Densities of Meson and Baryon States

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In tachyon-free string theories the ratio of the densities of bosonic and fermionic states must approach unity with increasing energy. This observation of Kutasov and Seiberg is considered in the context of a potential stringy description of the hadron spectrum. The densities of the meson and baryon states are approximately equal up to a mass of about 1.7 GeV/c^2, above which the density of baryon states appears greater. In a stringy picture of hadrons, this result implies the existence of a large number of as yet unseen meson states above 1.7 GeV/c^2. These are likely to be exotic, consisting of at least two quarks and two antiquarks, and to be accompanied at still higher masses by exotic baryons as well.

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The observed density of hadron states grows fast with energy [1]. This is seen already in the GeV region and has led to the early suggestion [2] of an exponentially growing hadron density. It has long been argued [3] that a stringlike picture, for which such an exponential rise is natural, underlies this phenomenon. From the point of view of quantum chromodynamics (QCD) such a picture is reasonable [4], especially in the limit of $N \rightarrow \infty$ colors [5].

Bosonic and fermionic hadrons, i.e., mesons and baryons, were lumped together in previous hadron density studies. In view of some interesting recent developments, it pays to consider the mesonic and baryonic components of the hadron spectrum separately and then to concentrate on their *difference* rather than on their sum. We have in mind Kutasov and Seiberg's [6] observation that forbidding the appearance of destabilizing tachyons in a string theory severely constrains the *difference* of the densities of bosons and fermions in that theory. It turns out that, though the density of bosons and that of fermions each rises exponentially with energy, their difference rises only like a low power of the energy, characteristic of a two-dimensional field theory, i.e., even less than in a four-dimensional field theory. It is well known [7] that (target) space-time supersymmetry eliminates tachyons. But this result of Kutasov and Sieberg shows that the converse is *not* true: Tachyon elimination does not require full-fledged supersymmetry. The just-described cancellation between the boson and fermion densities is all that is needed.

At present, it is far from clear precisely what type of string theory can be invoked to describe hadrons, and to what extent such a string theory can function as a viable approximation. Nevertheless, if a stringy approach (e.g., at large number of colors) to hadrons makes sense, then one should expect a cancellation of the exponential rise in the *difference* of the baryon and meson densities.

In the present Letter we show that the experimentally observed hadron spectrum does indeed seem to have an approximately equal number of bosons (mesons) and fermions (baryons), at least up to a certain mass. Above this mass, the number of baryons observed at present is greater. We speculate that this is a result of our poor techniques for observing high-mass meson states, and suggest possible ways to discover the missing particles.

One of our results will turn out to be the need for mesons which are "exotic," i.e., composed of at least two quarks and two antiquarks. We shall show that a self-consistent treatment demands the existence of *both* mesons and baryons beyond the known $\bar{q}q$ and qqq configurations.

Let us denote the densities of meson and baryon states at a certain energy E by $\rho_M(E)$ and $\rho_B(E)$. A string picture of hadrons leads one to expect $\rho_M(E)$ and $\rho_B(E)$ to each increase exponentially with energy with Hagedorn temperature of order 200 MeV or so. Yet, according to Ref. [6], the ratio $\rho_M(E)/\rho_B(E)$ should approach 1 exponentially fast as $E \rightarrow \infty$.

We now examine the relative densities of observed states. We restrict our attention to those states composed only of the three lightest quarks, u, d, and s. We expect the number of mesons and baryons to be equal flavor by flavor, so one could also perform the exercise for nonstrange mesons and baryons. Here, however, one would encounter significant statistical limitations. There are not enough states observed containing c or b quarks to permit a useful comparison with more than three flavors.

We count each hadronic state [1] with a total statistical weight

$$W_{\text{tot}} = (2I+1)(2J+1)W_C , \tag{1}$$

where I is the isospin, J is the total angular momentum, and $W_C = 1$ for a self-charge-conjugate multiplet (nonstrange meson) and 2 otherwise (i.e., a kaon or baryon). The number of states with a mass smaller than a certain mass M_0 is plotted against M_0 in Fig. 1. Let us point out some of the main features.

(1) Both the number of meson states and the number of baryon states do increase very fast as expected. Exponential growth is meaningful even at these low energies.

(2) The number of mesons starts out larger than the number of baryons for well-understood reasons. In the chiral-symmetry limit pions and kaons would be massless.

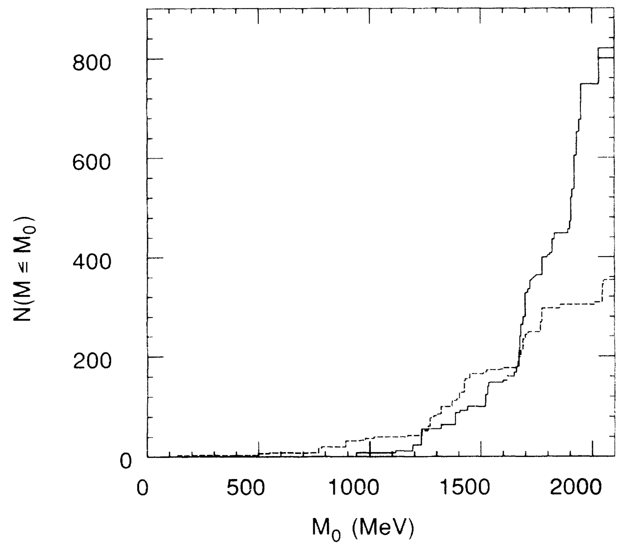

FIG. 1. Number of states with mass $M \leq M_0$ for mesons (dashed histogram) and baryons (solid histogram).

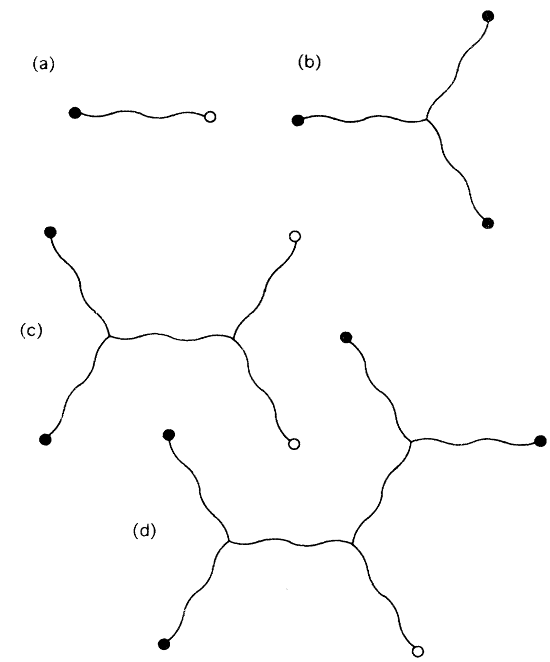

FIG. 2. String diagrams for (a) nonexotic mesons, (b) nonexotic baryons, (c) lowest exotic mesons, and (d) lowest exotic baryons. Solid circles denote quarks; open circles denote antiquarks.

The lowest vector mesons are expected to be lighter than the lowest baryons (e.g., in the constituent-quark picture).

(3) In the mass range from about 1.2 to 1.7 GeV/c^2, the number of baryon states nearly keeps pace with that of meson states. In this region, there exist phase-shift analyses both for baryonic and for many-mesonic systems.

(4) Above 1.7 GeV/c^2, the number of observed baryons begins to outstrip that of the mesons. Two significant jumps occur in the number of baryons states, corresponding to clusters of baryonic resonances which are well identified in phase-shift analyses. No such analyses exist for mesonic states. In part, we view the meson deficit as a consequence of this experimental shortcoming. There may be many conventional ($\bar{q}q$) mesons which remain to be discovered above 1.7 GeV/c^2. However, we feel this is not the whole story.

If one takes a string picture of hadrons seriously, one is led not only to $\bar{q}q$ mesons and qqq baryons [Figs. 2(a) and 2(b)], but to a whole host of "exotic" configurations such as those illustrated in Figs. 2(c) and 2(d) [8,9]. The string picture was motivated [10] to some extent by the dual resonance model, which was known [11] to require exotic configurations for self-consistency [11,12]. One can estimate the minimum energy for creation of each "string" in the examples of Fig. 2 by comparison with the known mesons and baryons.

In a quark model, the vast majority of the hadron states in the energy region of Fig. 1 are orbital excitations, rather than radial excitations. In other words, they lie on "leading" Regge trajectories. These are known to have equal slopes for baryons and mesons. At this rate baryons are bound to "win out" simply because a qqq system has more states than a $q\bar{q}$ system. As to the lower trajectories, for $q\bar{q}$ mesons they have the same Regge slope as the leading trajectory, but for baryons the slope can be lower. For instance, if the three quarks are attached at the ends of three equally long string segments whose other ends meet in one central node [as in Fig. 2(b)] then the Regge slope is [9] $\frac{2}{3}$ that of the meson slope. Similarly, for $qq\bar{q}\bar{q}$ mesons in a configuration [Fig. 2(c)] in which the string segment between the two nodes is very short, whereas the four quark or antiquark supporting segments are equal in size, the slope is $\frac{2}{4}$ that of the $q\bar{q}$ mesons. Under similar conditions the $qqqq\bar{q}$ baryons of Fig. 2(d) yield a Regge slope $\frac{2}{5}$ that of $q\bar{q}$ mesons, and so on. Whereas the different slopes of the $q\bar{q}$ mesons and qqq baryons may somewhat alleviate the imbalance due to the excess of qqq over $q\bar{q}$ states, it is hard to see how it can yield the close match expected if the Kutasov-Sieberg argument applies. Rather, we envision the fermion-boson balance as follows. As the mass increases, the excess of qqq states first tilts the balance in favor of fermions, but then the $q^2\bar{q}^2$ mesons begin to contribute with even more states. This now favors the bosons, but then the threshold for the $q^4\bar{q}$ baryons is reached, and so on.

One then estimates that the lowest configurations of exotic mesons [Fig. 2(c)] are quite likely to provide at least part of the missing contribution to the meson states implied by the region in Fig. 1 just above 1.7 GeV/c^2. The lowest configurations of exotic baryons [Fig. 2(d)] are likely to become important at masses of about 2–2.5

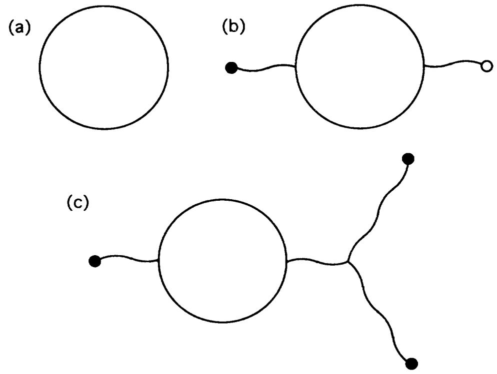

FIG. 3. Examples of diagrams involving string loops: (a) glueball; (b) "hybrid" meson; (c) "hybrid" baryon. Similar diagrams may be constructed for exotic states.

GeV/c^2.

To make up for the lack of experimental meson-spectroscopic data in the mass region just above 1.7 GeV/c^2, we can refer to quark-model calculations of the $q\bar{q}$ meson [13] and qqq baryon [14] spectra. For M_0 $=0.8$, 1.2, 1.6, 2, and 2.2 GeV/c^2 such calculations yield $N_{\text{mesons}}(M \leq M_0)=20$, 40, 176, 469, and 789 and $N_{\text{baryons}}(M \leq M_0)=0$, 24, 172, 1220, and 2544. Just as in the experimentally based Fig. 1, we see that these calculated $q\bar{q}$ meson and qqq baryon spectra are also closely matched for M_0 up to 1.6 GeV/c^2, but above this mass the baryons are *expected* to be more numerous. Although such calculations involve assumptions at variance with a stringy picture, in the low-mass region considered here these assumptions (which lead to fewer radial excitations) do not create a situation very different from that in a stringy picture. At higher masses these differences would make themselves felt much more. We presented these comparisons with the calculated hadron spectra in order to dispel any impression that some large number of $q\bar{q}$ meson states may somehow be expected to lie just above 1.7 GeV/c^2, so as to match the qqq baryons.

(5) We have not included glueballs (gluonic bound states) in our discussion. Glueballs are the lowest in a tower of states involving one or more closed string insertions [9] in a $q^N\bar{q}^N$ or $q^{N+3}\bar{q}^N$ configuration ($N=0,1,2,\ldots$), as shown in Fig. 3. Such states could presumably be related to the "hybrid" mesons and baryons (such as $q\bar{q}g$ and $qqqg,\ldots$) of QCD. If this is the case, it would be interesting to determine the exact nature of the relationship between hybrid states, on the one hand, and the configurations depicted in Figs. 2 and 3, on the other. (In this context, see, e.g., Ref. [15].)

(6) That the fermion-boson balance can at all be provided by mesons and baryons is predicted on the number n of colors being odd. Were the number of colors n to be even, the singlet baryons would be bosons just like the mesons. From the point of view of the large-n expansion, it appears that the stability of the stringy picture which one obtains is sensitive to whether one approaches large n through even or odd values. Fortunately the experimental value is $n=3$, which is odd.

In short, then, what we suggest is that the absence of tachyons *à la* Kutasov and Seiberg leads to a close correspondence between the meson and baryon spectra and that this correspondence involves, along with the familiar $q\bar{q}$ mesons and qqq baryons, also their $q^{N+1}\bar{q}^{N+1}$ and $q^{N+3}\bar{q}^N$ ($N=1,2,3,\ldots$) counterparts.

We wish to thank Nathan Seiberg for raising the issue addressed in item (6) above.

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