Cosmological QCD Z(3) Phase Transition in the 10 TeV Temperature Range?

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At high temperatures, in the presence of quarks, QCD has one stable and two metastable Z(3) vacua. We propose a scenario in which the Universe, after inflationary expansion for example at the grand unified scale, lies in one of these metastable vacua. We estimate that the metastable vacuum decayed at $T \sim 10$ TeV. The inhomogeneities caused by this phase transition are over one-tenth of the horizon radius. As a result of a large pressure difference between stable and metastable vacua their density contrast is also large.

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Pure glue QCD has three degenerate Z(3) vacua at high temperatures where the theory is deconfining. However, in the presence of fundamental-representation fermions two of these vacua become metastable and only the one with real expectation value of the Wilson line remains stable. The purpose of this Letter is to investigate what cosmological consequences this interesting vacuum structure of QCD may have.

Quantitatively, the vacuum structure can be studied by calculating the effective potential in a constant temporal background gauge field [1,2]. Note that at finite temperatures, even a constant field $A_0 = A_0^3 T_3 + A_0^8 T_8$ cannot be trivially gauge transformed away because of the $1/T$ periodicity of the Euclidean space in the temporal direction. Thus the background gauge field can have dynamical significance. When considering transitions between the different Z(3) vacua, it suffices to concentrate on routes along the boundary curve of the allowed values of the Wilson line in the complex plane, i.e., to choose $A_0^3 = 0$. Employing the same notations as in [3,4], we parametrize the background gauge field in the following way:

$$A_0 = A_0^8 \frac{\lambda_8}{2} = \frac{2\pi T}{g} q \frac{\lambda_8}{\sqrt{3}} . \tag{1}$$

To the lowest order, that is, at the one-loop level, the effective potential [1,2] along the λ_8 direction is

$$V_{\rm eff}(q) = \tfrac{8}{3} \pi^2 T^4 V(q) , \tag{2}$$

where

$$V(q) = f(q) + N_f V_f(q) , \tag{3}$$

$$V_f(q) = \tfrac{3}{32} - f(\tfrac{1}{3} q + \tfrac{1}{2}) - \tfrac{1}{2} f(-\tfrac{2}{3} q + \tfrac{1}{2}) , \tag{4}$$

and

$$f(y) = [y(\mathrm{mod}\,1)]^2 \{1 - [y(\mathrm{mod}\,1)]\}^2 . \tag{5}$$

In Eq. (3) the first term is the gluonic and the second term the fermionic contribution to the potential $V(q)$. The constant $\frac{3}{32}$ is chosen so that the fermionic potential also vanishes in the stable vacuum, and N_f is the number of massless fundamental-representation fermions.

The full potential $V(q)$ for three different values of N_f, and the fermionic part of the potential, $V_f(q)$, are plotted in Fig. 1. When no dynamical fermions are present, we have the three equivalent vacua at $q = 0$, 1, and 2. When fermions are added to the system, the vacua at $q = 1$ and 2, which correspond to values $\exp(\pm 2\pi i/3)$ of the Wilson line, become metastable.

The $q = 0$ vacuum is the usual perturbative $A_0 = 0$ vacuum. In the $q = 1,2$ vacua, A_0^8 is a multiple of $2\pi T/g$. The physical differences of the vacua are most clearly seen by considering their pressures, equal to minus the free-energy density. From the effective potential the pressure difference between the stable vacuum ($p_{\rm stable}$) and either of the metastable vacua ($p_{\rm ms}$) is for all $T \gg T_c \approx 150$ MeV

$$p_{\rm stable} - p_{\rm ms} = \tfrac{14}{81} \pi^2 N_f T^4 . \tag{6}$$

This should be compared to the usual expression for the pressure in the Universe, which we identify as the pressure in the stable vacuum:

$$p_{\rm stable} = \tfrac{1}{90} \pi^2 g_* T^4 . \tag{7}$$

Between the grand unified and electroweak scales the

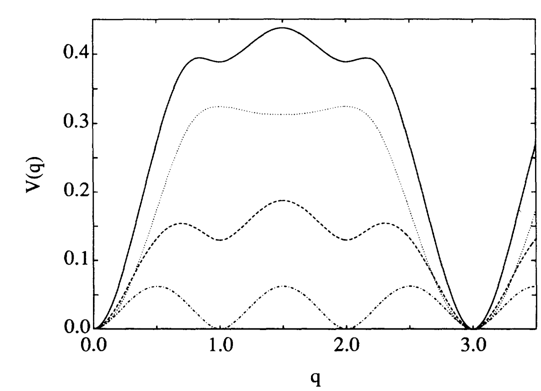

FIG. 1. The full dimensionless potential $V(q)$: dash-dotted curve, $N_f = 0$; dashed curve, $N_f = 2$; solid curve, $N_f = 6$. The fermionic part of the potential $5V_f(q)$: dotted curve. The third derivative of the gluonic ($N_f = 0$) potential is discontinuous when q is an integer, and $V_f'''(q)$ is discontinuous when $q(\mathrm{mod}3) = \frac{3}{4}, \frac{6}{4}$, or $\frac{9}{4}$.

effective number of relativistic degrees of freedom (g_*) in the minimal standard model is 106.75 and the number of massless SU(3) fermions is 6. In this range Eq. (6) means that the pressure in the metastable vacua is only 13% of the pressure in the stable vacuum. In fact, Eqs. (6) and (7) imply that in this range

$$p_{\rm ms} = \tfrac{1}{90}\pi^2 g_{\rm ms} T^4 , \quad (8)$$

where

$$g_{\rm ms} = 28 + \tfrac{7}{8} 6N_{\rm fam} + \tfrac{7}{8}(12 - \tfrac{160}{9})N_f . \quad (9)$$

In the metastable vacua adding quark flavors actually decreases the pressure (even when quarks are added together with leptonic members of family, i.e., $N_{\rm fam} = N_f/2$). However, the bosonic degrees of freedom in the minimal standard model save the pressure from becoming negative: $g_{\rm ms} = 13.42$ for $N_f = 6$.

Note the main differences between Eq. (6) and the corresponding quantity in usual first-order symmetry-breaking transitions: Equation (6) holds for all $T \gg T_c$ and only depends on T (and on quark masses), not on any symmetry-breaking scale. Thus the effective potential $V_{\rm eff}$ cannot cause any inflation. Inflation would need a constant vacuum energy density as an extra scale.

In the very early Universe domains of different Z(3) vacua have been created, for example when the grand unified theory broke to SU(3)×[SU(2)×U(1)]. However, if the metastable vacua were within one causal horizon, they would collapse in a time of the order of the Hubble time because of the huge pressure difference. But let us suppose that the scale factor increased exponentially due to inflation in the grand unified scale. Although the presence of domain walls associated with the different vacua may have had an influence on the inflation, it seems likely that the radius of a typical domain after the inflation would reach far beyond the horizon. In this case the whole causal region would stay in the metastable vacuum until it either tunnels to the stable vacuum or until the temperature drops close to that of the quark-hadron phase transition.

In the following, we will calculate numerically the decay rate of the metastable vacua, already estimated by Dixit and Ogilvie [4]. The general framework is discussed in [5,6].

We will employ an effective-action approach used in [3,4]. In addition to the quantum potential $V_{\rm eff}(q)$, the classical tension term for the gauge field contributes to the effective action as well. The validity of this approach—treating q as a constant when calculating the effective potential and otherwise as a field $q(x)$—has been discussed by Bhattacharya *et al.* [3]. At high temperatures we get the following effective three-dimensional action:

$$S[q] = \frac{16\sqrt{2}}{3}\pi^2 \frac{1}{g^3} \int d^3x' [\tfrac{1}{2}(\nabla' q)^2 + V(q)] , \quad (10)$$

where $\mathbf{x}'$ is a dimensionless space coordinate,

$$x_i' = (gT/\sqrt{2}) x_i . \quad (11)$$

The probability of tunneling per unit time per unit volume is [5,6]

$$\Gamma = \mathcal{M}\kappa 2^{-3/2} g^3 (S_b/2\pi)^{3/2} T^4 e^{-S_b} . \quad (12)$$

Essentially the tunneling rate is $T^4 \exp(-S_b)$, where the so-called bounce or bubble action $S_b \equiv S[q_b] - S[q_{0\rm ms}]$ is an O(3)-symmetric extremum of the action with the appropriate boundary conditions. With the function $q_b(r)$ we denote the bubble solution and with the constant $q_{0\rm ms}$ the value of the field in the metastable vacua. The preexponential factors arise from functional integration over the Gaussian fluctuations around this extremum. In Eq. (12) $\mathcal{M}$ is a dimensionless factor and κ is the dimensionless determinant,

$$\kappa = \left(\frac{\det'[-\nabla'^2 + V''(q_b(r'))]}{\det[-\nabla'^2 + V''(q_{0\rm ms})]} \right)^{-1/2} . \quad (13)$$

Here det′ denotes the determinant computed with the zero eigenvalues (from the three translations of the bubble center) omitted. In numerical estimates we shall assume that the factor $\mathcal{M}\kappa$ is of the order of unity.

Numerically computed values for the bubble action with different numbers of fermions are shown in Table I. Qualitatively, when N_f is small, the solution resembles a thin-wall bubble. However, for bigger values of N_f this is not the case. From Fig. 2 we can see that for $N_f = 6$ the bubble core is not even close to the true vacuum. Dixit and Ogilvie [4] have estimated values of the bubble action for SU(N) employing the thin-wall approximation. In the limit of small N_f their results coincide well with the exact numerical values presented in here. However, for the cosmologically most interesting case $N_f = 6$ their approximative method already overestimates the bubble action by 17%. Even this is surprisingly accurate, taking into account that the true bubble solution is far from a thin-wall bubble. For the tunneling rate relatively small differences are naturally important due to the exponentiation in Eq. (12).

Next, let us estimate when the metastable vacuum decays in cosmology. At time t the fraction of space still in

TABLE I. The bubble action, relevant for calculation of the decay rate of the metastable vacuum, for different numbers of fundamental-representation fermions.

N_f	$g^3 S_b$
2	1030
3	444
4	236
5	142
6	92.3

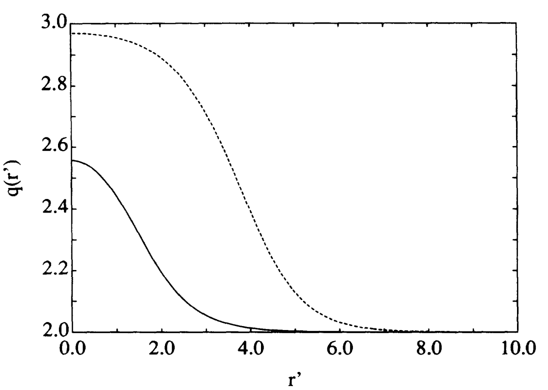

FIG. 2. Bubble profile $q_b(r')$ for SU(3). Dashed curve, $N_f=2$; solid curve, $N_f=6$. Of the metastable vacua we have chosen the one at $q=2$ and so the true vacuum is at $q=3$ (see Fig. 1). Using Eq. (11) we can deduce that the bubble radius is approximately $3/(gT/\sqrt{2})$ for $N_f=6$.

the metastable phase is [7]

$$f_{ms}(t)=\exp\left[-\int_0^t dt'\Gamma(t')V(t',t)\right], \qquad (14)$$

where $V(t',t)$ is the volume occupied at the time t by a bubble nucleated at the time t'. Here we assumed that the time scale for the essential part of the phase transition is shorter than the Hubble time so that expansion of the Universe could be neglected. The bubbles grow at roughly the speed of sound; on the other hand, the bubble action decreases rather slowly in time. So we could get a rough estimate for the phase-transition time t_{pt} solving the equation $\Gamma(t_{pt})t_{pt}^4=1$.

However, the phase-transition point can be solved more accurately [8]. Here we will follow the formulation of [9]. The phase-transition time t_{pt} is determined from the equation

$$8\pi v^3\Gamma(t_{pt})/[S_b'(t_{pt})]^4=1, \qquad (15)$$

where $S_b'\equiv dS_b/dt$ and v is the velocity of the bubble front for which we will use the sound velocity, $v=1/\sqrt{3}$. The average distance of the nucleation centers is

$$R_{nucl}=\frac{(8\pi)^{1/3}v}{-S_b'(t_{pt})}. \qquad (16)$$

In the present case we have, between the grand unified and electroweak scales when $N_f=6$,

$$S_b(t)=\frac{92.3}{g^3(T)},$$
$$-S_b'(t)t=\frac{92.3}{g(T)}\frac{3}{2}\beta_0,\quad \beta_0=\frac{11-2N_f/3}{16\pi^2}, \qquad (17)$$

where, in the metastable vacuum,

$$T^2t=\left(\frac{45}{16\pi^3}\right)^{1/2}\frac{M_{\text{Planck}}}{(g_{ms})^{1/2}}. \qquad (18)$$

Putting Eqs. (9), (12), (15), (17), and (18) together, we arrive at the following approximate equation, which determines the phase-transition temperature T_{pt}:

$$4\ln\left(\frac{M_{\text{Planck}}}{T_{pt}}\right)-\frac{92.3}{g^3(T_{pt})}+\frac{5}{2}\ln g(T_{pt})-12.7=0. \qquad (19)$$

To get an estimate for the phase-transition temperature T_{pt} we use the two-loop formula

$$g^{-2}(T)=2\beta_0\ln(T/\Lambda_T)+(\beta_1/\beta_0)\ln[\ln(T^2/\Lambda_T^2)],$$

$\beta_1=(102-38N_f/3)/(16\pi^2)^2$ and fix its Λ parameter by using results from particle physics experiments, and by using $\mu=3T$ as the momentum scale [10].

A careful analysis of the uncertainties in different experiments gives the following value for the strong coupling: $\alpha_s(M_Z)=0.113\pm0.004$ [11]. Using the central value $\alpha_s(M_Z)=0.113$ and $N_f=5$ at M_Z the above reasoning implies that $\Lambda_T=52$ MeV. Now we get from Eq. (19) $g(T_{pt})=0.91$ and $T_{pt}=19$ TeV.

In this estimate, the small fluctuation determinant in Eq. (12) was put to unity. If $\mathcal{M}\kappa=0.1$ or 10, we would, respectively, get 17 or 22 TeV for the phase-transition temperature. In other words, T_{pt} is not very sensitive to the numerical value of the determinant. However, the phase-transition temperature does depend rather strongly on the fixed value of the coupling: $\alpha_s(M_Z)=0.10$ and 0.14 would give $T_{pt}=9.5$ and 53 TeV, respectively. Using $\mu=T$ as momentum scale would give $T_{pt}=45$ TeV instead of 19 TeV.

We also give values for the phase-transition temperature for different numbers of quarks: $N_f=5$, $T_{pt}=450$ GeV; $N_f=4$, $T_{pt}=20$ GeV; $N_f=3$, $T_{pt}=1.7$ GeV; $N_f=2$, $T_{pt}=290$ MeV. Their ratios are roughly like those of quark masses, without any obvious reason. Here we again employed the relation $\mu=3T$ and used $\alpha_s(M_Z)=0.113$ and $N_f=5$ at M_Z. Within our scenario only the six-quark case is relevant.

Next, we will solve the average distance of the nucleation centers immediately after the phase transition, R_{nucl}, from Eqs. (16) and (17). We will compare it to the horizon radius, which at $T=19$ TeV is $R_{hor}=2t\approx1.1\times10^{-6}$ m. We find that

$$\frac{R_{nucl}}{R_{hor}}=0.0106\frac{g(T_{pt})v}{\beta_0}\approx0.13. \qquad (20)$$

The critical bubble radius, i.e., the bubble radius immediately after the nucleation is, from Fig. 2, only $2\times10^{-14}R_{hor}$.

The nucleation distance R_{nucl} obtained is quite large. This situation that a few bubbles have time to expand and fill the horizon before a significant number of new bubbles are nucleated is quite contrary to the situation in the electroweak or quark-hadron phase transition (supposing they are of first order), where a vast number of bubbles are nucleated within a short period of time. The explanation is that in the QCD Z(3) phase transition the tunneling rate increases slowly with time, only as the exponential of a power of $\log(t)$, whereas the more familiar cases the rate increases very rapidly after the critical temperature T_c.

Observable consequences from this phase transition at ~ 10 TeV may ultimately have arisen via the significant density inhomogeneities produced: They are both large in relative scale [Eq. (20)] and, because the pressures of the true and metastable vacuum differed by such a large amount, large in intensity. These inhomogeneities may have affected later phase transitions. For example, at the QCD phase-transition temperature $T_c \approx 150$ MeV the distance $R_{\rm nucl} \approx 1.4\times 10^{-7}$ m is redshifted to 2 cm. This happens to be practically equal to the value for the nucleation distance in the quark-hadron phase transition, obtained from a pure-glue lattice Monte Carlo determination of the surface energy between the quark and hadron phases [12], but now the density contrast is likely to be larger.

The main uncertainty in the scenario is clearly the creation of domains of metastable vacua. This depends on so far unknown phenomena at energy scales at which QCD became dynamically independent; at present one can only assume that our part of the Universe had been in a metastable vacuum originally.

Finally, it is interesting to note that $T_{\rm pt}$ is quite close to the sphaleron scale $\sim \pi M_W/\alpha_W \approx 10$ TeV. This closeness seems purely coincidental.

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