Grand Unification and the Supersymmetric Threshold

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It is shown that grand unification of the minimal supersymmetric standard model does not lead to a prediction for the scale of the superpartner masses, even if the inputs α, $\sin^2\theta$, and α_s are known exactly.

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The minimal supersymmetric grand unified theories [1] predict larger values for the unification scale and $\sin^2\theta$ than their nonsupersymmetric counterparts [2]. This was originally taken to be progress on understanding the nonobservation of proton decay, but apparently gave a prediction for $\sin^2\theta$ somewhat above the favored experimental result of about 0.21. However, during the mid-1980s the experimental value of $\sin^2\theta$ increased, and now the precise data from the CERN e^+e^- collider LEP imply that the $\sin^2\theta$ prediction of simple supersymmetric grand unified theories is highly successful.

Recent comparisons of the theory with data [3] have led to the speculation that the gauge-coupling unification in supersymmetric theories is so precise that more information can be extracted from the analysis than usual. For example, the meeting of α_1 and α_2 apparently determines α_G and the unification mass M_G very accurately. Does the requirement that α_3 meet these couplings at the same point determine M_S, the scale of the superpartner masses? If so then more accurate determinations of α_s would lead to a prediction of M_S via the one-loop equation $\ln(M_S/M_Z)=4\pi(3-15s^2+7\alpha/\alpha_s)/19\alpha$, where $s^2=\sin^2\theta$, and α and α_s are the inputs at scale M_Z. In this Letter we compute the superheavy threshold corrections that are present in all grand unified theories, and show that uncertainties from the superheavy particle spectrum invalidate any attempt to pinpoint M_S in terms of α_s.

Superheavy particles with mass $\sim M_G$ necessarily lead to threshold corrections in the unification relations. Any supersymmetric grand unified theory based on a gauge group G which contains, or is, conventional SU(5) necessarily has the following three types of superheavy particles: V (twelve gauge boson supermultiplets, which lie in four color triplets), H [the heavy components of the SU(5) chiral multiplet in which the Higgs doublets lie], and Σ [the remnants of the superheavy Higgs multiplet which induced the breaking of the gauge group G to SU(3)×SU(2)×U(1) at M_G]. These superheavy particles produce threshold corrections which depend on the spectrum. We assume, for illustration, that the members of each of the three types are degenerate with masses M_V, M_H, and M_Σ, respectively. In most models there will be further superheavy multiplets and a more complicated spectrum of superheavy particles. This is likely to increase the size of the threshold corrections: We ignore the possibility that there is a finely tuned cancellation among many terms, and we simply compute the threshold corrections from the V, H, and Σ multiplets. We find a one-loop result:

$$\ln\frac{M_S}{M_Z}=I+\frac{6}{19}\ln\frac{M_V}{M_\Sigma}-\frac{18}{19}\ln\frac{M_V}{M_H}, \tag{1}$$

where I is the quantity given in terms of the measured inputs,

$$I=\frac{4\pi}{19\alpha}\left(3-15s^2+7\frac{\alpha}{\alpha_s}\right)=-2.1\pm2.6\pm1.0, \tag{2}$$

where we have used $\alpha^{-1}=127.8\pm0.2$. The first uncertainty comes from the strong coupling, $\alpha_s=0.115\pm0.007$ [4], and the second from $\sin^2\theta=0.2334\pm0.0008$. At one-loop order there are no nonlogarithmic correction terms in supersymmetric theories [5] and we have ignored two-loop beta function contributions. These lead to a change in the numerical prediction for M_S, but do not affect our main point. In Eq. (1) M_S is the effective scale of superpartner masses; it is actually a weighted sum over the superpartner spectrum. *We stress that the superheavy corrections of Eq. (1) are independent of G, the representation which breaks G, and the representation of G which contains the Higgs doublets.*

The crucial point is that two of the three mass parameters M_V, M_Σ, and M_H are unknown. The largest is equal to the unification mass M_G, which is determined to be close to 10^{16} GeV. It is likely that this is M_V which is proportional to the large gauge coupling $g_G=0.7$. On the other hand, M_Σ and M_H could be many orders of magnitude smaller since they are proportional to unknown Yukawa couplings. The only bound we have is on M_H from proton stability: $M_H>10^{10}$ GeV. One might also argue that it is unlikely that the superheavy logarithms are larger than 5–10, otherwise the present success of supersymmetric unification would be accidental. It is obvious that the unknown logarithms in Eq. (1) imply that no meaningful result for M_S can be obtained from this equation, even if $I(\alpha,\alpha_s,\sin^2\theta)$ were known exactly. In particular, the coefficients of these logarithms are ~ 1 and are of opposite sign. Furthermore, we can conceive of no feasible experiment which could determine M_Σ and M_H. It is conceivable that one day masses such as M_Σ and M_H could be computed from a more fundamental theory.

The main point of this Letter is that better measurements of α_s will not lead to a precise determination of the superpartner mass threshold given present theoretical knowledge. However, cetain other conclusions can also be pointed out. It is not possible to say that the data distinguish between grand unified models with similar predictions; for example, between flipped SU(5) and SU(5). It is hardly worth refining the prediction for superpartner masses by including nondegeneracies in the superpartner spectrum. Logarithms analogous to those in Eq. (1) also enter the prediction for the unification scale M_G, which therefore cannot be precisely determined just from α and $\sin^2\theta$. Finally, we point out that our analysis applies to all string models where there is a grand unification scale sensibly lower than the scale of compactification. It does not apply to the case that the gauge group at the compactification scale is SU(3) $\times$SU(2)$\times$U(1), since in this case there is no four-dimensional grand unified theory. However, even in this case superheavy threshold corrections are present [6].

The $\ln(M_V/M_H)$ and $\ln(M_V/M_\Sigma)$ corrections are also present in the nonsupersymmetric case. However, their coefficients are a factor of 6 smaller than in the supersymmetric case, so that these terms alone are insufficient to fix up the relation between α_s and $\sin^2\theta$. The minimal supersymmetric standard model is the best motivated model with successful unification.

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