Coherent Bremsstrahlung at pp or $\bar{p}p$ Colliders

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We consider radiation caused by the collective electromagnetic field of one beam deflecting particles of the other. We find the number of photons for a single collision is $dN_\gamma \approx N_0 dE_\gamma/E_\gamma$ in the energy range $E_\gamma < E_c = 4\hbar c\gamma^2/l$, where l is the length of the bunch. At the Superconducting Super Collider, for example, $N_0 = 50$ and $E_c = 6$ keV. Specific features of this radiation can be used for a fast control over collisions and for measuring beam parameters. A background due to synchrotron radiation in the magnetic field of a collider is estimated.

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1. Introduction.— If the photon energy is large enough, one deals with the usual bremsstrahlung process, which is described in textbooks in detail. In this case the number of produced photons dN_γ is proportional to the bremsstrahlung cross section $d\sigma_{pp\to pp\gamma}$ and the number of particles N_i in the colliding beams: $dN_\gamma \propto N_1 N_2 d\sigma_{pp\to pp\gamma}$. As a rule, the bremsstrahlung of protons is not considered due to its small cross section.

However, if the photon energy E_γ becomes small enough the radiation due to the interaction of one proton with the different particles of an opposing bunch becomes coherent, and the number of the produced photons becomes large. This occurs when a coherence length $4\hbar c\gamma^2/E_\gamma$ becomes of the order of the length of the opposing bunch, l, or larger (here $\gamma = E_p/mc^2$ is the Lorentz factor of a proton). In this region the number of photons dN_γ is proportional to $N_1 N_2^2$ (here the subscripts 1 and 2 refer, respectively, to the bunch whose radiation is being calculated and to the target bunch). We call such a radiation *coherent bremsstrahlung* (CBS).

The general formulas for CBS were obtained in Ref. [1], and the main results for e^+e^- colliders were given in Ref. [2]. In the present paper we perform calculations for pp or $\bar{p}p$ colliders. Here the photon energy $E_\gamma \ll E_p$ and the energy losses due to CBS are negligible (they can be described by classical formulas). The more interesting subjects are the number of CBS photons and their energy and angular distributions.

2. Applicability conditions and main results.— The parameters used for calculations were taken from Ref. [3]. Some of them are given in rows 1–3 of Table I. We assume that the proton bunches at the interaction point are round and have a Gaussian distribution of the particles with mean-square radii $\sigma_{1x} = \sigma_{1y} = \sigma_1$ and $\sigma_{2x} = \sigma_{2y} = \sigma_2$; the length of bunch 2 is $l = \sigma_{2z}$. The angular spread of the particles in the beams θ_b is rather small, $\theta_b \ll 1/\gamma$. Therefore, one can neglect the change of the transverse bunch sizes during collision. For the sake of simplicity, we consider below only collisions with a crossing angle equal to 0.

Our calculations are valid only if the proton deflection angle θ_d in the field of the opposing bunch is small compared with $1/\gamma$ (cf. [1,2]),

$$\theta_d \gamma \sim \frac{r_p N_2}{\sigma_2} \ll 1, \quad r_p = \frac{e^2}{m_p c^2} = 1.5\times 10^{-16}\,\mathrm{cm}. \qquad (1)$$

This condition is fulfilled for all the colliders given in Table I.

The problem under discussion is related to the beamstrahlung problem treated by many authors for linear e^+e^- colliders (see, e.g., Refs. [4]). For these colliders the condition $\theta_d \gg 1/\gamma$ holds and their beamstrahlung is close to the usual synchrotron radiation in a uniform magnetic field. However, proton colliders have a quite different range of parameter values [see Eq. (1)] and the formulas for the photon distributions from Refs. [4] are not valid for them (see also [5]).

We find [see Eqs. (6), (7), and (9)] for pp or $\bar{p}p$ colliders that there are $dN_\gamma \approx N_0 dE_\gamma/E_\gamma$ photons of CBS for a single collision of the beams, or $d\dot{N}_\gamma \approx \dot{N}_0 dE_\gamma/E_\gamma$ photons per second in the energy range $E_\gamma < E_c = 4\hbar c\gamma^2/l$. The quantities N_0, $\dot{N}_0$, and E_c for different colliders also are given in Table I.

3. Energy and angular distribution.— The number of produced photons dN_γ is expressed via the Born amplitude M_{fi} of the $pp\to pp\gamma$ process and the form factor $F_2(\mathbf{q})$ of the second bunch [$F_i(\mathbf{q})$ is the Fourier transform of the ith bunch density; as usual, $F_i(0) = N_i$]. If the first bunch were treated as an unbounded plane wave, then we would have the well-known expression that dN_γ is proportional to $|M_{fi}(\mathbf{q})F_2(\mathbf{q})|^2$. This expression corre-

TABLE I. Calculation parameters for various colliders.

Parameter	SSC	LHC	$S\bar{p}pS$	Tevatron
E_p (TeV)	20	8	0.45	1
$\sigma_\perp$ (μm)	4.8	10	70	43
l (cm)	6	7.5	20	50
E_c (eV)	6000	770	0.9	1.8
N_0	50	2×10^4	770	170
$\dot{N}_0$ ($10^9\,\mathrm{s}^{-1}$)	2.6	10^3	0.2	0.047

sponds to the scattering of the protons of the first bunch on one "composite particle"—the second bunch—with the internal structure described by the form factor of this bunch.

In our problem, however, this formula does not work and it is very important to take into account that the first bunch has a finite size. To do that we treat the initial state as finite wave packets. As for an elementary interaction, it is described by the Born approximation.

Since the effect under discussion is dominated by small momentum transfer ($q_z \sim 1/l$, $q_\perp \sim 1/\sigma_\perp$) we considerably simplify the expressions for the matrix elements in the basic formula. In Ref. [1] it was shown that with a good enough accuracy CBS can be described in an approximation such that the collective electromagnetic field of the target bunch may be interpreted as a flux of equivalent photons distributed with some density on a frequency spectrum. As a result, the number of the CBS photons for a *single* collision of the bunches is equal to

$$dN_\gamma = \frac{\alpha}{\pi}\frac{d\omega}{\omega}J(\omega)d\sigma_{\gamma p},\quad \alpha = \frac{e^2}{\hbar c} = \frac{1}{137}, \tag{2}$$

where $d\sigma_{\gamma p}$ is the cross section of Compton scattering of the equivalent photon with frequency ω on the proton, and the function $J(\omega)$ is defined by the form factors of the bunches,

$$J(\omega) = 4\pi\int \frac{\mathbf{q}_\perp\cdot\mathbf{q}'_\perp}{\mathbf{q}_\perp^2\mathbf{q}'^2_\perp}F_2(\mathbf{q})F_2^*(\mathbf{q}')F_1(\mathbf{q}'-\mathbf{q})\frac{d^2q_\perp d^2q'_\perp}{(2\pi)^4}, \tag{3}$$

$q_z = q'_z = \omega/c$. Here the longitudinal momenta of the equivalent photons $\hbar q_z = \hbar q'_z$ equal $\hbar\omega/c$. The equivalent photon energy $\hbar\omega$ is related to the energy E_γ and the emission angle θ of the final photon by the well-known relation (it was taken into account that $E_\gamma \ll E_p$)

$$\hbar\omega = (E_\gamma/4\gamma^2)[1+(\gamma\theta)^2]. \tag{4}$$

The Compton cross section is

$$d\sigma_{\gamma p} = 2r_p^2\frac{dE_\gamma}{E_\gamma}\frac{d\varphi}{(1+\gamma^2\theta^2)^3}[1+\gamma^4\theta^4 - 2\gamma^2\theta^2(\xi_3\cos 2\varphi + \xi_1\sin 2\varphi)]. \tag{5}$$

Here φ is the azimuthal angle of the emitted photon and the Stokes parameters ξ_i of the equivalent photons will be given below.

Using Eqs. (2)-(5) we obtain the energy and angular distribution of CBS photons,

$$dN_\gamma = \frac{3}{2}N_0\frac{dE_\gamma}{E_\gamma}\frac{dz}{(1+z)^4}\frac{d\varphi}{2\pi}[1+z^2-2z(\xi_3\cos 2\varphi+\xi_1\sin 2\varphi)]\exp[-(E_\gamma/E_c)^2(1+z)^2],$$

$$z=(\gamma\theta)^2,\quad E_c = 4\hbar c\gamma^2/l,\quad N_0 = \frac{8}{3}\alpha r_p^2 J(0) = \frac{4}{3\pi}\alpha N_1\left(\frac{r_pN_2}{\sigma_1}\right)^2\ln\frac{(\sigma_1^2+\sigma_2^2)^2}{2\sigma_1^2\sigma_2^2+\sigma_2^4}I(R). \tag{6}$$

Here R is the distance between the bunch axes. For a central collision (at $R=0$) we have

$$I(0)=1;\quad \xi_1=\xi_3=0. \tag{6a}$$

In this case, azimuthal asymmetry of CBS photons is absent.

The main contribution to this number of photons is given by the region $\theta \lesssim 1/\gamma$. It should be noted that at $E_\gamma > E_c$ additional angular narrowing appears due to the factor $\exp[-(E_\gamma/E_c)^2(1+\gamma^2\theta^2)^2]$.

4. Spectrum of CBS photons.—Integrating the distribution (6) over the angles θ and φ, we obtain the energy distribution of CBS photons,

$$dN_\gamma = N_0\Phi(E_\gamma/E_c)dE_\gamma/E_\gamma,$$

$$\Phi(x) = \tfrac{3}{2}\int_0^\infty \frac{1+z^2}{(1+z)^4}\exp[-x^2(1+z)^2]dz. \tag{7}$$

Some values of $\Phi(x)$ are given in Table II. At $x>3$ the following asymptotics is valid (with the accuracy better than 15%):

$$\Phi(x) = \frac{3}{4x^2}\left(1-\frac{5}{2x^2}+\frac{37}{4x^4}+\cdots\right)\exp(-x^2). \tag{8}$$

If a collider has n_b bunches and the bunch collision rate is f, the number of CBS photons emitted per second is

$$d\dot{N}_\gamma = \dot{N}_0\Phi(E_\gamma/E_c)dE_\gamma/E_\gamma;\quad \dot{N}_0 = N_0 n_b f. \tag{9}$$

The quantities $\dot{N}_0$ for different colliders are given in Table I.

The energy distribution has a very sharp dependence on the bunch length l; therefore one can determine this length by measuring the photon spectrum. [It can be

TABLE II. Some values of $\Phi(x)$.

x	0	0.2	0.4	0.6	0.8	1	1.5	2	2.5
$\Phi(x)$	1	0.65	0.44	0.29	0.18	0.10	0.019	0.0023	1.7×10^{-4}

seen from Eq. (3) that the photon spectrum is determined by the squared form factor of the longitudinal density of bunch 2. It depends strongly on the form of this density. In particular, had we used a density of the form $(1/2l)\times\exp(-|z|/l)$, instead of the Gaussian one, then Eq. (7) would have been valid but with another form $\Phi(E_\gamma/E_c)$. At $E_\gamma \gg E_c$, $\Phi(x)=8/35x^4$.]

5. *Collisions with nonzero impact parameter of the bunches.*—If the bunch-1 axis is shifted a distance R from the bunch-2 axis, the luminosity $L(R)$ (as well as a number of events for usual reactions) decreases very quickly,

$$L(R)=L(0)\exp[-R^2/2(\sigma_1^2+\sigma_2^2)]\,. \tag{10}$$

On the contrary, the number of CBS photons decreases slowly and for small values of R this number even increases slightly. For equal beams (at $\sigma_1=\sigma_2=\sigma_\perp$) we obtain from Eq. (3) the function $I(R)$ given in Eq. (6):

$$\begin{aligned} I(R)&=I_1(R/\sigma_\perp)+I_2(R/\sigma_\perp)\,,\\ I_i(x)&=\frac{1}{4\ln(\frac{4}{3})}\left(\int_0^1 dy\,y+\int_1^{4/3} dy\,(4-3y)\right)\\ &\quad\times\frac{f_i(x,y)}{2-y}\exp(-x^2y/4)\,,\\ f_1(x,y)&=1,\quad f_2(x,y)=1+x^2(1-y/2)\,. \end{aligned} \tag{11}$$

The corresponding curves are given in Fig. 1. The maximum of the function $I(R)$ equals 1.06 at $R=1.5\sigma_\perp$; after that, this function decreases, but even at $R=8\sigma_\perp$ the number of photons decreases by a factor of only 9, whereas the luminosity decreases by 7 orders of magnitude. At $\sigma_\perp \ll R \ll l$ fields of bunch 2 are $|\mathbf{E}|\approx|\mathbf{B}| \propto 1/R$ and, therefore, $dN_\gamma(R)\propto 1/R^2$. More exactly, one can obtain from Eq. (11),

$$I(R)=\frac{2}{\ln(\frac{4}{3})}\left(\frac{\sigma_\perp}{R}\right)^2 \quad \text{for } \sigma_\perp^2 \ll R^2 \ll l^2\,. \tag{12}$$

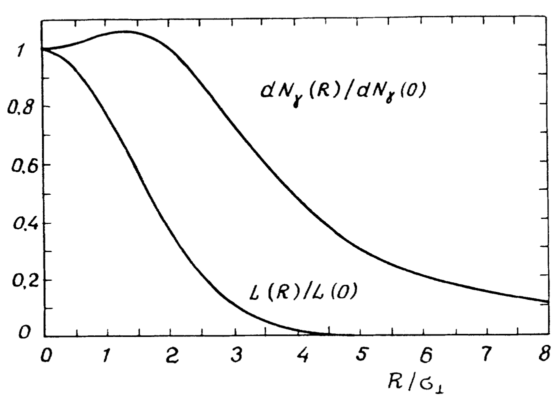

FIG. 1. Ratio of the number of CBS photons $dN_\gamma(R_y)/dE_\gamma$ for a distance between beam axes of R to that, $dN_\gamma(0)/dE_\gamma$, at $R=0$ vs $R/\sigma_\perp$, where $\sigma_\perp$ is the transverse size of the bunch. The lower curve is the same for the luminosity L.

Such a slow decrease in the number of CBS photons can be used for a fast control over the impact parameter between beams.

6. *Azimuthal asymmetry.*—If the impact parameter between beams is nonzero, an azimuthal asymmetry of CBS photons appears which can also be used for operative control over beams. For definiteness, let the bunch-1 axis be shifted in the vertical direction a distance R_y from the bunch-2 axis. When R_y increases, bunch 1 is shifted into the region where the electric field of bunch 2 is directed almost in a vertical line. As a result, the equivalent photons obtain a linear polarization. The mean values of the Stokes parameters of the equivalent photons in this case are expressed via the functions $I_i(x)$ from Eq. (11),

$$\xi_3(R_y)=\frac{I_1(x)-I_2(x)}{I_1(x)+I_2(x)}\,,\quad x=\frac{R_y}{\sigma_\perp}\,,\quad \xi_1=\xi_2=0\,. \tag{13}$$

The behavior of this function is given in Fig. 2; moreover,

$$-\xi_3(R_y)=\begin{cases}\dfrac{(R_y/\sigma_\perp)^2}{12\ln(\frac{4}{3})} & \text{for } R_y^2 \ll \sigma_\perp^2,\\ 1-2(\sigma_\perp/R_y)^2 & \text{for } R_y^2 \gg \sigma_\perp^2.\end{cases} \tag{14}$$

Negative values of $\xi_3(R_y)$ are in accordance with the vertical direction of the mean electric field of bunch 2 in the place where bunch 1 is located.

Let us define the azimuthal asymmetry of the emitted photons by the relation

$$A=\frac{dN_\gamma(\varphi=0)-dN_\gamma(\varphi=\pi/2)}{dN_\gamma(\varphi=0)+dN_\gamma(\varphi=\pi/2)}\,, \tag{15}$$

where the azimuthal angle φ refers to the horizontal direction. It follows from distribution (6) that this quantity does not depend on photon energy and equals

$$A=-\xi_3(R_y)\frac{2(\gamma\theta)^2}{1+(\gamma\theta)^4}\,. \tag{16}$$

This asymmetry can be essential for angles $\theta\sim 1/\gamma$,

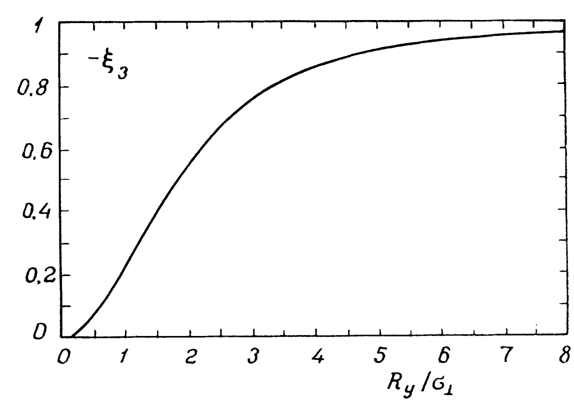

FIG. 2. Degree of equivalent photon polarization vs $R_y/\sigma_\perp$, where R_y is a vertical shift of the bunch-1 axis.

where $A \sim -\xi_3(R_y)$.

7. Estimate of the synchrotron-radiation background.— Let us perform a simple estimate of the background from the synchrotron radiation (SR) of protons in the external magnetic field of a collider. The bunch 1 traveling through the uniform magnetic field B emits during time Δt a number of photons,

$$dN_\gamma^{\rm SR} = N_1 \frac{dI}{E_\gamma}\Delta t = \frac{\sqrt{3}}{2\pi}\alpha N_1 F(E_\gamma/\hbar\omega_c)\frac{dE_\gamma}{E_\gamma}\frac{eB}{mc}\Delta t\,, \quad (17)$$

where dI is the classical intensity of radiation, $F(x)$ is a well-known function (see [6], Sec. 74), and $\omega_c = 3eB\gamma^2/2mc$ is a critical frequency. Because the angular spread of CBS photons is $\lesssim 1/\gamma$, we should consider those SR photons which are emitted in the same angular interval. Therefore, for Δt we can take the time of deflection over the angle $1/\gamma$ in a given magnetic field, i.e., $\Delta t \sim mc/eB$. That results in

$$dN_\gamma^{\rm SR} \sim \frac{\sqrt{3}}{2\pi}\alpha N_1 F(E_\gamma/\hbar\omega_c)\frac{dE_\gamma}{E_\gamma}\,. \quad (18)$$

As a rule, $E_c^{\rm CBS} \gg \hbar\omega_c^{\rm SR}$, and it leads to a considerable suppression of SR at $E_\gamma \sim E_c$. We obtain the following estimate for the signal to background ratio (S/B) in the region $E_\gamma = E_c \gg \hbar\omega_c$:

$$\frac{dN_\gamma^{\rm CBS}}{dN_\gamma^{\rm SR}} \sim 0.05\left(\frac{r_p N_2}{\sigma_\perp}\right)^2 \frac{e^x}{\sqrt{x}}, \quad x = \frac{E_c^{\rm CBS}}{\hbar\omega_c^{\rm SR}}\,. \quad (19)$$

The results for different colliders are given in Table III; for B we use the maximal values of the magnetic field from Ref. [4]. Even in this case the SR background is small for the SSC and $S\bar{p}pS$. An essential decrease of the SR background will obtain if one uses a smaller magnetic field in the first (after the interaction point) deflection magnets.

TABLE III. Estimated signal to background ratio for several colliders.

	SSC	LHC	$S\bar{p}pS$	Tevatron
B (T)	6.6	10	2	4.4
$E_c^{\rm CBS}/\hbar\omega_c^{\rm SR}$	21	11	21	3.8
$(S/B)_{E_\gamma=E_c}$	100	0.2	100	5×10^{-6}

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[1] I. F. Ginzburg, G. L. Kotkin, S. I. Polityko, and V. G. Serbo (to be published).

[2] I. F. Ginzburg, G. L. Kotkin, S. I. Polityko, and V. G. Serbo (to be published).

[3] Particle Data Group, J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990).

[4] V. E. Balakin and N. F. Solyak, Novosibirsk Report No. INP 82-123, 1982 (unpublished); M. Jacob and T. T. Wu, Phys. Lett. B **197**, 253 (1987); R. Blancenbecler and S. D. Drell, Phys. Rev. D **36**, 277 (1987); J. S. Bell and M. Bell, Part. Accel. **22**, 301 (1988); P. Chen and K. Yokoya, Phys. Rev. Lett. **61**, 1101 (1988); V. N. Baier, V. M. Katkov, and V. M. Strakhovenko, Nucl. Phys. **B328**, 387 (1989).

[5] P. Chen, *An Introduction to Beamstrahlung and Disruption,* Lectures Notes in Physics (Springer-Verlag, Berlin, 1988).

[6] L. D. Landau and E. M. Lifshitz, *Field Theory* (Nauka, Moscow, 1973).