Parity Violation in the Compound Nucleus: The Role of Distant States

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We explain why the measured parity-violating asymmetries for $p_{1/2}$ compound-nuclear resonances tend to have a common sign. We show that the asymmetry is a sum of two terms: an average term dominated by admixtures of distant levels and a term that fluctuates from resonance to resonance dominated by admixtures of nearby compound-nuclear states. The average asymmetry involves single-particle transition amplitudes of the parity-violating interaction, while the fluctuating asymmetry retains a statistical character. Our theoretical estimate of the average asymmetry agrees with a value extracted from experimental data.

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A recent communication from the TRIPLE Collaboration [1] on the measurement of parity-violating asymmetries in the scattering of low-energy (1–400 eV) polarized neutrons from ^{232}Th pointed out that measured asymmetries were overwhelmingly of the same sign. All seven of the asymmetries in ^{232}Th are positive, as are thirteen of fifteen of all known asymmetries. The probabilities of these outcomes occurring by chance, if the asymmetries had random signs, are 1.6% and 0.6%, respectively. Thus, it would appear that there is clear evidence for a nonstatistical behavior for this parity-violating process involving compound-nuclear levels, which otherwise exhibit statistical behavior. In this Letter we shall explain why asymmetries tend to have a common sign. We show that configurations several MeV distant in energy are important [2].

The scattering cross sections of low-energy neutrons from nuclei exhibit strong closely spaced compound-nuclear resonances. Parity violation is observed at $p_{1/2}$ resonances due to admixtures of opposite parity $s_{1/2}$ states produced by the parity-violating part of the weak interaction. (p and s designate the orbital angular momentum carried by the neutron and $\frac{1}{2}$ is the total angular momentum.) Parity-violating effects are observable at low-energy $p_{1/2}$ resonances because the neutron decay amplitudes of the admixed $s_{1/2}$ states are larger than the decay amplitudes of $p_{1/2}$ resonances. The longitudinal asymmetry for the mth resonance, P_m, is defined as the fractional difference of the resonance cross sections for neutrons polarized parallel and antiparallel to their momenta, $\sigma_R(+)$ and $\sigma_R(-)$:

$$P_m = \frac{\sigma_R(+) - \sigma_R(-)}{\sigma_R(+) + \sigma_R(-)} . \tag{1}$$

We develop an expression for the parity-violating asymmetry of $p_{1/2}$ resonances as the sum of two terms: a fluctuating asymmetry and an average asymmetry. The fluctuating asymmetry has zero mean and is dominated by admixtures of those $s_{1/2}$ states whose energies are closest to the $p_{1/2}$ resonance under study. The fluctuating asymmetry has been discussed recently by Johnson, Bowman, and Yoo [3]. The average asymmetry will be shown to arise from correlations between matrix elements of the parity-violating interaction, V_{PV}, and decay amplitudes. These correlations involve single-particle configurations that are several MeV away.

The parity-violating asymmetry has been expressed as a perturbation expansion [4]:

$$P_m = -2 \sum_n \frac{\langle n | V_{PV} | m \rangle}{E_n - E_m} \frac{\gamma_n}{\gamma_m} . \tag{2}$$

The state $|m\rangle$ and the states $|n\rangle$ are a $p_{1/2}$ resonance and the $s_{1/2}$ states admixed into it, E_m and E_n are their respective energies, and γ_m and γ_n are their neutron decay amplitudes [5]. The statistical point of view treats the expansion coefficients of compound-nuclear states as independent Gaussian random variables each having zero mean. Parity mixing in the compound nucleus was thus viewed as a totally statistical process; the decay amplitudes and mixing matrix elements were taken to be random variables. Since compound-nuclear resonances are linear combinations of many components, the correlations between γ_n, γ_m, and $\langle n|V_{PV}|m\rangle$ are small and were neglected [6,7].

The existence of a tendency for asymmetries to have a common sign *per se* leads to the conclusion that states with large energy denominators play an important role. Even if the small correlations between γ_n, γ_m, and $\langle n|V_{PV}|m\rangle$ are taken into account, the energy denominator in Eq. (2) weighs contributions from $s_{1/2}$ resonances above and below E_m with opposite sign, leading to no sign preference for asymmetries. Therefore, terms with the smallest energy denominators cannot produce an average asymmetry and contributions from distant states must be important.

To show how the average asymmetry arises, we express the decay amplitudes for $s_{1/2}$ and $p_{1/2}$ resonances as the decay amplitudes for single-particle configurations times the amplitudes for finding these configurations in the compound-nuclear resonances [8]:

$$\gamma_n = \gamma^s \langle n | G + s \rangle, \quad \gamma_m = \gamma^p \langle m | G + p \rangle,$$
$$\frac{\gamma^s}{\gamma^p} = \frac{-i\sqrt{3}}{kR} . \tag{3}$$

Here k is the neutron wave number ($k \sim \sqrt{E}$) and R is the nuclear radius. The single-particle configurations $|G+s\rangle$ or $|G+p\rangle$ are formed by the addition of one s or p neutron to the target ground state, and γ^s and γ^p are the decay amplitudes for these single-particle configurations ($|\gamma^s/\gamma^p| = 1.2\times10^{+3}$ for a 1-eV neutron on ^{232}Th) [9]. The observed neutron decay amplitudes of compound-nuclear resonances are always much smaller than the decay amplitudes of a single-particle configuration because the wave functions for compound-nuclear resonances are linear combinations of a large number N of multiparticle-multihole configurations. The single-particle configurations $|G+p\rangle$ or $|G+s\rangle$ appear as a small component of a large number of stationary states spread over a range of energy Γ, the spreading width. N can be estimated from the relation [9]

$$N = \pi\Gamma/2D\,, \tag{4}$$

where D is the level spacing of compound-nuclear resonances. Taking $D = 17$ eV for $p_{1/2}$ neutron resonances in ^{232}Th and $\Gamma = 1.5$ MeV we obtain $N = 1.4\times10^5$. The correlations between γ_n, γ_m, and $\langle n|V_{\rm PV}|m\rangle$ are of order $1/\sqrt{N}$ and were therefore neglected.

In order to exhibit the correlations implicit in Eq. (2) we first insert and sum over complete sets of states $\{|j\rangle\}$ and $\{|l\rangle\}$, of which $|G+s\rangle$ and $|G+p\rangle$ are members:

$$P_m = \frac{-2\gamma^s}{|\langle m|G+p\rangle|^2\gamma^p}\sum_{jln}\frac{\langle G+s|n\rangle\langle n|j\rangle\langle j|V_{\rm PV}|l\rangle\langle l|m\rangle\langle m|G+p\rangle}{E_n - E_m}\,. \tag{5}$$

The terms with $|j\rangle = |G+s\rangle$ and $|l\rangle = |G+p\rangle$ will all have the same sign, while all other terms fluctuate in sign when considered as functions of j, l, m, or n. First, retaining only the terms having the same sign in Eq. (5) yields an expression for the average asymmetry $\bar{P}$:

$$\bar{P} = -2\frac{\gamma^s}{\gamma^p}\sum_n \frac{|\langle G+s|n\rangle|^2\langle G+s|V_{\rm PV}|G+p\rangle}{E_n - E_m}\,. \tag{6}$$

The expression $|\langle G+s|n\rangle|^2$ is the probability of finding the state $|n\rangle$ in the configuration $|G+s\rangle$.

In order to understand the behavior of the parity-violating asymmetry it is useful to consider some aspects of the relationship between the independent-particle and compound pictures of the nucleus [9]. The sum of probabilities $|\langle G+s|n\rangle^2$ (or $|\langle G+p|n\rangle|^2$) per energy interval is called the s-wave (or p-wave) strength function. In the independent-particle model of the nucleus s and p single-particle configurations alternate and are separated in energy by one major shell spacing, $\sim$6.1 MeV in ^{232}Th. The part of the strength function arising from the single-particle configuration $|G+s\rangle$ (or $|G+p\rangle$) may be thought of as a Lorentzian distribution of width Γ, the spreading width, centered at E_s (or E_p), the excitation energy of the configuration. The sum of the probabilities $|\langle G+s|n\rangle|^2$ over the states is unity if E_s is well above the Fermi surface. Strong p-wave resonances occur whenever E_p is near neutron threshold energy, E_t (A = 20–40, 80–140, and 220–240). The occurrences of strong s-wave resonances (A = 40–80 and 140–200) interleave the occurrences of strong p-wave resonances.

The resonances for which nonzero parity-violating asymmetries have been measured occur in nuclei showing strong p-wave resonances; $E_m \sim E_t \sim E_p$. For such nuclei $\bar{P}$ can be approximated as

$$\bar{P} = -2\frac{\gamma^s}{\gamma^p}\sum_s \frac{\langle G+s|V_{\rm PV}|G+p\rangle}{E_s - E_t}\,. \tag{7}$$

Here s enumerates the configurations $|G+s\rangle$ that have large matrix elements and $|E_s - E_t| \sim$ (one shell spacing).

Equation (7) can be used to estimate the size of the average asymmetry since $\bar{P}$ no longer depends on properties of individual compound-nuclear states. We note that Eq. (7) involves matrix elements of $V_{\rm PV}$ between single-particle configurations, reminiscent of a direct reaction.

The most complete experimental data for parity-violating asymmetries are available for the nucleus ^{232}Th. We estimate P for ^{232}Th by evaluating the energy denominators and matrix elements in Eq. (7). The neutron threshold energy E_t = 4.8 MeV in ^{233}Th. There are two single-particle configurations that admix with the $4p_{1/2}$ configuration: the $4s_{1/2}$ and the $5s_{1/2}$ configurations. From deformed shell-model calculations [10] we estimate the excitation energy of the $4s_{1/2}$ configuration in ^{233}Th to be 3.1 MeV. This gives $E_s - E_t = -1.7$ MeV. This energy difference is smaller than one major shell spacing due to the effects of deformation. For the $5s_{1/2}$ configuration we take $E_s - E_t$ to be one major shell spacing, 6.1 MeV.

In order to estimate the matrix elements we make use of the single-particle character of the matrix element in Eq. (7) and replace $V_{\rm PV}$ with an effective one-body operator [11]. The lowest-order T-invariant parity-odd rank-zero operator for a single nucleon is $\boldsymbol{\sigma}\cdot\mathbf{p}$, where $\boldsymbol{\sigma}$ are the Pauli operators and $\mathbf{p}$ is the nucleon momentum. We take $V_{\rm PV} = \varepsilon\boldsymbol{\sigma}\cdot\mathbf{p}c$, where c is the speed of light and ε is a dimensionless constant. For simplicity, we take harmonic-oscillator wave functions for the single-particle configurations. The matrix elements of $\boldsymbol{\sigma}\cdot\mathbf{p}c$ are [9]

$$\begin{aligned}\langle G+5s_{1/2}|\boldsymbol{\sigma}\cdot\mathbf{p}c|G+4p_{1/2}\rangle &= -i152\ \text{MeV}\,,\\ \langle G+4s_{1/2}|\boldsymbol{\sigma}\cdot\mathbf{p}c|G+4p_{1/2}\rangle &= +i161\ \text{MeV}\,.\end{aligned} \tag{8}$$

Both of the matrix elements are of the same order as the Fermi momentum (P_F) times c, 250 MeV. The two configurations contribute with the same sign yielding $\bar{P} = \varepsilon 2.9\times10^{+5}$.

We estimate the strength of the weak interaction as given by the constant ε in two ways. The nuclear single-particle potential, which accounts for many gross nuclear properties, is also the one-body reduction of a sum of two-body operators. The value of the strong one-body potential at nuclear density is about 50 MeV. We estimate the ratio of the weak matrix elements to strong matrix elements as $G_F P_F^2/G_s \sim 10^{-6}$, where G_F is the weak coupling constant and $G_s \sim 1$ is the strong coupling constant. This yields $\langle s|V_{\rm PV}|p\rangle \sim 50$ eV, $\varepsilon \sim 4\times 10^{-7}$, and $\bar{P} \sim 10\%$ at 1 eV neutron energy. A more formal estimate may be obtained by starting with the meson-exchange two-body weak potential of Desplanques, Donoghue, and Holstein (DDH) [12]. This potential involves weak meson-nucleon coupling constants, which are estimated using current algebra techniques, relations between strangeness-changing hyperon decays and strangeness-conserving processes, and weak SU(6) symmetry. Adelberger and Haxton (AH) [13] give the one-body reduction of the meson-exchange potential for a Fermi gas. We estimate the range of the constant ε to be $-0.9 < 10^{+7}\varepsilon < 1.2$ from the ranges of meson couplings given by DDH applied in the AH expression. This yields $-15 < i\langle G + 4s_{1/2}|\varepsilon\boldsymbol{\sigma}\cdot\mathbf{p}c|G + 4p_{1/2}\rangle < 21$ eV and $-2.5\% < \bar{P} < +3.5\%$. Short-range repulsion in the nucleon-nucleon interaction would decrease the above estimate, but other effects particular to the medium may increase the estimate.

Continuing the development that led to Eq. (6) results in an expression suitable for the analysis of experimental data. To obtain such an expression, recall that the correlations between γ_n, γ_m, and $\langle n|V_{\rm PV}|m\rangle$ originate from only one term in the expansion of the matrix elements $\langle n|V_{\rm PV}|m\rangle$ in Eq. (5). Removing these terms changes the value of each matrix element by a negligible amount. The fractional change is $\sim 1/\sqrt{N}$. The asymmetry can therefore be written as the sum of a fluctuating term having zero mean and an average term:

$$P_m = -2\sum_n \frac{\langle n|V_{\rm PV}|m\rangle}{E_n - E_m}\frac{\gamma_n}{\gamma_m} + B\left(\frac{1\ \mathrm{eV}}{E}\right)^{1/2}. \qquad (9)$$

The quantities γ_m, γ_n, and $\langle n|V_{\rm PV}|m\rangle$ can be treated as uncorrelated random variables. Experimental data for a given nucleus can be described by two quantities: an average asymmetry parameter B and a mean-squared matrix element $M^2 = \overline{|\langle m|V_{\rm PV}|n\rangle|^2}$ as defined by Bowman *et al.* [6].

In order to test the above ideas quantitatively we determine a value of B from the ^{232}Th experimental data of Frankle *et al.* [1] and compare it with the theoretical estimates of $\bar{P}$. The analysis of data on parity-violating asymmetries for p-wave resonances by the likelihood method must be modified to include B. In addition, q, the probability that a p-wave resonance included in the analysis has angular momentum $\frac{1}{2}$, must be evaluated. Although Bowman *et al.* [6] showed M to be insensitive to the value assumed for q and took $q = \frac{1}{3}$, B does depend on q. The density of $p_{1/2}$ resonances is the same as the known density of $s_{1/2}$ resonances, $\frac{1}{17}$ eV^{-1} in ^{233}Th. The $p_{1/2}$ resonances are twice as strong as the $p_{3/2}$ resonances [14]. While 67 $p_{1/2}$ and $p_{3/2}$ resonances are expected in the neutron energy interval 0 to 400 eV, only 23 were strong enough for Frankle *et al.* to analyze. We assume the $p_{1/2}$ and $p_{3/2}$ neutron widths to have Thomas-Porter distributions and obtain $q = 0.44$. A likelihood analysis then yields $B = 8.0^{+6.2}_{-6.0}\%$ and $M = 1.2^{+0.5}_{-0.4}$ meV. (Frankle *et al.* obtained $M = 1.4^{+0.5}_{-0.4}$ meV with $B = 0$.) The experimental value of B corresponds to

$$i\langle G + 4s_{1/2}|\varepsilon\boldsymbol{\sigma}\cdot\mathbf{p}c|G + 4p_{1/2}\rangle = 48^{+37}_{-36}\ \mathrm{eV}$$

and is in agreement with the above estimates of $\bar{P}$: a few percent at 1 eV.

Our model predicts that the sign of $\bar{P}$ will be the same for all nuclei that show strong p-wave resonances near neutron threshold, $E_p \sim E_t$. In the one-body approximation $V_{\rm PV} \sim \boldsymbol{\sigma}\cdot\mathbf{p}c$. For harmonic-oscillator wave functions, the matrix element of $\boldsymbol{\sigma}\cdot\mathbf{p}c$ for the s configuration above the p configuration in energy has a phase $-i$ and that below has a phase $+i$ [9]. According to Eq. (7) the sign of $\bar{P}$ is the same for all such nuclei.

In conclusion, we have shown why parity-violating asymmetries predominantly have a common sign. The direct mixing of distant $s_{1/2}$ single-particle configurations into $p_{1/2}$ compound-nuclear resonances produces an average parity-violating asymmetry of approximately the same size as the fluctuating asymmetry produced by the mixing of nearby compound-nuclear states. All nuclei showing strong $p_{1/2}$ resonances near neutron threshold are predicted to have the same sign for $\bar{P}$. The size of the underlying single-particle parity-violating mixing matrix elements needed to explain the size of the average asymmetry in ^{232}Th is tens of eV. Matrix elements of this magnitude seem to emerge in a natural way from dimensional arguments as well as meson-exchange models of the weak nucleon-nucleon force. The observed parity-violating asymmetries in compound nuclei, where the level spacing is tens of eV, are seen not to be dominated by the mixing of nearest levels. This conclusion suggests a reexamination of the approximation that the nearest levels play a dominant role in the parity-violating phenomena in light nuclei.

We have given an expression suitable for the analysis of experimental data. There are two quantities that can be determined for each nucleus: the average asymmetry parameter B and the root-mean-squared matrix element of the weak parity-violating interaction, M. More precise and more extensive data are needed to determine better these parameters and their variation with atomic mass. As we better understand parity-violating effects in complex nuclei, we may improve our quantitative understanding of the weak nucleon-nucleon force in the nuclear medium. The experimental determination and interpretation of a handful of data have already increased our understanding of parity-violating mechanisms in nuclei.

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[2] The problem of the common sign of parity-violating asymmetries is considered in two recent works. N. Auerbach [Phys. Rev. C (to be published)] emphasizes the role of the 0^- spin-flip-dipole giant resonance as a doorway state for the $\boldsymbol{\sigma}\cdot\mathbf{p}$ operator. V. V. Flambaum [Phys. Rev. C **45**, 437 (1992)] emphasizes the role of the nuclear surface using commutator relations. S. Nogura and B. Deplanques [Nucl. Phys. **A457**, 189 (1986)] and D. F. Zaretskii and K. K. Sirotkin {Yad. Fiz. **45**, 1302 (1987) [Sov. J. Nucl. Phys. **45**, 808 (1987)]} compare single-particle and compound-nuclear mechanisms. V. E. Bunakov *et al.*, {Yad. Fiz. **49**, 988 (1989) [Sov. J. Nucl. Phys. **49**, 613 (1989)]} consider various mechanisms of parity violation in neutron-induced reactions. V. E. Bunakov, E. D. Davis, and H. A. Weidenmüller [Phys. Rev. C **42**, 1718 (1990)] emphasize statistical considerations. A. Griffiths and P. Vogel [Phys. Rev. C **43**, 284 (1991)] have argued against the validity of the approximation of nearest-neighbor dominance.

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