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Synchronization and Computation in a Chaotic Neural Network

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Chaos generated by the internal dynamics of a large neural network can be correlated over large spatial scales. Modulating the spatial coherence of the chaotic fluctuations by the spatial pattern of the external input provides a robust mechanism for feature segmentation and binding, which cannot be accomplished by networks of oscillators with local noise. This is demonstrated by an investigation of synchronized chaos in a network model of bursting neurons responding to an inhomogeneous stimulus.

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Coherent modes in extended chemical, biological, and fluid systems have been the focus of considerable interest [1,2]. Theoretical studies of these phenomena include systems of coupled limit-cycle oscillators [1], the complex Ginzburg-Landau system [1-3], and the lattices of coupled maps [4,5]. Recent experiments in neurobiology have renewed interest in the cooperative dynamical properties of large neuronal systems, in particular, the emergence of synchronized patterns of neural activity and their computational role. Large-scale spatiotemporal patterns of activity in the frequency range of 30-70 Hz have been found in the olfactory system, the visual cortex, and other brain areas [6-9].

Temporally modulated neuronal responses to oriented stimuli have been observed in the cat visual cortex. Local groups of neurons responding to a common stimulus display synchronized activity. Neurons responding to separate stimuli are also phase locked, even when the distance between them is large, provided their stimuli have similar features (e.g., similar orientation and direction of motion) [6,7]. It has been suggested that the selective synchronization of neural activity serves as a mechanism of binding spatially distributed features into a coherent object [6,10].

The observed synchronization of neural activity and its proposed function raise several important issues. The synchronized neurons have different levels of responses, depending on, e.g., their preferred orientation (PO) (i.e., the stimulus orientation that elicits the maximal response of the neuron) or stimulus velocity. These differences presumably serve to encode the local features of the external stimulus. It is thus important to investigate mechanisms that are capable of synchronizing activity in spatially inhomogeneous neural systems, without destroying the response properties of individual neurons. Second, one needs to understand how the properties of the external stimuli can modulate rapidly the pattern and degree of synchronization across the system.

Models of *oscillatory* neural networks with phase-locked activity capable of feature linking have been studied recently [11-14]. In these models synchronization of the oscillations is mediated by (mostly excitatory) long-range connections. Desynchronization of weakly coupled neurons is achieved by local noise.

Both the experimental results and the suggested relevance of synchronization to global operations, such as object segmentation, imply that there is an efficient mechanism for rapid desynchronization of the relative temporal phase of large internally synchronized groups of neurons. However, systems of noisy oscillators are incapable of generating such a large-scale rapid and reversible desynchronization. This is because the amplitude of the effective noise that acts on the phase of an internally synchronized group of say N oscillators is only $\delta/\sqrt{N}$, where δ is its local amplitude, and thus is negligible for large N. Consequently, even weak coupling between two large assemblies of oscillators will eventually synchronize them. More importantly, even when such assemblies are not interacting at all, the time taken to desynchronize the *initial* relative phase between them is extremely long, i.e., of $O(N/\delta^2)$. To overcome this problem within the context of oscillatory networks one has to introduce an *ad hoc* spatially correlated noise [13,14].

In this Letter we study a chaotic neural network model that exhibits synchronization at large spatial scales, modulated by the distributed features of the external stimulus. The advantage of chaos over external noise is the fact that the spatial correlations of the deterministic noise are not fixed but depend on the dynamic state of the system. As a consequence, the external stimulus can modulate the spatial scale of the dynamic noise. In particular, depending on the pattern of the input the system can break into large weakly coupled clusters, each exhibiting a globally chaotic activity. The chaotic fluctuations rapidly desynchronize the relative phases of the different clusters.

We study a network of coupled Hindmarsh-Rose neurons [15] described by the following equations:

$$\dot{X}_i = Y_i - aX_i^3 + bX_i^2 - Z_i + I_i + \sum_{\substack{j=1 \\ j\neq i}} J_{ij} S_j(t) , \tag{1}$$

$$\dot{Y}_i = c - dX_i^2 - Y_i , \tag{2}$$

$$\dot{Z}_i = r[s(X_i - x_0) - Z_i] . \tag{3}$$

Each neuron is characterized by three time-dependent variables: the *membrane potential* X_i, the *recovery variable* Y_i, and a slow *adaptation current* Z_i. The external inputs are given by I_i. The effect of the firing activity of the jth neuron on the ith neuron is modeled in Eq. (1) by an impulse current to the ith neuron, proportional to the synaptic strength J_{ij}, generated when the jth neuron is *active*. The activity of a neuron is denoted by the (0,1) variable $S_i(t) \equiv \Theta(X_i(t) - x^*)$, where x^* is some threshold potential taken here to be 0, and Θ is the Heaviside step function. The Hindmarsh-Rose *single neuron* model and its extensions exhibit a dynamical behavior similar in several aspects to the properties of real neurons [15-17].

For sufficiently low values of I the neuron is in a stable quiescent state with $X = x_0 < 0$. As I increases there is a bifurcation to a low-frequency *repetitive firing* state consisting of a train of regularly spaced spikes. Further increase in I leads to a sequence of bifurcations through *bursting* states consisting of periodic bursts of two or more spikes per burst. The long time scale associated with the burst is provided by the time constant of the adaptation current, Z, i.e., $1/r$, which is assumed to be small, of $O(10^{-2})$. For large values of I the system is in a high-frequency repetitive firing state. For intermediate values of r and I there is also a *chaotic* state of irregular bursts [17]. For concreteness we will fix the parameters to the values $a = 1.0$, $b = 3.0$, $c = 1.0$, $d = 5.0$, $s = 4.0$, $x_0 = -1.6$, and $r = 0.006$, as in Ref. [17].

We first consider a network consisting of N neurons with different values of I_i coupled globally by excitatory interactions, $J_{ij} = J/N$. The values of I_i are uniformly distributed between 1 and 5. The global nature of the state of the network can be characterized by the mean field, $I_{\text{syn}}(t) \equiv N^{-1} \sum_{i=1}^{N} S_i(t)$, which is proportional to the total synaptic current, and is thus the force mediating the interactions between the neurons. Simulations of the above network with $0 < J < 6.0$ reveal three phases.

(i) Asynchronous stationary state.—For small values of the coupling J, I_{syn} is constant in time, except for small finite-size fluctuations of the order $1/\sqrt{N}$. The neurons are not synchronized, and the whole effect of the interaction is to shift the value of the local current acting on them by a constant and spatially uniform amount I_{syn}. The time-averaged firing rates $\Gamma_i \equiv \langle S_i(t) \rangle$ are displayed in Fig. 1 against the local external currents I_i. It exhibits a series of discontinuities marking the boundaries between different firing states. An example of two neurons, with periodic bursts of two and three spikes respectively, is shown in Fig. 2(a). The bursts of the two neurons are not phase locked, and have slightly different frequencies. For the parameters given above this globally stationary phase exists for $0 \lesssim J \lesssim 0.8$.

(ii) Synchronized oscillations.—For intermediate coupling strength, in our case for $0.8 \lesssim J \lesssim 3.5$, $I_{\text{syn}}(t)$ is periodic in t except for small finite-size noise, and the activity of most of the neurons is phase locked to this periodic driving force. The local activities may also have an aperiodic component which, however, is not synchronized across the system, and therefore contributes only a small finite-size noisy component to $I(t)$.

(iii) Synchronized chaos.—When the coupling is strong, $3.5 \lesssim J$, all the active neurons are chaotic and furthermore their chaotic fluctuations are spatially correlated. An example is shown in Fig. 2(b) where the syn-

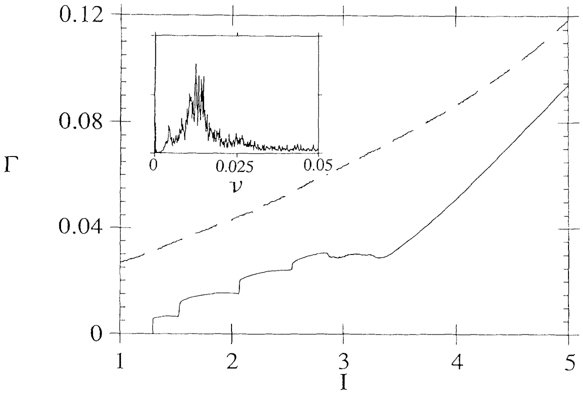

FIG. 1. The time-averaged firing rates plotted against the local external inputs. The continuous line is for $J = 0.5$. The plateau at 0 corresponds to neurons in a quiescent state. The next plateaus correspond to periodically bursting neurons with 1, 2, 3, and 4 spikes per burst, respectively. The nonsmooth regime corresponds to chaotic neurons. The last, linear part corresponds to repetitively firing neurons. The dashed line is for $J = 5.0$. Results from simulations of Eqs. (1)-(3) with $N = 800$. For parameters see text. Inset: The power spectrum of I_{syn} for $J = 5.0$ (in arbitrary units).

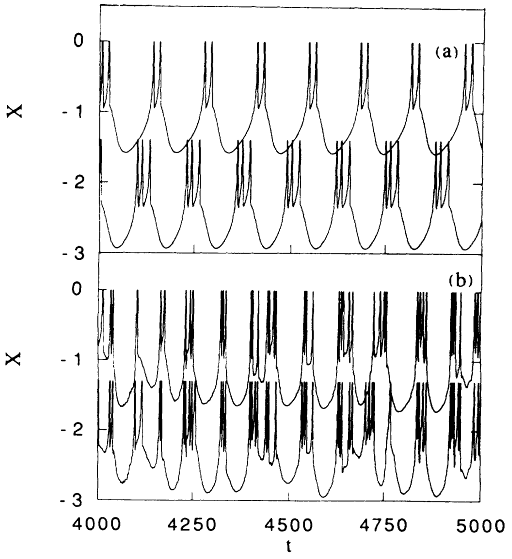

FIG. 2. Membrane potentials of single neurons. Network the same as in Fig. 1. (a) $J = 0.5$. Upper trace for neurons with $I_i = 1.75$; lower trace for $I_i = 2.25$. (b) The same neurons but with $J = 5.0$. Vertical scale of second neurons was shifted downwards by 1.4. The tips of the spikes were clipped.

chronized chaotic bursts of the same pair of neutrons as in Fig. 2(a) are clearly seen. In this phase the neural activity no longer consists of bursts with fixed numbers of spikes, yielding a smooth monotonic dependence of the average rate $\Gamma(I_i)$ as shown in Fig. 1. The synchronized chaotic activity results in a mean field that displays substantial chaotic fluctuations, as indicated by its power spectrum, Fig. 1 (inset). Studying sizes $N < 6000$ we have checked that the amplitude of the chaotic fluctuations in $I_{\rm syn}(t)$ remains constant for large N, in this regime of J.

To investigate the role of synchronized chaos in the processing of sensory information we need to consider more structured networks. A neuron is labeled by $(\mathbf{r},\theta)$, where $\mathbf{r}$ represents its spatial location in the network, which maps also to a location in the external sensory space. The coordinate θ represents a feature coordinate, e.g., the preferred orientation PO of the neuron. The interaction between neurons has a long spatial range, relative to the distance between nearby neurons, and is nonzero only for pairs of neurons with $\theta \simeq \theta'$. The external sensory input is characterized by a feature density $\theta_0(\mathbf{r})$ which varies slowly with $\mathbf{r}$, except at lines of discontinuities representing boundaries between different "objects." At the vicinity of a site $\mathbf{r}$ only neurons with $\theta \simeq \theta_0(\mathbf{r})$ are vigorously excited. Under a suitable choice of network parameters, the active neurons in regions where $\theta_0(\mathbf{r})$ varies smoothly will be synchronized through their interactions. On the other hand, since only neurons with similar θ are coupled, the lines of discontinuity in $\theta_0(\mathbf{r})$ will break the desynchronization, thus leading to the formation of weakly interacting synchronized clusters whose boundaries separate between objects. The weakly interacting clusters will be rapidly desynchronized by the global chaotic noise of each cluster.

To demonstrate the operation of such a network, we consider two clusters, each comprising N neurons responding to a common oriented stimulus [13]. The neurons in each cluster are labeled by their PO, θ, distributed uniformly between $-\pi/2$ and $+\pi/2$. Neurons in the same cluster interact with a constant coupling, $J_{ij} = N^{-1} J_{\rm intra}$, $J_{\rm intra} = 21.0$, if the difference in their PO is less than $\pi/4$ and zero otherwise. The interaction between neurons belonging to different clusters is $J_{ij} = J_{\rm inter}$, $J_{\rm inter} = 1.5$, if they have similar PO; otherwise it is zero. A stimulus with an orientation θ_0 is modeled as a local input given by $I(|\theta_0 - \theta|)$, where $I(|\theta|)$ varies linearly from a minimal value at $|\theta| = \pi/2$, $I(\pi/2) = -10.0$, to a maximal value at 0, $I(0) = 3.4$.

This system of two interacting clusters has been simulated with $N = 160$. When the intracluster interaction $J_{\rm intra}$ is varied, we find that each cluster displays the same three phases as the fully connected system described above. In particular for values of $J_{\rm intra} \gtrsim 15$ the clusters are in a synchronized chaotic state. Thus for the chosen value $J_{\rm intra} = 21.0$ the clusters are well within their synchronized chaotic state. The time-averaged firing rate of the neurons within each cluster is maximal at $\theta = \theta_0$ and drops linearly with $|\theta - \theta_0|$. The firing rate vanishes for $|\theta - \theta_0| \gtrsim 0.35\pi$. Note that despite the presence of strong intracluster interactions the dispersion of activity levels is strong.

In spite of the intercluster interactions, the above dynamical properties of each cluster are largely insensitive to the orientation of the stimulus of the other cluster. However, the intercluster interactions are strong enough to synchronize them. This synchronization depends on $|\theta_0 - \theta_0'|$, where θ_0 and θ_0' denote the orientations of the stimuli acting on the two clusters, respectively. Because the intercluster coupling exists only between neurons with similar PO the synchronization of the two clusters strongly decreases with $|\theta_0 - \theta_0'|$ and vanishes for $|\theta_0 - \theta_0'| \gtrsim 0.3\pi$. This is clearly demonstrated in Fig. 3 where the time-dependent correlation of the mean fields of the two clusters,

$$C(\tau) \equiv \langle [I_{\rm syn}(t) - \langle I_{\rm syn}\rangle][I'_{\rm syn}(t+\tau) - \langle I'_{\rm syn}\rangle]\rangle ,$$

is shown for the case of $\theta_0 = \theta_0'$ and for $\theta_0 - \theta_0' = 0.28\pi$. The dependence of the strength of the correlations on $\theta_0 - \theta_0'$ is shown in the inset.

As stated above, a major advantage of the chaotic dynamics is in the speed by which synchronization or desynchronization is achieved. The time for synchronization of interacting clusters is roughly proportional to $J_{\rm inter}^{-1}$. Indeed we find that synchronization between clusters with $\theta_0 \simeq \theta_0'$ occurs within a few time units. Desynchronization of noninteracting clusters (from an initial synchronized state) occurs rapidly by virtue of the

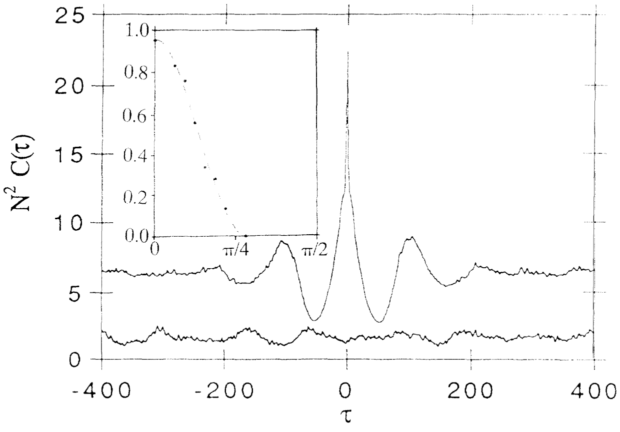

FIG. 3. Time dependence of the correlations of two clusters for the case of similar stimuli $\theta_0 = \theta_0'$ (upper) and dissimilar ones $\theta_0 - \theta_0' = 0.28\pi$ (lower). For clarity the vertical scale of the lower case has been shifted downward by 5. Parameters are given in text. Inset: The strength of the correlation between the two clusters vs the angular difference between their stimuli. The vertical axis is the equal-time correlation, $C(0)$, normalized by the geometric mean of the variances of the mean fields of the two clusters.

exponential growth of a perturbation of the initial states of the chaotic clusters, the rate of which is measured by the Lyapunov exponent, λ. Hence the dephasing time will be roughly λ^{-1}. In the present model we find numerically that the clusters are completely desynchronized within 3–5 bursts of activity.

In conclusion, we would like to point out that the main mechanism for generating the synchronized chaotic state in our model network is the long-range positive interactions in a population of neurons with a distribution of local driving currents. Our results hold even for values of the parameter r of Eq. (3) such that a *single* neuron does not show a chaotic behavior for any value of I. We also note that unlike the models of Refs. [3–5], the interaction in our model is not diffusive but is similar to an impulse force, which is more appropriate for approximating the effect of chemical synaptic couplings.

Finally, the existence and possible functional role of chaos in brain activity have been previously discussed [9]. It is important to note that encoding information in the actual temporal chaotic pattern of activity of a neural system is probably not feasible as it is inherently unstable. Our proposal here is that representing information in the statistical features of the chaotic state, in our case the *spatial patterns of coherence* of a chaotic system, may be a robust and efficient process.

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