{F\downarrow})$ [6] and the four matrix elements (4) fully determine the functions (1). On the other hand, by applying a charge or spin probe, those functions can be calculated as in a Fermi liquid [8]—the transport properties are described by kinetic equations that govern the flow of pseudoparticles, which are the transport carriers. This can be simply understood by recalling that at finite fields all the gapless excitations can be described by particle-hole processes in the pseudoparticle bands [6]. Then, following the form of the functions (1), for low momentum and low frequency the delta peaks which constitute the charge and spin dynamical form factors are produced by these processes. Since these factors embody all properties that are relevant to the scattering of the probes, it follows that the electronic degrees of freedom couple to the probes through the pseudoparticles of the one-dimensional theory. After deriving the amplitudes of the matrix elements (4) by calculating expressions for the functions (1), the use of a boundary condition allowed the evaluation of their relative phases. The final result is

$$\langle c|\hat{n}_{k\uparrow}|0\rangle=1+\Phi_{cc}(2k_F,2k_F)-\Phi_{cc}(2k_F,-2k_F)-\langle c|\hat{n}_{k\downarrow}|0\rangle, \tag{5}$$

$$\langle c|\hat{n}_{k\downarrow}|0\rangle=\Phi_{cs}(2k_F,k_{F\downarrow})-\Phi_{cs}(2k_F,-k_{F\downarrow}), \tag{6}$$

$$\langle s|\hat{n}_{k\uparrow}|0\rangle=\Phi_{sc}(k_{F\downarrow},2k_F)-\Phi_{sc}(k_{F\downarrow},-2k_F)-\langle s|\hat{n}_{k\downarrow}|0\rangle, \tag{7}$$

$$\langle s|\hat{n}_{k\downarrow}|0\rangle=1+\Phi_{ss}(k_{F\downarrow},k_{F\downarrow})-\Phi_{ss}(k_{F\downarrow},-k_{F\downarrow}), \tag{8}$$

where the pseudoparticle phase shifts Φ in units of 2π, which were studied in Ref. [1], define the S matrix for the elementary excitations [6]. Equations (5)–(8) show that the electronic matrix elements (4) are fully determined by that S matrix. The matrices (5)–(8) can also be expressed as simple combinations of the elements of the dressed charge matrix [9]. In Figs. 1(a) and 1(b) we plot the matrix elements (5)–(8) as a function of U/U_c for $n=0.7$ and several magnetic fields. For $H\neq 0$ these

elements are analytical functions of the on-site interaction. Unlike a Fermi liquid, when $0<U<U_c$ they are not simply given by 1 or 0. Although the pseudoparticles refer to eigenstates, they are not dressed electrons: In contrast to a Fermi liquid, when $k\rightarrow 0$ the operators (2) do not fully project the ground state $|0\rangle$ on the corresponding single-pair excitation. In addition, $\det^2\mathbf{N}_0<1$. Then the overlap is only partial and the diagonal elements of (4) are smaller than 1 (and the off-diagonal elements are different from 0). Such behavior implies $Z=0$ [9]—only when the overlap becomes total as $k\rightarrow 0$ does the excitation refer to a real electron and the single-particle spectral function show a peak. For $U\rightarrow 0$ the matrix (4) tends continuously to the unit matrix, which is nothing but the corresponding matrix for the noninteracting system. Also in the limit $U\rightarrow U_c$ the two matrix elements involving the charge pseudoparticles tend to 1 and 0, which are the values for up-spin electrons. On the other hand, the on-site electronic interactions induce a change in the structure and nature of the eigenstate spectrum and there is a crossing of states as $U\rightarrow 0$. Nonetheless, in the case of a Landau liquid we are mainly concerned with the excitations of lowest momentum and energy. The continuous manner in which the matrix (4) tends to the unit matrix as $U\rightarrow 0$ [and (5) and (6) tend to 1 and 0 as $U\rightarrow U_c$] establishes a generalized adiabatic continuity principle which is valid for the gapless charge and spin excitations of electronic Landau liquids. We emphasize that this continuous behavior is not required when the corresponding excitation branch collapses in one side of the transition [as in Fig. 1(b) when $U/U_c\rightarrow 1$].

Figures 1(a) and 1(b) confirm that the zero magnetic field case has a different physics: In the limit $H\rightarrow 0$ the matrix elements (4) and (5) are singular as $U\rightarrow 0$, and the above adiabatic principle is not valid (this singular behavior, however, does not show up in the static susceptibilities and critical exponents [9]). The generalized definition of a Landau liquid requires that all gapless excitations can be described by particle-hole processes in pseudoparticle bands. At $H=0$ the Hubbard chain is not a Landau liquid because the spin string excitations, which involve two spin pseudoparticles and have a gap for $H>0$, become gapless [6]. Furthermore, the spin pseudoparticle particle-hole excitations collapse at $H=0$. This implies that at $H=0$ the S matrix for gapless excitations acquires a different nature. Nonetheless, the limit $H\rightarrow 0$ of the finite field expressions provides the correct values for the zero-field critical exponents, static susceptibilities, and stiffnesses.

The combination of the kinetic equations with the conservation law for charge and spin allows the derivation of the charge and spin currents. As the energy [1], these can be written as a functional of the pseudomomentum deviations. To first order in the deviations we obtain

$$\langle J^{(\rho)}(x,t)\rangle=\frac{N_a}{2\pi}\int_{-\pi}^{\pi}dq\,\delta_c(q;x,t)J_c^{(\rho)}(q)\,, \tag{9}$$

$$\langle J^{(\sigma_z)}(x,t)\rangle=\frac{N_a}{2\pi}\left[\int_{-\pi}^{\pi}dq\,\delta_c(q;x,t)J_c^{(\sigma_z)}(q)+\int_{-k_{F\uparrow}}^{k_{F\uparrow}}dp\,\delta_{\downarrow}(p;x,t)J_s^{(\sigma_z)}(p)\right], \tag{10}$$

respectively. We note that in this framework the deviations depend on the position x and time t. The elementary current spectra of the pseudoparticles $J_c^{(\rho)}(q)$, $J_c^{(\sigma_z)}(q)$, and $J_s^{(\sigma_z)}(p)$ are independent of the form of these deviations and involve the velocities and f functions introduced in Refs. [1,7,9]. Equation (9) shows that the spin pseudoparticles carry no charge. Since in the domain of energy where the only contributing states are the single and multipair excitations the one-dimensional liquid has only forward scattering [6], these states are not expected to be connected to the ground state by current operators. On the other hand, there are other higher-energy charge and spin states which bring about the dissipative part of the spectra. Each of the latter states has a gap, which implies that the charge and spin conductivity spectra in the nondissipative region, which is defined by the smallest of these gaps, are given simply by

$$\mathcal{R}\sigma^{(\eta)}(\omega)=(e^{(\eta)})^2 2\pi D^{(\eta)}\delta(\omega)\,, \tag{11}$$

where $e^{(\rho)}=-e$, $e^{(\sigma_z)}=\frac{1}{2}$, $-e$ is the electronic charge, and $D^{(\rho)}$ and $D^{(\sigma_z)}$ are the charge and spin stiffnesses which were calculated previously for zero magnetic field [4,10]. The spectrum (11) agrees with numerical results [11]. Based on the small momentum and frequency expressions for the functions (1) we could evaluate these important quantities for arbitrary magnetic fields. The result is

$$\begin{aligned}2\pi D^{(\rho)}&=J_c^{(\rho)}(2k_F)/(-e)\\&=v_c|\langle c|\hat{n}_k^{(\rho)}|0\rangle|^2+v_s|\langle s|\hat{n}_k^{(\rho)}|0\rangle|^2\,,\end{aligned} \tag{12}$$

$$\begin{aligned}2\pi D^{(\sigma_z)}&=[J_c^{(\sigma_z)}(2k_F)-2J_s^{(\sigma_z)}(k_{F\downarrow}]/\tfrac{1}{2}\\&=v_c|\langle c|\hat{n}_k^{(\sigma_z)}|0\rangle|^2+v_s|\langle s|\hat{n}_k^{(\sigma_z)}|0\rangle|^2\,.\end{aligned} \tag{13}$$

The exclusive dependence of $D^{(\rho)}$ on $J^{(\rho)}(2k_F)_c$ confirms that the charge is transported by the "charge pseudoparticles." On the other hand, the form of $D^{(\sigma_z)}$ shows that spin transport results from the coherent superposition of charge and spin pseudoparticle currents. In the limit $H\rightarrow 0$, $J_c^{(\sigma_z)}(2k_F)$ vanishes and spin transport results exclusively from the flow of spin pseudoparticles. The second terms on the right-hand side of Eqs. (12) and (13) have a remarkably simple form and a profound physical meaning. The charge and spin stiffnesses are a linear combination of the velocities which determine the linear

term of the specific heat [6], weighted by the squares of the matrix elements for charge- and spin-density fluctuations, respectively. These are simply obtained from Eqs. (5)–(8), their amplitudes being 1 for $U \rightarrow 0$. The parameter $\det^2 \mathbf{N}_0$, which is 1 in the limits $U \rightarrow 0$ and $U \rightarrow U_c$, is otherwise smaller than 1. This is a consequence of the fact that the above-mentioned dissipative states of higher energy and zero momentum remove spectral weight from the single-pair excitations, which can be confirmed by the study of the sum rule for the conductivity spectra [12]. For $H > 0$, as at $H = 0$ [4,10], the ratio $(2N_a D^{(\eta)})/|\langle T \rangle|$ gives the fraction of spectral weight associated to the δ peak of $\sigma^{(\eta)}(\omega)$, and the kinetic energy $\langle T \rangle$ is related to the ground-state energy by $\langle T \rangle = t\partial E/\partial t$ [12]. The δ peak exhausts the sum rules in the limits $U \rightarrow 0$ and $U \rightarrow U_c$. A detailed study of the dependence of the stiffnesses on U, H, and n will be presented elsewhere. In the limit $H \rightarrow 0$, Eqs. (12) and (13) reproduce the results obtained previously [4,10].

The renormalized charge and spin of the pseudoparticles at the pseudo-Fermi points are defined as

$$e_c = J_c^{(\rho)}(2k_F)/v_c, \quad s_c = J_c^{(\sigma_z)}(2k_F)/v_c, \quad e_s = 0,$$

and

$$s_s = J_s^{(\sigma_z)}(k_{F\downarrow})/v_s .$$

While $s_s = -\frac{1}{2}$, or close to $-\frac{1}{2}$, e_c and s_c are rather field dependent. Their limiting values are presented in Table I, where ξ_0 is the parameter that regulates the critical exponents [7] and thermodynamic quantities [6] at $H = 0$. While the spinon excitations gain a gap, at finite fields the holon excitations change their character because the charge pseudoparticles acquire spin for $H > 0$. Their spin changes continuously from $s_c = 0$ as $H \rightarrow 0$ to $s_c = \frac{1}{2}$ as $H \rightarrow H_c$. We emphasize that the generalized adiabatic continuity principle introduced above does not imply that for $U \rightarrow 0$ the charge and spin pseudoparticles map on up-spin and down-spin electrons, respectively; although in that limit the matrix (4) transforms continuously to the unit matrix, the charge pseudoparticles are still dressed by the down-spin electrons in such a way that their effective charge and spin are given by the values of Table I (and the spin pseudoparticles remain chargeless). On the other hand, in the limit $U \rightarrow U_c$ that principle is corroborated by the almost "Fermi-liquid"-like character, i.e., e_c and s_c tend to the corresponding values for up-spin electrons.

In this Letter we have defined the new generalized Landau liquids from the point of view of the two-particle spectral properties. The nontrivial spectral properties of the Luttinger phases arise from the fact that the parameters that regulate the low-energy physics [1,6,9]—phase shifts (or elements of the dressed charge matrix [9]) and f functions—are controlled by two-particle matrix elements between the ground state and $k \rightarrow 0$ pseudoparticle particle-hole excitations. In contrast to a Fermi liquid, where the single-pair quasiparticle states become electron-hole excitations as $k \rightarrow 0$, the two-electron operators partially project the ground state on higher-energy dissipative states, which remove part of the spectral weight. As illustrated in Fig. 1, unlike a Fermi liquid [see Eq. (3)], these matrix elements are interaction dependent. Nonetheless, on the boundaries to the Fermi-liquid phases they tend continuously to the values (3). We expect the existence of such $Z = 0$ two-dimensional Landau liquids [5]. This implies separate (chargeless) spin and (not necessarily spinless) charge transport carriers confined to the condensed-matter system. Although from the electronic point of view these pseudoparticles are complicated collective modes, within the many-body system they obey a simple Landau liquid physics [1,6,9].

TABLE I. Limiting values of the renormalized charge e_c and spin s_c ($\xi_0 = \sqrt{2}, 1$ for $U \rightarrow 0, \infty$, respectively, and as $U \rightarrow 0$, $v_c \rightarrow 2t \sin k_{F\uparrow}$, $v_s \rightarrow 2t \sin k_{F\downarrow}$).

	$H \rightarrow 0$	$U \rightarrow U_c$	$U \rightarrow 0$
e_c	$-e\xi_0^2$	$-e$	$-e(1 + v_s/v_c)$
s_c	0	$\frac{1}{2}$	$\frac{1}{2}(1 - v_s/v_c)$

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