Negative Compressibility of Interacting Two-Dimensional Electron and Quasiparticle Gases

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Direct measurements of the compressibility K of the 2D electron gas at both zero magnetic field B and in the high-field extreme quantum limit, $\nu < 1$, are reported. A new method is used to determine both the magnitude and *sign* of K. At $B=0$ we find K^{-1} changes sign and becomes negative as the 2D density is reduced. At high B, K^{-1} exhibits negative anomalies near both $\nu=0$ and 1. Compressibility features near the fractional fillings $\nu=\frac{1}{3}$ and $\frac{2}{3}$ give thermodynamic evidence for dilute interacting quasiparticle gases.

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The importance of exchange and correlation contributions to the total energy, and hence the thermodynamics, of interacting electron systems has long been theoretically appreciated [1]. For example, in the low-density regime, where interactions dominate the kinetic energy, the exchange energy alone is sufficient to produce a *negative* compressibility for the electron gas [2,3]. For two-dimensional electron systems (2DES) at high magnetic fields, interaction effects have spectacular transport consequences, e.g., the fractional quantum Hall effect [4] (FQHE). By contrast, the thermodynamic consequences [5] of the same interactions have proven experimentally elusive, with few results reported [6,7]. In this Letter we report direct measurements of the compressibility K of a high-mobility 2DES at both zero and high magnetic field B. Using a novel technique based upon a double layer 2DES, we can directly extract the sign and magnitude of K as a function of electron density N. Regions of negative K are observed at both $B=0$ and in the high-field extreme quantum limit. Near the FQHE $\frac{1}{3}$ and $\frac{2}{3}$ states we observe compressibility anomalies characteristic of dilute gases of quasielectron and quasihole excitations.

A traditional method for observing K relies on measuring the capacitance [6–8] between the 2DES and a gate electrode. For thin 2D systems the inverse capacitance C^{-1} is the sum of two terms: a large geometric contribution due to the spatial separation of the gate and 2D plane, and a much smaller term proportional to $\partial\mu/\partial N$, the rate of change of chemical potential with density. This derivative determines the compressibility: $K^{-1} = N^2\partial\mu/\partial N$. The overwhelming dominance of the geometric capacitance seriously limits the usefulness of this technique; even the *sign* of K at $B=0$ cannot be reliably determined. The present technique avoids this problem entirely, with the observed signal being directly proportional to K^{-1}. The basic concept is illustrated in the inset to Fig. 1. A gate is used to apply an electric field to two closely spaced 2D electron gases. These layers are assumed to be isolated except via external electrical connections to the individual 2D systems. These connections maintain nearly zero chemical-potential difference between the two layers. Classically, the gate electric field E_0 would be entirely screened by the "top" 2DES with no field E_p penetrating through and affecting the "bottom" 2DES. But as the density of states of the top layer is finite, some field penetration does occur [9]. For two infinitely thin 2D sheets separated by a distance a, the ratio, in equilibrium, of differential changes in E_p and E_0 is given by

$$\delta E_p/\delta E_0 = d_t/(a+d_t+d_b)\,, \tag{1}$$

where d_t and d_b are distance parameters defined by the compressibilities of the two 2DES: $d_{t,b} = (\kappa\varepsilon_0/e^2) \times \partial\mu_{t,b}/\partial N_{t,b}$. The field δE_p forces charge to flow on or off of the bottom 2DES layer in response to a change in gate voltage. This current produces a measurable voltage $V_{\rm sig}$ across a small external resistor (or capacitor). By applying both a small ac voltage and a larger dc bias V_g to the gate, the penetrating field $\delta E_p \propto V_{\rm sig}$ can be measured as a function of the top layer density N_t. As Eq. (1) shows, the penetration also depends on the compressibility of the bottom 2DES. Usually, however, the penetration is weak ($a \gg d_t$ and d_b) and the bottom layer density remains close to its nominal value for all gate voltages until the top layer depletes. For noninteracting 2D electrons in GaAs, $d_t = d_b = 25$ Å, and since a is of order several hundred Å, the typical penetration would be only a few percent, and would be positive. Our results show that interactions render this estimate qualitatively incorrect.

The sample used in this study is a modulation-doped GaAs/AlGaAs heterostructure grown by molecular-beam epitaxy. Two 200-Å-wide GaAs quantum wells are separated by and embedded in the alloy $Al_{0.3}Ga_{0.7}As$. The barrier width is 175 Å and is centered 0.48 μm below the sample top surface. The wells contain nearly equivalent 2D electron gases with nominal densities of 1.5×10^{11} cm^{-2} and mobilities around 3×10^6 cm^2/V s. A square mesa, 250 μm on a side and covered by an aluminum gate electrode, is produced using standard photolithography. Four 40-μm-wide arms extend from this region; each is terminated by an electrical contact to one or the other of the 2D layers. The technique employed to produce these separate contacts has been described elsewhere [10]. We emphasize that these layers are electrically isolated; tunneling and/or defect-related interlayer conduction is negligible.

Figure 1 displays the normalized ac penetration field $\delta E_p/\delta E_0$ versus dc bias voltage V_g applied to the gate. These normalized data are obtained from the measured signal voltages V_{sig} by dividing by the signal observed when the top 2DES is fully depleted (for $V_g < -1$ V). Obviously, when the top 2DES is depleted $\delta E_p = \delta E_0$. (Before depletion the data are further divided by the factor 1.09 to account for the slightly increased gate capacitance when the top 2DES is intact.) The upper horizontal axis gives the conversion between gate bias and top layer density. This is obtained from the observation, at low magnetic field, of Landau-level structure in the penetration field. The data shown, taken at $T = 1.2$ K, were obtained using an ac gate excitation of 20 mV rms at 1 kHz. We stress that the signal exhibits no significant resistive component at this frequency, except very close to depletion. The data show no significant changes upon cooling to 0.3 K, lowering the frequency, or reducing the excitation amplitude.

As the data show, the differential penetration field becomes *negative* for densities below about $N_c \sim 1.1 \times 10^{11}$ cm^{-2}. From the small magnitude of the penetration prior to depletion, and the large center-to-center spacing of the two 2D layers, $a = 375$ Å, Eq. (1) implies that d_t, and hence the inverse compressibility K^{-1} of the top layer, is changing sign at this critical density. A number of quantitative effects, including the finite thickness of the 2D sheets, any small stray capacitance between gate and the bottom 2DES, etc., can influence the critical density N_c, but the qualitative result in Fig. 1 is robust. Indeed, it is entirely expected owing to the increasing importance of interactions at low 2D density.

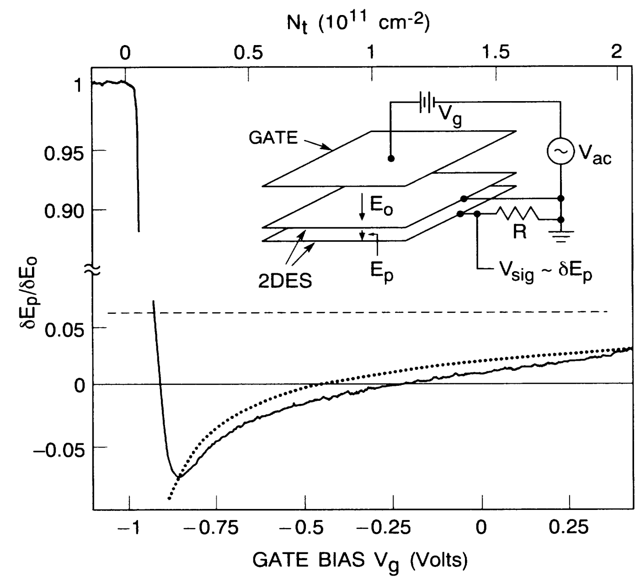

FIG. 1. Normalized penetrating electric field $\delta E_p/\delta E_0$ vs gate bias V_g at zero magnetic field and $T = 1.2$ K. Note broken vertical axis. Upper horizontal axis gives density N_t of top 2DES. Dotted curve calculated using Tanatar and Ceperley's [12] compressibility; dashed horizontal line is the noninteracting result. Inset: Experimental arrangement.

To lowest approximation, Coulomb interactions produce a negative exchange contribution to the chemical potential of the electron gas. This term, inversely proportional to the mean interelectron spacing [in units of the Bohr radius a_B, $r_s^{-1} = a_B(\pi N)^{1/2}$], exceeds the kinetic energy at low density. With K_0 the noninteracting compressibility, an ideal 2DES in the Hartree-Fock (HF) approximation [11] exhibits K_0/K falling linearly with r_s, becoming negative at $r_s = 2.22$, or $N_{c,\mathrm{HF}} = 6.5 \times 10^{10}$ cm^{-2} for GaAs. In this approximation K_0/K, and thus $\partial\mu/\partial N$, diverges as $N^{-1/2}$ at low density. Estimates of the correlation energy by Tanatar and Ceperley [12] yield a critical density 20% higher than the HF result and a slight deviation from the linear r_s dependence. The dotted curve in Fig. 1 is calculated using Tanatar and Ceperley's [12] results and Eq. (1) setting $a = 375$ Å, the center-to-center distance between the quantum wells. Quantitative agreement should not be expected since the finite thickness of the quantum wells, which creates both band-bending effects and a softened Coulomb interaction, has not been accounted for. Nevertheless, the data and ideal 2DES theory are in good qualitative agreement. The horizontal dashed line in the figure, calculated for a noninteracting 2DES, makes clear the importance of many-body effects on the compressibility.

Applying a magnetic field perpendicular to the 2D planes dramatically modifies the measured compressibility. Here we investigate K^{-1} at very high fields, in the extreme quantum limit where the Fermi level is in the lowest Landau level. Large magnetic fields, however, create an additional experimental problem: The conductivity σ_{xx} of the 2DES becomes very small. This requires the measurement frequency to be greatly reduced in order to eliminate spurious resistive signals [13]. This is especially important when the Fermi level moves into a gap, as in the integer or fractional quantum Hall effects. Figure 2 shows the normalized penetration field at $B = 7.5$ T along with the quadrature signal arising from in-plane conductivity effects. These data were obtained at $T = 1.2$ K using 20-mV, 43-Hz excitation. The upper horizontal axis is now the Landau-level filling factor of the top 2DES, $\nu_t = hN_t/eB$. (The bottom 2DES remains near $\nu_b = 0.8$.) Strictly speaking, since the penetration field affects the conversion between gate bias and the density N_t, the filling-factor scale should be slightly nonlinear, especially near $\nu_t = 0$ and 1. This small correction has been ignored for our present illustrative purposes. The large peak near $V_g = 0.25$ V indicates that the top 2DES is entering the $\nu_t = 1$ integral quantum Hall state. As the quadrature signal suggests, the data at $\nu_t = 1$ are heavily influenced by the vanishing 2D conductivity. Except here, and close to $\nu_t = 0$, the quadrature signal is negligible and the capacitive signal reliably determines the thermodynamic compressibility. As the figure shows,

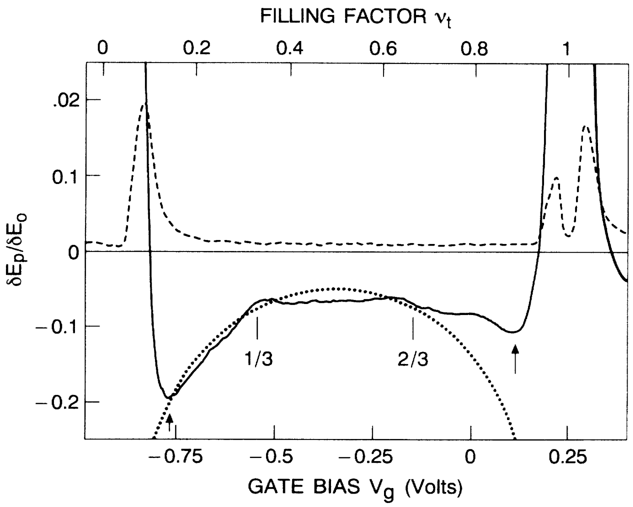

FIG. 2. Solid curve: Normalized penetration field vs gate bias and Landau-level filling factor ν at $B=7.5$ T and $T=1.2$ K. Arrows indicate negative anomalies near $\nu=0$ and 1. Dashed curve: associated resistive signal; same scale as for the penetration field. Dotted curve: Efros' [16] calculation.

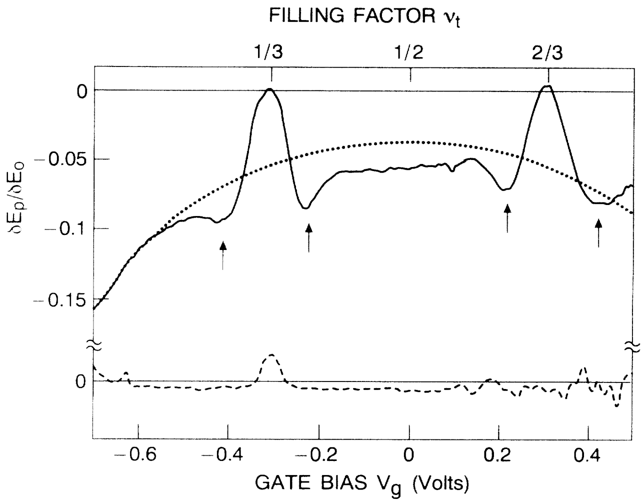

FIG. 3. Solid curve: Normalized penetration field vs gate bias and filling factor at $B=12$ T and $T=0.35$ K. Anomalies associated with quasiparticle interactions indicated by arrows. Dashed curve: associated resistive signal, to same scale but vertically displaced. Dotted curve: Efros' [16] calculation, which does not include FQHE structure.

the penetration field is negative [14] throughout the extreme quantum limit $0<\nu_t<1$. The deep minimum at low density ($\nu_t \to 0$) is accompanied by a similar, though weaker, dip as $\nu_t \to 1$. (A similar observation near $\nu=1$ has been reported [7] for the Si metal-oxide-semiconductor system.) Both features suggest divergences that are cutoff: the lower by depletion of the top 2DES and the higher by the onset of the $\nu_t=1$ QHE. Note that both minima occur well before significant resistive signals have developed.

Also indicated in Fig. 2 are the locations of the fractional filling factors $\nu_t=\frac{1}{3}$ and $\frac{2}{3}$. Only slight modifications of the penetration field are observed there. By increasing the magnetic field and lowering the temperature these features can be greatly enhanced. Figure 3 displays the penetration field versus filling factor ν_t at $B=12$ T and $T=0.35$ K using a measurement frequency of 3 Hz. At this magnetic field, and at zero gate bias, each 2DES is near filling fraction $\nu=\frac{1}{2}$. As the data indicate, substantial structure is observed around both the $\frac{1}{3}$ and $\frac{2}{3}$ states. Near the center of the $\frac{1}{3}$ state a small resistive component persists, but careful study of its frequency dependence [13] proves that it is not polluting the capacitive component. Hence, the data in Fig. 3 represent thermodynamic signatures of the FQHE states at $\nu_t=\frac{1}{3}$ and $\frac{2}{3}$.

These high-field results suggest a plausible, unifying picture. To begin, the overall negative K^{-1} in the extreme quantum limit, $\nu_t<1$, is anticipated since the kinetic energy is quenched; i.e., in the absence of interactions (and disorder) the chemical potential would simply be half the cyclotron gap, independent of density, and thus $K^{-1}=0$. Including interactions would, in loose analogy with $B=0$, be expected to produce $K^{-1}<0$, at least so long as an incompressible FQHE state does not develop. At low density, $\nu_t \to 0$, we again expect a negative divergence in $\partial\mu/\partial N$ due to interaction effects. But, via particle-hole symmetry, an analogous divergence can be expected as $\nu_t \to 1$, for then the system may be viewed as a dilute gas of interacting *holes* along with a filled Landau level. Using the total-energy interpolation formula of Fano and Ortolani [15], Efros [16] has calculated the compressibility in the $\nu<1$ regime, neglecting the FQHE. His results, along with Eq. (1) and $a=375$ Å, yield the dotted curve in Fig. 2. The qualitative agreement is again good.

For the FQHE states at $\nu=\frac{1}{3}$ and $\frac{2}{3}$ the data in Fig. 3 offer a new perspective. The fundamental prediction of the standard model of the FQHE is a cusp in the total energy at certain filling factors, ν_0. This implies an ideal 2DES at $T=0$ will exhibit incompressibility exactly at $\nu=\nu_0$. For the present experiment, incompressibility ($K^{-1}\to\infty$) would be manifested by $\delta E_p/\delta E_0 \to 1$. While upward peaks in $\delta E_p/\delta E_0$ are observed, Fig. 3 reveals them to be rather weak. We attribute this to disorder and inhomogeneity in the sample. Note that the present technique averages over a large area and does not select, as transport measurements do, an optimal path through the sample.

For ν slightly away from ν_0, a gas of quasiparticles should exist along with the condensed FQHE ground state. If quasiparticle interactions are neglected, the total energy would be *linear* in filling factor, suffering only a change in slope at $\nu=\nu_0$. This assumption would lead to K^{-1}, and thus $\delta E_p/\delta E_0$, being zero. As the arrows in

Fig. 3 indicate, however, K^{-1} exhibits distinct minima above and below both the $\frac{1}{3}$ and $\frac{2}{3}$ states. We attribute these features to interactions among the quasiparticles and suggest a direct analogy with our prior discussion of the cutoff "divergences" seen as $N_t \rightarrow 0$ at $B=0$ and as $\nu_t \rightarrow 0$ or 1 in the extreme quantum limit. On approaching any fractional state the density of quasielectron or quasihole excitations tends to zero. Thus, just as in these prior cases, Coulomb interactions will produce [17] a negatively divergent $\partial\mu/\partial N$. At sufficiently low quasiparticle density disorder will dominate the interactions and truncate the divergences. In all the above cases, $\nu_0=0$, 1, $\frac{1}{3}$, and $\frac{2}{3}$, the functional form of the divergence is predicted [17] to be the same: $\partial\mu/\partial N \propto |\nu-\nu_0|^{-1/2}$. Our observation of these compressibility features constitutes strong thermodynamic evidence for existence of the dilute quasiparticle gases central to theory of the FQHE.

To summarize, a new technique for directly probing the compressibility of 2D electron systems has revealed regimes of negative thermodynamic compressibility of the interacting 2DES. Negative "divergences" in K^{-1} have been observed at $B=0$ and in the extreme quantum limit when ν_t approaches 0, 1, $\frac{1}{3}$, and $\frac{2}{3}$. A unifying qualitative picture of these features centers on the increasing importance of interactions as the (quasi)particle density is reduced. Our results are in good qualitative agreement with existing theoretical predictions.

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