Nuclear-Matter Properties Based on a Relativistic Model of the Nucleon-Nucleon Interaction

A. Amorim
Centro de Física Nuclear da Universidad de Lisboa, Av. Gama Pinto 2, 1699 Lisboa, Portugal

J. A. Tjon
Institute for Theoretical Physics, University of Utrecht, 3508 TA Utrecht, The Netherlands
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Saturation properties of nuclear matter are determined within a Dirac-Brueckner approach. Relativistic covariance is implemented by constructing the complete representation of the nucleon-nucleon amplitude in the Dirac space. As a dynamical model the relativistic one-boson-exchange model is used, including also the negative-energy-state components. A consistent treatment is discussed of nucleon propagation in the nuclear medium. The compressibility obtained for symmetric matter is rather low as compared to other calculations and more in agreement with the value suggested by astrophysical studies.

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Considerable attention has been paid in the past to the saturation properties of nuclear matter, which are essential ingredients in star evolution models [1] and heavy-ion scattering studies [2]. The failure to be able to describe these properties using nonrelativistic two-body nucleon-nucleon forces has motivated investigations of both many-body methods in microscopic calculations and the nature of the nuclear interaction. In particular, encouraging results have been presented within a relativistic framework using a meson-theory-based nucleon-nucleon (NN) interaction. Relativistic nuclear-matter calculations require the knowledge of the two-nucleon amplitude in the nuclear-matter frame. As this does not coincide with the two-nucleon center-of-momentum frame one, either a Bethe-Salpeter-type equation is approximated by averaging out the total momentum dependency [3] or the boosting of the two-nucleon amplitude is done by expressing it in a covariant form. In general, to determine the covariant form of the NN amplitude in a nonambiguous way, information is needed for both positive- and negative-energy Dirac components. To avoid such a complete but complex analysis, a five-covariant ansatz [4,5] is usually made for the Dirac structure of the two-nucleon interaction, so that the representation can be reconstructed from the scattering matrix in the positive-energy sector only.

In this Letter we present results based on a relativistic one-boson-exchange (OBE) model, which does not have the above ambiguity, leading to a complete representation of the two-nucleon amplitude in the Dirac space. The resulting IA2 representation (IA refers to the impulse approximation) has been shown to give a reasonable description of the elastic proton-nucleus scattering observables at medium energies [6]. To study nucleon propagation in the nuclear medium we make use of the Bethe-Brueckner-Goldstone equations [7], which can readily be generalized to the relativistic field-theoretical case using covariant Green's functions. Let us consider the ground state of nuclear matter, which can be parametrized by the space-time homogeneous value of the baryonic current $B^\mu=\rho_0 u^\mu$, where ρ_0 is the density in the nuclear-matter frame and u^μ the unit vector given by $u^\mu=(1,\mathbf{0})$ in this frame. The filled-Fermi-sphere free propagator that corresponds to the state of definite density which minimizes the energy is given by

$$S^0(p)=S_F^0(p)+2\pi i(\not p+m)\delta^{(p\cdot u>0)}(p^2-m^2)\theta(\mathbf{p}_f^2+p_\perp^2)\,,$$

where S_F^0 is the bare vacuum Feynman propagator and the transverse momentum $p_\perp^\mu$ is given by $p_\perp^\mu=p^\mu-(p\cdot u)u^\mu$. This reduces to $p_\perp^\mu=(0,\mathbf{p})$ in the nuclear-matter frame. The second term on the right-hand side amounts to a twist of the boundary conditions for the Dirac positive-energy states inside the Fermi sphere. As a result of invariance under the Lorentz group and time and space reflections, the nucleon self-energy in the medium has the form $\Sigma=\Sigma^s-\Sigma^u\gamma\cdot u-\Sigma^v\gamma\cdot p$. Rewriting it as

$$\Sigma=\Sigma^s-\Sigma^0\gamma\cdot u-\Sigma^v\gamma\cdot p_\perp\,, \tag{1}$$

with $\Sigma^0=\Sigma^u+\Sigma^v p\cdot u$, we get for the full nucleon propagator in nuclear matter

$$S(p)=\frac{\bar p_u\not u+(1+\Sigma^v)\not p_\perp+\bar m}{\bar p_u^2+(1+\Sigma^v)^2p_\perp^2-\bar m^2+i\epsilon}+2\pi i[\bar p_u\not u+(1+\Sigma^v)\not p_\perp+\bar m]\delta^+(\bar p_u^2+(1+\Sigma^v)^2p_\perp^2-\bar m^2)\theta(\mathbf{p}_f^2+p_\perp^2)\,, \tag{2}$$

where $\bar p^\mu=p^\mu+\Sigma^0u^\mu$, $\bar m=m+\Sigma^s$, and $\bar p_u=\bar p\cdot u$. In determining the contribution of the valence nucleons, using Eq. (2)

the factor $1+\Sigma^v$ can be factored out according to

$$[\bar{p}_u \slashed{u} + (1+\Sigma^v)\slashed{p}_\perp + \bar{m}]\delta^{(+)}(\bar{p}_u^2 + (1+\Sigma^v)^2 p_\perp^2 - \bar{m}^2) = \frac{E^* \slashed{u} + \slashed{p}_\perp + m^*}{2E^*}\delta(\bar{p}_u - (1+\Sigma^v)E^*)\,, \tag{3}$$

where the effective mass $m^* = (m+\Sigma^s)/(1+\Sigma^v)$ and the energy $E^* = (m^{*2} - p_\perp^2)^{1/2}$ are introduced. From Eq. (3) we see that the mass-shell condition is given by $p\cdot u = (1+\Sigma^v)E^* - \Sigma^0$. The factorization is useful in the calculation of both the nucleon self-energy from the antisymmetrized NN effective t matrix Γ and the binding energy. Neglecting the vacuum fluctuation contribution we have for the self-energy contribution

$$\Sigma(p_1) = \int \frac{d\mathbf{p}_2}{(2\pi)^3 2E_2^*}\,\mathrm{Tr}_2[-\Gamma(p_1,p_2;p_1,p_2)(\slashed{u}E_2^* + \slashed{p}_{\perp 2} + m^*)]\theta(\mathbf{p}_f^2 + p_{\perp 2}^2)\,. \tag{4}$$

The NN effective t matrix in nuclear matter is determined from the quasipotential approximation of the Pauli-blocked Bethe-Salpeter equation. For the NN interaction the fully relativistic OBE model from Ref. [8] has been taken, characterized by the exchange of π, ϵ, ω, ρ, δ, and η mesons. The intermediate-state two-nucleon Green's function can readily be constructed from Eq. (2). For simplicity we assume that we may approximate the nucleon propagator by the quasiparticle pole contribution. Moreover, the invariants Σ^a are also assumed not to be strongly dependent on the momentum and in the actual calculations their values are taken at the Fermi surface. It should be noted that these approximations to Eq. (2) are consistent with the sum rule as derived from the canonical anticommutation relations for the Heisenberg fields [9,10]. Using the Blankenbecler-Sugar prescription we get in the NN c.m. frame

$$S^{(1)}\otimes S^{(2)} \to \frac{i\pi\delta(p'^0)}{1+\Sigma^v}\,\frac{[E_f\gamma^0 - \mathbf{p}'\cdot\gamma + m^*]^{(1)}[E_f\gamma^0 + \mathbf{p}'\cdot\gamma + m^*]^{(2)}}{(E_{p'}^* + E_f)^2(E_{p'}^* - E_f - i\epsilon)}\bar{Q}\,, \tag{5}$$

where $E_f = [(\bar{p}_1+\bar{p}_2)^2]^{1/2}/(1+\Sigma^v)$ is the total invariant energy of the final state. Furthermore, p' is the relative four momentum, $E_{p'}^* = (\mathbf{p}'^2 + m^{*2})^{1/2}$, and $\bar{Q}$ is the angle averaged Pauli-blocking operator, while we have neglected the terms involving $\Sigma^v\mathbf{u}$. The quasipotential equation is solved using the helicity basis with positive- and negative-energy spinors corresponding to mass m^*. From these helicity amplitudes the covariant form of the NN t matrix in terms of the 44 invariants [6] can be constructed, which is then used to calculate the self-energy contribution employing Eq. (4). In this way a self-consistent solution is constructed at a given Fermi momentum.

The energy-momentum tensor resulting from the free Lagrangian is bilinear in the fields. Making use of the equations of motion and the dressed mass-shell condition for the valence part of fermion propagators, the expectation value of the energy reduces to performing traces involving only the baryon propagators. For the energy density in the nuclear-matter frame we get

$$\mathcal{E} = \lambda \int d\mathbf{p}\,\theta(p_f - p)\mathrm{Tr}\left\{(m + \gamma\cdot\mathbf{p})\frac{\slashed{u}E^* + \slashed{p}_\perp + m^*}{2E^*}\right\}, \tag{6}$$

where λ is the isospin degeneracy, which is 2 for symmetric nuclear matter and 1 for neutron matter. From this the binding energy and compressibility can be determined.

The $1/(1+\Sigma^v)$ factor appearing in the intermediate-state propagator, in the case of the full calculation where Σ^v is positive, suppresses the contributions which are higher order in the NN interaction [10]. Two versions of the OBE model have been studied, both giving a reasonable description of the free nucleon-nucleon phase shifts up to 300-MeV laboratory energy. In Table I are given the various coupling constants used in the two models. The results of our self-consistent calculations in the case of model B for the binding energy, effective mass, and self-energy contribution as a function of the Fermi momentum are displayed in Fig. 1. The system clearly exhibits saturation at a Fermi momentum of 1.29 fm^{-1}. Moreover, we see a very similar behavior as compared to previous relativistic calculations, except that the self-energy invariant Σ^v is substantially larger in our case. This requires a proper treatment of this term, which is usually neglected. It has recently been suggested that be-

TABLE I. Meson coupling constants and masses (in MeV) of models A and B. A cutoff form factor $\Lambda^2/(\Lambda^2-q^2)$ is used at all vertices.

	π	η	δ	ϵ	ω	Vector ρ^v	Tensor ρ^t/ρ^v	Cutoff Λ (MeV)
$g^2/4\pi$ (model A)	14.2	3.09	0.75	7.6	11.0	0.43	6.8	1150
$g^2/4\pi$ (model B)	14.16	2.0	1.6	8.27	11.7	0.43	5.1	1140
μ (MeV)	139	548	960	570	783	763	763	

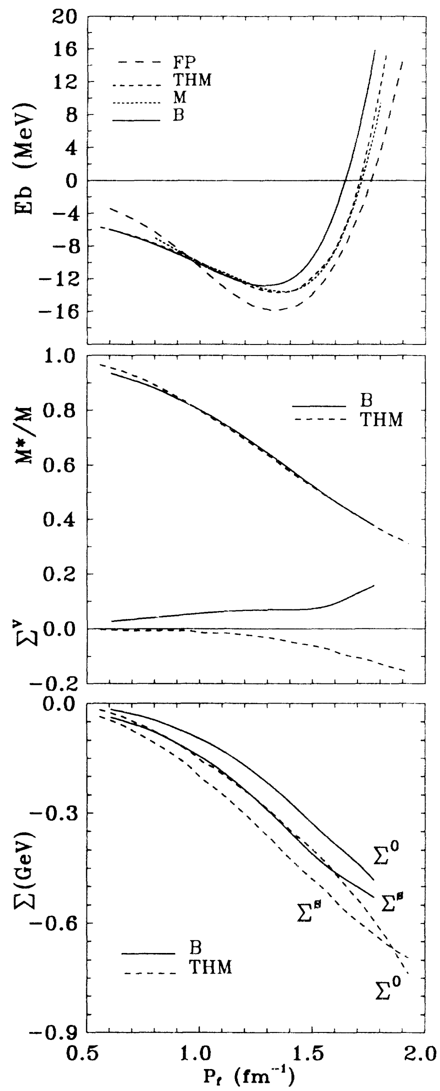

FIG. 1. The nucleon binding energy (in MeV), relative effective mass, and self-energy parts (in GeV) for symmetric nuclear matter as a function of the Fermi momentum for model B (solid curve). For comparison the results of ter Haar and Malfliet [5] (THM), Machleidt [3] (M), and Friedman and Pandharipande [11] (FP) are also plotted.

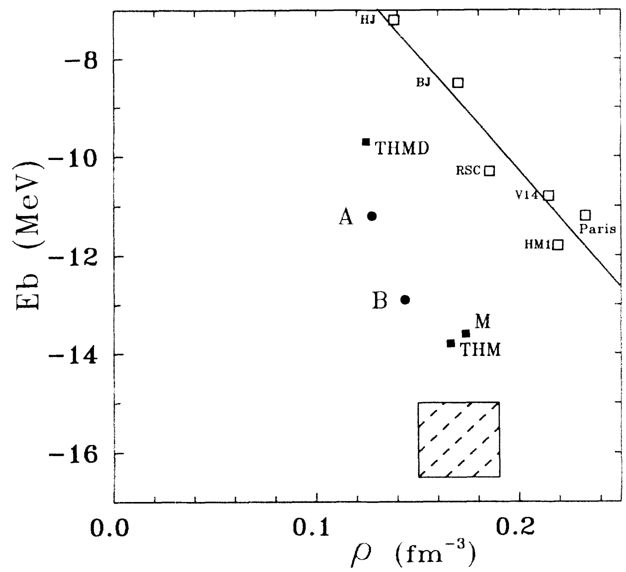

FIG. 2. The binding energies (in MeV) at the saturation point for the complete calculation using interactions A and B, together with the Coester line obtained in nonrelativistic two-nucleon-interaction-only models. The results of ter Haar and Malfliet [5] with and without considering the propagation of the Δ resonance, labeled respectively THMD and THM, and the results of Machleidt [3] (M) are also displayed.

sides the nucleon also the meson masses are modified in the presence of a nuclear medium [12]. We have studied its effect by introducing a density dependence in the meson masses through $\mu/\mu_0 = m^*/m$ with μ_0 the meson mass in free space. In so doing, it is found that the Dirac-Brueckner equations do not support a self-consistent solution any more. Clearly an additional ingredient is needed to be able to explain saturation of nuclear matter if such a density-dependent effect in the meson propagators is included [13].

In Fig. 2 are shown the calculated binding energies as a function of density at the saturation point. From this we see that the results may lie on a "relativistic Coester line," which is compatible with the experimental saturation point. The calculated compression modulus is, in general, low. For model B it is given by 170 MeV. A linear extrapolation of the compression modulus to the experimental saturation point gives a value on the order of 200 MeV. This value is substantially lower than found by ter Haar and Malfliet, leading to a softer equation of state, which is favored by astrophysical calculations of supernova explosions [14], but smaller than suggested by the giant monopole resonance data [15] (see, however, Ref. [16]).

In Fig. 3 are shown the results obtained for neutron matter. The predictions for the binding energy and equation of state are close to those obtained by Friedman and Pandharipande [11] in a nonrelativistic calculation, using an effective three-body force. Using the pseudovector representation from Ref. [19] does not lead to a self-consistent solution of the Dirac-Brueckner equations. The sensitivity found may be due to the fact that in the neutron-matter case there is no isospin averaging and as a result it is more sensitive to how well the pion is treated in the approximate representation. In Fig. 3 the results of the full calculation are also compared with the ones of Malfliet [17] based on a five-term representation. From this we may conclude that it is essential to have additional information on the NN interaction in the negative-energy sector in order to be able to obtain reliable predictions for the neutron-matter case. The considered relativistic OBE model may be a possibility for this. The rela-

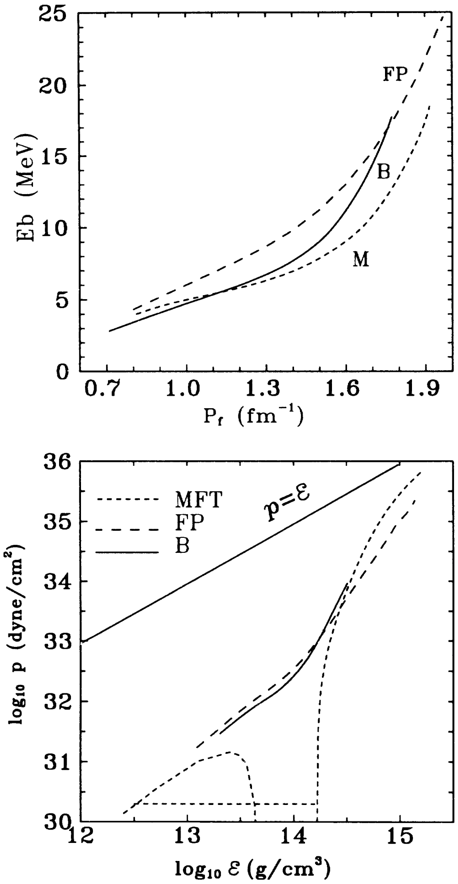

FIG. 3. The binding energy (upper panel, in MeV) in neutron matter using model B. For comparison the results of Malfliet [17] (M) and Friedman and Pandharipande [11] (FP) are shown. In the lower panel, the neutron-matter equation of state obtained in the full calculation is compared with the mean-field prediction [18] (MFT) and with the results obtained by Friedman and Pandharipande [11] (FP). The causal limit $p=\mathcal{E}$, when the speed of sound in the medium approaches the speed of light, is also displayed.

tively stiff equation of state obtained suggests a strong dependence of the compressibility on the asymmetry of nuclear matter. This would allow for a simultaneous explanation of neutron-star mass data and supernovae calculations [14].

In conclusion, we have studied the saturation properties of nuclear matter by solving self-consistently the Dirac-Brueckner equations in the full Dirac space. The ambiguity in the relativistic structure of the *NN* amplitude as derived from the *NN* interaction between physical states only is resolved using a relativistic OBE model. Since the tensor part Σ^v of the self-energy is not negligible, a consistent treatment of the nucleon propagator in the medium is needed. In so doing, reasonable binding energies and equations of state are found.

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