Flux-Sensitive Correlations of Mutually Incoherent Quantum Channels

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A two-particle state can be sensitive to a magnetic flux through exchange even if it is composed of single-particle states which on the average are statistically uncorrelated and exhibit no direct interference. Consequently, two-particle observables can be sensitive to a flux even if all single-particle observables are flux insensitive. We investigate exchange effects in current cross correlations of small conductors subject to a magnetic field.

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In a 1963 paper Goldberger, Lewis, and Watson [1] analyzed scattering geometries in which a target is illuminated by two mutually incoherent sources and in which the intensity cross correlation is measured with the help of two detectors. Remarkably, the intensity cross correlation yields phase information despite the fact that the sources are entirely independent. A striking demonstration of this effect could be obtained for charged carriers since an Aharonov-Bohm (AB) flux [2] can be used to modulate the phase of the correlation function [3]. In this paper we illustrate the origin of this effect by calculating the density-density correlation for electronic beams in the presence of an AB flux. We demonstrate that the interference structure in the Fermi hole [4] can be modulated with an AB flux even if none of the single-particle states is sensitive to the flux. Experimental tests of these predictions can be carried out in vacuum using electron microscopes or field emission from tunneling microscope tips as sources. A discussion which focuses on the realization of such an experiment in conductors has, however, a number of advantages. First, a simple description of sources (current contacts) and detectors (voltage contacts) exists [5] and has been experimentally tested [6]. Second, the current correlations for this approach have been calculated in the framework of second quantization [3,7,8] and can be compared with discussions using wave packets impinging on the conductor [9] and with a semiclassical analysis [10]. Third, the S-matrix description of electric conduction makes the constraints on any electron current pattern due to current conservation and microreversibility particularly clear and prevents us from proposing a geometry which does not obey these basic constraints. We analyze the current correlations across two contacts of a small conductor in which a magnetic field permits the phase of the correlation function to change.

The phase information in the correlation function of mutually incoherent beams points to the interesting possibility that in addition to *direct interference* of partial waves quantum mechanics offers a second mechanism to generate *interference via exchange*. These two possibilities are discussed with the help of Fig. 1. Figure 1(a) shows the typical AB geometry [2,11] in which a wave emitted by a source S reaches a sink S. The wave is split into two partial waves p_1 and p_2 which encircle the flux and are recombined. If there are many quantum channels (incident waves specified by different quantum numbers) connecting the source and sink, then each quantum channel can be treated as if it were incoherent with all other channels. In Fig. 1(a) the flux sensitivity of the electronic states is a consequence of *direct interference* of the partial waves. In Fig. 1(b) two waves emanate from different sources. One channel is guided on an upper arc around the flux and one channel on a lower arc around the flux. There is no elastic scattering from one of these states to the other. Neither of the two waves by itself encircles the flux. We deal with mutually incoherent quantum channels that exhibit no direct interference and consequently exhibit no response to a change in the AB flux. However, if there is an overlap of the wave functions in the regions O_1 and O_2 to the left and right of the flux, we must satisfy the Pauli principle and consider two- (or many-) particle states. Interestingly, the *two-particle state* is sensitive to the AB flux even if the single-particle states are not. To show this we now analyze Fig. 1(b) in detail.

Assume that the waves shown in Fig. 1(b) represent transmission channels of a conductor. We assume that the potential provided by the conductor is fixed in time and does not fluctuate. Regions O_1 and O_2 are very nar-

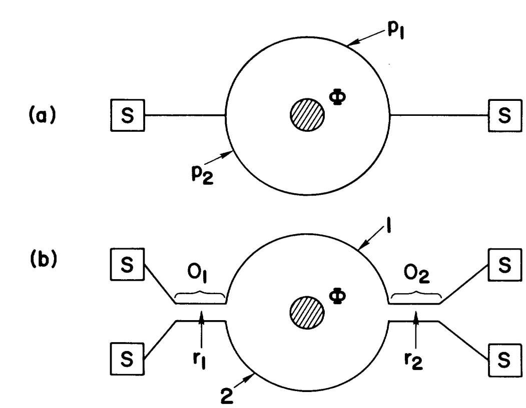

FIG. 1. (a) Direct interference: Wave transmitted from source (S) to sink (S) is split into partial waves encircling the flux. (b) Exchange interference: Two waves transmitted from sources to sinks without encircling the flux but with an overlap in regions O_1 and O_2.

row portions of the conductor in which the transverse wave functions overlap. Direct interference caused by interchannel scattering induced by disorder is assumed to be absent. These stringent conditions are very nearly satisfied at a quantized conductance plateau of a quantum point contact formed in a two-dimensional constricted high-mobility electron gas [12]. Two of these contacts separated by an intervening circular region [13] provide a realization of the current pattern of Fig. 1(a): The large separation of the electron states in the circular region between the contacts can be achieved with a magnetic field with a cyclotron radius which is small compared to the diameter of the circular region but is large compared to the lateral width of the contacts. Here we wish to make a conceptual point and for clarity deal with electrons guided by scalar potentials (no magnetic field). We assume that the transverse motion of the electrons in channels 1 and 2 is quantized and described by wave functions $\chi_1(y)$ and $\chi_2(y)$. Here y is the transverse coordinate. The longitudinal motion along the narrow wire is given by $e^{ik_{1,2}x}$. At a point $r_1=(x_1,y_1)$ region O_1 the single-particle wave functions are

$$\psi_1(r_1)=e^{ik_1x_1}\chi_1(y_1)\,, \tag{1}$$

$$\psi_2(r_1)=e^{ik_2x_1}\chi_2(y_1)\,. \tag{2}$$

At a point $r_2=(x_2,y_2)$ in region O_2 the wave functions are

$$\psi_1(r_2)=e^{ik_1x_2}\chi_1(y_2)e^{i\phi_1}e^{i\Theta_1}\,, \tag{3}$$

$$\psi_2(r_2)=e^{ik_2x_2}\chi_2(y_2)e^{i\phi_2}e^{i\Theta_2}\,, \tag{4}$$

where $\phi_{1,2}$ take into account the excess path due to motion along the arcs and

$$\Theta_{1,2}=\frac{e}{\hbar c}\int_{r_1}^{r_2}\mathbf{ds}_{1,2}\cdot\mathbf{A} \tag{5}$$

is the phase due to the AB flux. In Eq. (5) the integral for Θ_1 is along the upper arc and Θ_2 is along the lower arc and $\mathbf{A}$ is the vector potential. Clearly, $\Theta\equiv\Theta_1-\Theta_2=2\pi\Phi/\Phi_0$, where Φ is the flux and $\Phi_0=hc/e$. Note that the absolute squares of the single-particle wave functions $|\psi_{1,2}|^2$ are independent of the flux. However, the two-particle wave function

$$\Psi(r_1,r_2)\equiv\psi_1(r_1)\psi_2(r_2)-\psi_1(r_2)\psi_2(r_1)$$
$$=e^{i\phi_0}[\chi_1(y_1)\chi_2(y_2)-\chi_1(y_2)\chi_2(y_1)e^{i\phi}]\,, \tag{6}$$

where $\phi_0\equiv k_1x_1+k_2x_2+\phi_2+\Theta_2$ and

$$\phi\equiv(k_1-k_2)(x_2-x_1)+(\phi_1-\phi_2)+\Theta\,, \tag{7}$$

depends through ϕ in an essential way on the flux. Next we compare the probability density $W^{(2)}=|\Psi(r_1,r_2)|^2$ of the two-particle state with the symmetrized single-state density $W^{(1)}$. The difference $\Delta W=W^{(2)}-W^{(1)}$ is a measure of correlations not contained in the single-particle states. We find

$$\Delta W(r_1,r_2)=|\Psi(r_1,r_2)|^2-[|\psi_1(r_1)|^2|\psi_2(r_2)|^2+|\psi_1(r_2)|^2|\psi_2(r_1)|^2]=-\psi_1^*(r_1)\psi_2(r_1)\psi_2^*(r_2)\psi_1(r_2)-\text{c.c.}\,, \tag{8}$$

and using the single-particle states as given above,

$$\Delta W(r_1,r_2)=-2\chi_1(y_1)\chi_2(y_2)\chi_1(y_2)\chi_2(y_1)\cos(\phi)\,. \tag{9}$$

Thus the portion of $W^{(2)}$ which is sensitive to the flux has an amplitude that is a product in which each transverse wave function occurs twice but is taken at different locations. The flux sensitivity is due to an exchange effect. In the two-particle state we cannot distinguish between carriers which passed the flux in the upper or lower arc of Fig. 1(b).

The exchange amplitude in Eq. (9) is zero if the transverse wave functions do not overlap (have no common support). Suppose that we are interested not in the density $|\Psi(r_1,r_2)|^2$, but only in the line density $\int dy_1dy_2|\Psi(r_1,r_2)|^2$. In this case the orthogonality of the transverse wave function guarantees that the exchange term vanishes. Exchange effects in the line density can only occur from states k which belong to the same quantum channel (i.e., if $\chi_1\equiv\chi_2$).

So far we have considered well-defined states k_1 and k_2 and their two-particle state. If the states k_1 and k_2 are produced by sources (electron reservoirs), the phase of each of these states is a random variable without mutual correlation. Furthermore, due to thermal agitation, the probability of occupation (whether these states are actually filled or are empty) is also a fluctuating quantity. Second quantization provides a convenient framework to discuss such fluctuations. The field operator is $\hat\Psi(r)=\sum_{\alpha i}\psi_{\alpha i}(k_{\alpha i},r)\hat a_{k_{\alpha i}}$, where $\hat a_{k_{\alpha i}}$ annihilates an electron with wave vector k_i in the source α. The average density $\langle n(r)\rangle=\langle\hat\Psi^\dagger(r)\hat\Psi(r)\rangle$ is simply $\sum_{\alpha i}|\psi_\alpha(k_{\alpha i})|^2f_\alpha(k_{\alpha i})$. Here we have assumed that two waves $k_{\alpha i}$ and $k_{\beta j}$ are on the statistical average uncorrelated, $\langle\hat a^\dagger_{k_{\alpha i}}\hat a_{k_{\beta j}}\rangle=\delta_{\alpha,\beta}\delta_{k_i,k_j}f_\alpha(k_{\alpha i})$. Following Ref. [4] we calculate the equal time correlation $\langle\Delta n(r_1)\Delta n(r_2)\rangle$ of the density fluctuations $\Delta n(r_1)=n(r_1)-\langle n(r_1)\rangle$. For $r_1\neq r_2$, the density-density correlation function is

$$\langle\Delta n(r_1)\Delta n(r_2)\rangle=\sum_{\alpha\beta ij}\Delta W(r_1,r_2;k_{\alpha i},k_{\beta j})f_\alpha(k_{\alpha i})f_\beta(k_{\beta j})\,. \tag{10}$$

It is determined by two-particle excitations. ΔW is given by Eq. (8) with the wave function indices 1 and 2 replaced by $k_{\alpha i}$ and $k_{\beta j}$. For a three-dimensional electron gas evaluation of Eq. (10) without a flux gives the Fermi correlation hole. At $kT=0$ and at equilibrium, it is proportional to the electron density and decays with distance $r=|r_1-r_2|$ like $\cos^2(k_Fr)/r^4$. For the quasi-one-dimensional situation of Fig. 1(b) it decays with distance like $1/s^2$ with $s=x_2-x_1+(\pi-1)R$. R is the radius of the arcs in Fig. 1(b). Most importantly, there is now an exchange contribution to the correlation function which is proportional to $\sin^2(k_Fs/2)\cos(2\pi\Phi/\Phi_0)$. The interference structure of the Fermi hole is modulated by the AB flux.

Exchange interference survives statistical averaging: It is sensitive to an AB flux even if the single-particle channels are mutually incoherent.

Next, we investigate current-current fluctuations at the contacts of a conductor. What counts experimentally are not current densities but the total current at a contact. At a contact the current density is integrated over the cross section of the contact. As in our discussion of the line density given above, the integration results in the elimination of many exchange effects. At equilibrium the current correlations, via the fluctuation dissipation theorem, are related to the equilibrium transport coefficients. In the zero-frequency limit, the equilibrium transport coefficient is a single-particle observable and consequently is a sum of incoherent single-particle contributions. This leaves open the possibility that exchange correlations appear in a nonequilibrium situation, i.e., if a net current is driven through the sample. Indeed, for a two-terminal conductor [3,7–10], the current fluctuations $\langle(\Delta I_1)^2\rangle = \langle(\Delta I_2)^2\rangle$ at zero temperature are proportional to the chemical potential difference $|\mu_1 - \mu_2|$ and are proportional to $\mathrm{Tr}(\mathbf{r}^\dagger \mathbf{r} t^\dagger \mathbf{t})$. Here $\mathbf{t}$ and $\mathbf{r}$ are the matrices of transmission amplitudes and reflection amplitudes. The cross correlation of the currents at the two ports is $\langle \Delta I_1 \Delta I_2\rangle = -\langle(\Delta I_1)^2\rangle$ due to current conservation [3]. Therefore, the exchange terms invoke both reflection and transmission amplitudes. It is not possible to observe exchange effects without introducing reflection. Typically reflection gives rise to direct interference and therefore, in a two-terminal structure, the distinction between direct interference and exchange interference is difficult. Thus it is necessary to consider a multiprobe geometry.

Consider a multiprobe conductor with contacts labeled $\alpha = 1,2,3,\ldots$. Denote the scattering matrix that determines the outgoing current amplitudes in contact α in terms of the incoming amplitudes at contact β by $\mathbf{s}_{\alpha\beta}$. At $kT = 0$, in a frequency interval $\Delta\nu$, the spectral density of the current-current correlations [7] is given by

$$\langle \Delta I_\alpha \Delta I_\beta \rangle = 2\Delta\nu (e^2/h) \sum_{\gamma,\delta(\gamma\neq\delta)} \int dE\, f_\gamma (1 - f_\delta) \mathrm{Tr}(\mathbf{s}^\dagger_{\alpha\gamma} \mathbf{s}_{\alpha\delta} \mathbf{s}^\dagger_{\beta\delta} \mathbf{s}_{\beta\gamma}) . \tag{11}$$

In Eq. (11) the Fermi function f_α is 1 for energies below μ_α and is 0 for energies above μ_α. For $\alpha = \beta$, Eq. (11) gives the mean square current fluctuations at a contact. For $\alpha \neq \beta$, Eq. (11) determines the cross correlation of the current fluctuations. Taking into account that the matrices $\mathbf{s}_{\alpha\beta}$ are submatrices of a large matrix $\mathbf{S}$ which is unitary, it can be shown that the terms in Eq. (11) linear in the Fermi distribution add up to zero. Hence, the current cross correlation depends, like the density-density correlation in Eq. (10), on the products $f_\gamma f_\delta$ of Fermi functions. Like the density correlation Eq. (10), the current cross correlations are a measure of the population of two-particle states.

Let us now consider a geometry in which direct interference effects are small. Figure 2 schematically depicts a two-dimensional conductor in a quantizing magnetic field [14]. At the upper edge two well-separated edge states [15] provide channels along which carriers can be propagated with a transmission probability close to 1 [16] from contact 1 to contact 2. A large separation of the edge states can be achieved if the potential near the edge of the conductor can be arranged to vary smoothly compared to the cyclotron energy [17]. Two interior contacts [18] (3 and 4) are located between the two edge channels. To find an exchange effect it is necessary for the interior contact to couple with both edge channels simultaneously. As a consequence the interior contacts also lead to *channel mixing*: A wave incident in channel 1 can, via the interior contact, transmit a partial wave into channel 2 which with the help of contact 4 can be recombined with the partial wave in channel 1. Mixing gives rise to direct interference and gives rise to an ordinary AB effect. However, in the geometry of Fig. 2 the mixing of channels can be made smaller than the *direct* coupling of the edge channels to the interior contact. If the interior contact couples weakly to the edge states with a transmission probability T, then channel mixing is proportional to T^2.

Let us denote the scattering matrix which relates the incoming current amplitudes a_i in the edge states $(i = 1,2)$ and the interior contact $(i = 3)$ to the "outgoing" current amplitudes a_j' by a_{ji}. A matrix which is unitary and symmetric and has the desired properties can be specified as follows [19]. The direct coupling elements are $a_{31} = a_{32} = \sqrt{T/2}$, the mixing element is $a_{21} = \frac{1}{2}(\sqrt{1-T} - 1)$, the matrix element for transmission along the edge states is $a_{11} = a_{22} = \frac{1}{2}(\sqrt{1-T} + 1)$ and reflection back into the interior contact is determined by $a_{33} = -\sqrt{1-T}$. With these specifications applied to both interior contacts, the calculation of the overall scattering matrix of the conductor is now carried out readily. To simplify the notation we keep only the channel indices (as given in Fig. 2) for the overall scattering matrix elements and omit the probe indices. To leading order in T the relevant matrix elements are $s_{31} = (T/2)^{1/2}$, $s_{32} = (T/$

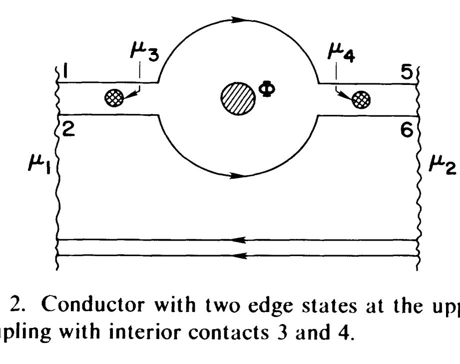

FIG. 2. Conductor with two edge states at the upper boundary coupling with interior contacts 3 and 4.

$2)^{1/2}$, and

$$s_{41}=(T/2)^{1/2}e^{i\phi_1}e^{i\Theta_1}+O(T^{3/2}), \quad (12)$$

$$s_{41}=(T/2)^{1/2}e^{i\phi_2}e^{i\Theta_2}+O(T^{3/2}). \quad (13)$$

To order T all transmission probabilities $T_{ij}=|s_{ij}|^2$ are independent of Φ.

Suppose now that contacts 1 and 2 of the conductor in Fig. 2 are both kept at the potential μ and contacts 3 and 4 are both kept at $\mu_0<\mu$. Using Eq. (11) we find to lowest order in T

$$\langle(\Delta I_3)^2\rangle=2e\Delta\nu(e/h)T(\mu-\mu_0)+O(T^2). \quad (14)$$

The same result holds for $\langle(\Delta I_4)^2\rangle$. The cross correlation $\langle\Delta I_3\Delta I_4\rangle$ is a sum of four terms: $s^*_{31}s_{31}s^*_{41}s_{41}=\frac{1}{4}T^2$, $s^*_{32}s_{32}s^*_{42}s_{42}=\frac{1}{4}T^2$, the exchange term $s^*_{31}s_{32}s^*_{42}s_{41}$, and its complex conjugate. The current correlation is

$$\langle\Delta I_3\Delta I_4\rangle=-2e\Delta\nu(e/h)(\mu-\mu_0)T^2\cos^2(\phi_1-\phi_2+\Theta), \quad (15)$$

where $\Theta=2\pi\Phi/\Phi_0$. Note that we have obtained Eq. (15) from a scattering matrix which contains no direct interference. The flux dependence of the current-current correlation function, Eq. (15), again, is a consequence of exchange. Carriers from quantum channels 1 and 2 are scattered into the same outgoing channel in either contact 3 or 4. It is not possible to distinguish the path which a carrier took to arrive in this outgoing channel. Thus it is plausible that the system responds as if carriers pass the flux on either side. A possible experimental verification of this effect requires a careful investigation of the conductances (transmission probabilities) and the cross correlation as a function of the coupling parameter T. Equation (15) is proportional to T^2 since it is related to four scattering amplitudes. To order T^2 the transmission probabilities do exhibit direct interference: For instance, T_{41} contains precisely the term $T^2\cos^2(\phi_1-\phi_2+\Theta)$. An experiment should compare the conductances with a cross-correlation function normalized by $[\langle(\Delta I_3)^2\rangle\times\langle(\Delta I_4)^2\rangle]^{1/2}$. The normalized cross correlation exhibits flux-sensitive terms proportional to T. The large separation of the edge states required here can be achieved with the help of one or two gates near the boundary of the conductor [17]. Variation of the gate voltage would permit the position of the edge states to shift with respect to the interior contacts. Conductances which are suitable for the experiment would thus emerge.

In conclusion we emphasize that in addition to direct interference, quantum mechanics leads to interference via exchange. Experimental demonstration of an interference pattern created via exchange would be very desirable.

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