Resonant Damping of Toroidicity-Induced Shear-Alfvén Eigenmodes in Tokamaks

Fulvio Zonca[a] and Liu Chen

Plasma Physics Laboratory and Department of Astrophysical Sciences, Princeton University, Princeton, New Jersey 08543

(Received 5 September 1991)

An analytical theory of toroidicity-induced shear-Alfvén eigenmodes (TAE) is presented. The full two-dimensional problem is treated using a variational approach and the radial eigenmode structure is analyzed. We show that TAE suffer a significant damping due to the strong absorption occurring at the resonances with the shear-Alfvén continuous spectrum. The resonant damping is shown to be larger than the electron Landau damping and therefore constitutes an important dissipation mechanism in determining the threshold for TAE instabilities driven by alpha and energetic particles in tokamak experiments.

PACS numbers: 52.35.Bj, 52.55.Dy, 52.55.Fa

With the tokamak fusion research program approaching the ignition experiment, the issue of the collective processes related to the energetic-alpha-particle dynamics in ignited plasmas has received increasing attention. The performance of the future reactors is crucially dependent on the good confinement of the 3.5-MeV alphas produced by the d-t fusion reactions and on the efficiency of the current-drive systems. Also in the latter case the dynamics and the confinement of high-energy ions can be of major importance, as in ITER (International Thermonuclear Experimental Reactor), for which a $\sim$1-MeV neutral-beam current drive has been proposed. These energetic ions will have speed comparable to the Alfvén velocity and hence could excite, among others, shear-Alfvén waves. Although the shear-Alfvén continuous spectrum is strongly damped because of phase mixing, in a two-dimensional toroidal geometry discrete eigenmodes, known as toroidicity-induced shear-Alfvén eigenmodes (TAE) or gap modes, do also exist [1,2]. The TAE are a very effective scattering mechanism for the high-energy alpha particles. Low-amplitude modes [3] with $\delta B/B \approx 5\times 10^{-4}$ can already cause the loss of the fusion alphas, and therefore the linear stability of the TAE is a relevant issue for fusion reactors.

Recent theories [4-7] have studied the linear stability of TAE driven by the resonant interaction with fast alpha particles considering the electron Landau damping as the only dissipation mechanism. The results show that the instability threshold for these modes is very low and that they can become unstable in ignited tokamaks. The most unstable modes have high toroidal mode number approximately given by [4,5] $n \approx r_0^2/sq_0R_0\rho_\alpha$. Here ρ_α is the Larmor radius of the alpha particles, s the magnetic shear defined as $s \equiv r_0 q_0'/q_0$, R_0 the major radius, q_0 the safety factor at r_0, r_0 the radial position of the model rational surface defined by $m_0 = nq(r=r_0)$, and m_0 the poloidal mode number. For ITER and BPX (Burning Plasma Experiment) we have typically $n \approx 10$. In this Letter, we report our recent results [8] about TAE damping due to the coupling with the shear-Alfvén continuous spectrum, suggested as a possible dissipation mechanism for these modes in Ref. [4]. We show that, for typical parameters, this damping mechanism is much larger than the electron Landau damping. As a result, the instability threshold for TAE is higher than previously estimated. In the following, we will focus on the general approach, which treats the full two-dimensional problem with a variational technique, as well as on the underlying physical picture rather than mathematical details, which will be given in a future publication [9].

In Ref. [1], it is shown that for low-β [$\beta \equiv$(kinetic pressure)/(magnetic pressure)] tokamaks with concentric circular magnetic surfaces, TAE can be described by the equation

$$\left\{ \frac{d^2}{d\theta^2} + \frac{\omega^2}{\omega_A^2}(1+2\epsilon\cos\theta) - F \right\}\Phi = 0\,, \tag{1}$$

where Φ is a scalar field related to the Fourier transform of the perturbed magnetic flux $\delta\phi$, $F = s^2/(1+s^2\theta^2)^2$, and $\omega_A \equiv v_A/qR$ is the Alfvén frequency, v_A being the Alfvén speed. In the large-θ limit, Eq. (1) describes the motion of a free particle in a small perturbing periodic potential. In the $\epsilon \to 0$ limit the solutions would be just plane waves $\approx \exp(\pm ik\theta)$, $k = \omega/\omega_A$. In a one-dimensional periodic lattice of period L, the traveling waves will combine in standing-wave solutions when the Bragg-reflection condition is satisfied, $2L = l\lambda$, $\lambda \equiv 2\pi/k$, $l = 1,2,3,\ldots$. In our case $L = 2\pi$. Then we must have $k = \omega/\omega_A = l/2$ for standing waves $\approx \sin(l\theta/2)$, $\cos(l\theta/2)$. For the two lowest-energy states the eigenfrequencies are $(\omega_A^2/4)(1\pm\epsilon)$. In real space this corresponds to the formation of small but finite gaps at the intersections of the shear-Alfvén frequencies of neighboring poloidal harmonics, i.e., at $k = \omega/\omega_A = \pm\frac{1}{2}$. Finally in Eq. (1) the role of F is to create a potential well which can localize a discrete eigenmode inside the gap with a frequency $(\omega_A^2/4)(1-\epsilon) < \omega^2 < (\omega_A^2/4)(1+\epsilon)$.

In the high-n limit the individual TAE eigenfunctions are highly localized in the radial direction between two mode rational surfaces and many poloidal harmonics are coupled together to determine the global radial envelope structure. Spatial (radial) variation of ω_A further localizes the envelope. Outside its localization region, the envelope is exponentially decaying. In general, different poloidal harmonics can meet the resonance condition where the TAE eigenfrequency matches the local shear-Alfvén continuous spectrum. Wave damping then occurs due to the finite absorption occurring at each resonance. The magnitude of the damping depends on the tunneling of the envelope out of its radial localization region. The tunneling becomes exponentially small in the high-n limit, which reinforces the fact that the most unstable solutions are obtained for large n.

Equation (1) can be reexpressed in the (r,θ) coordinates. Then the magnetic flux perturbation $\delta\phi$ satisfies the equation

$$B_0\hat{\mathbf{b}}\cdot\nabla\left[\frac{1}{B_0}\nabla_\perp^2\hat{\mathbf{b}}\cdot\nabla\delta\phi\right]+\omega^2\nabla\cdot\left[\frac{1+2\epsilon\cos\theta}{\bar{v}_A^2}\nabla_\perp\delta\phi\right]=0. \tag{2}$$

Here, for $n\gg 1$, the coupling with the compressional Alfvén wave has been neglected, B_0 is the equilibrium magnetic field, $\hat{\mathbf{b}}\equiv\mathbf{B_0}/B_0$, and $1/\bar{v}_A^2$ is the flux surface average of $4\pi\rho(r)/B_0^2$. Equation (2) can be derived as the Euler-Lagrange equation from the following Lagrangian:

$$\delta L=\int d\mathbf{r}\left\{|\nabla_\perp\hat{\mathbf{b}}\cdot\nabla\delta\phi|^2-\frac{\omega^2(1+2\epsilon\cos\theta)}{\bar{v}_A^2}|\nabla_\perp\delta\phi|^2\right\}, \tag{3}$$

looking only at internal modes. Equations (2) and (3) can be further simplified in the high-n limit, where we can assume localized modes.

We focus our attention on a mode excited at the rational surface r_0, with $m_0=nq(r_0)$. Because of the toroidicity-induced mode coupling between different poloidal harmonics, we can write

$$\delta\phi=e^{i(n\phi-m_0\theta)}\sum_j\delta\phi_j(r)e^{-ij\theta}. \tag{4}$$

In the high-n limit, the poloidal-mode-number spreading can be considered small, i.e., $|j|\ll m_0$ in Eq. (4). We also have

$$\hat{\mathbf{b}}\cdot\nabla=\frac{i}{qR}[n(q-q_0)-j]=\frac{i}{qR}(t-j), \tag{5}$$

where $t\equiv(r-r_0)/\Delta r_s$, Δr_s being the distance between two neighboring rational surfaces, i.e., $\Delta r_s\equiv 1/nq'(r_0)$.

It is then possible to show that Eqs. (2) and (3) reduce to

$$s^2\frac{\partial}{\partial t}[(t-j)^2-\Omega^2 f(t)]\frac{\partial}{\partial t}\delta\phi_j-[(t-j)^2-\Omega^2 f(t)]\delta\phi_j=\epsilon_0\Omega^2\left[s^2\frac{\partial^2}{\partial t^2}-1\right](\delta\phi_{j+1}+\delta\phi_{j-1}), \tag{6}$$

$$\frac{\delta L}{2\pi R_0}=k_\theta^2\frac{2\pi r_0\Delta r_s}{q_0^2R_0^2}\int dt\sum_j\left\{[(t-j)^2-\Omega^2 f(t)]\left[s^2\left|\frac{\partial}{\partial t}\delta\phi_j\right|^2+|\delta\phi_j|^2\right]\right.$$
$$\left.-\epsilon_0\Omega^2\left[\delta\phi_j\delta\phi_{j-1}^*+s^2\frac{\partial}{\partial t}\delta\phi_j\frac{\partial}{\partial t}\delta\phi_{j-1}^*+\text{c.c.}\right]\right\}. \tag{7}$$

Here, $\epsilon_0\equiv 2(r_0/R_0+\Delta')$, Δ' being the derivative of the Shafranov shift, $k_\theta\equiv m_0/r_0$ is the poloidal mode number of the excited mode, Ω is the mode frequency normalized to the local Alfvén frequency $\Omega=\omega/(\bar{v}_{A0}/q_0R_0)$, and $f(t)$ is a slowly varying function of t which accounts for the radial profile of the Alfvén frequency ω_A; i.e., $f(t)\equiv\omega_{A0}^2/\omega_A^2$ and $f(0)=1$. Equations (6) and (7) are the differential and variational formulations of a truly two-dimensional eigenvalue problem, which in general is difficult to solve.

In the $n\to\infty$ limit, the problem can be reduced to one dimension using the standard approach of the ballooning transformation, based on the translational invariance of the $\delta\phi_j(t)$ [10]. The translational invariance argument, however, breaks down when the coupling with the shear-Alfvén continuous spectrum is taken into account, due to the singular nature of the poloidal harmonics at their resonant surface. Therefore, the problem must be handled as a two-dimensional one and the radial eigenmode structure (the envelope) becomes essential to describe the physics of the coupling between the discrete and continuous spectra.

The Lagrangian of Eq. (7) can be divided into two pieces: $\delta L=\delta L_{\text{NR}}+i\delta L_{\text{R}}$, with a nonresonant contribution δL_{NR} coming from the regular slowly varying poloidal harmonics, and a resonant contribution δL_{R} due to the fast variation of the singular poloidal harmonics at the resonances. We will make the assumption that $|\delta L_{\text{R}}|\ll|\delta L_{\text{NR}}|$ so that at lowest order the problem is described by $\delta L_{\text{NR}}=0$ and can be handled via the usual ballooning transformation approach. At next order δL_{R} gives the damping due to the resonant absorption effects. The consistency of the assumption about the smallness of δL_{R} will be shown *a posteriori* to be a consequence of the exponential decay of the radial envelope outside its localization region. First, we solve the Euler-Lagrange equation (6), neglecting the Alfvén resonances. Let $\Omega=\Omega_0+i\gamma$. The poloidal harmonics $\delta\phi_j(t)$ can be assumed to have the form $\delta\phi_j(t)=A(t)\phi(t,t-j)$, where $\phi(t,t-j)$ is a function of a fast variable $t-j$ and a slow variable t, while $A(t)$ is an envelope function of the slow variable t only. Following the ballooning transformation approach, the function $\phi(t,t-j)$ is written in terms of its Fourier transform:

$$\phi(t,t-j)=\int_{-\infty}^{+\infty}d\theta\, e^{-i(t-j)\theta}\Phi(t,\theta). \tag{8}$$

Substituting $\delta\phi_j(t)$ in Eq. (6) with $A(t)$ expressed in the eikonal form $A(t)=\exp[i\int^t\theta_k(t)dt]$, we find the TAE eigenmode equation in the ballooning representation,

$$\left\{\frac{\partial}{\partial\theta}[1+s^2(\theta-\theta_k)^2]\frac{\partial}{\partial\theta}+\Omega_0^2 f(t)(1+2\epsilon_0\cos\theta)[1+s^2(\theta-\theta_k)^2]\right\}\Phi=0. \tag{9}$$

Imposing the boundary conditions $|\Phi|\to 0$ as $|\theta|\to\infty$, Eq. (9) yields the following local dispersion relation:

$$F(\Omega_0^2 f(t),\theta_k(t))=0, \tag{10}$$

where

$$F(\Omega_0^2 f(t),\theta_k(t))=\hat{\gamma}+\tfrac{1}{2}s\pi[\Omega_0^2 f(t)-\tfrac{1}{4}]-\tfrac{1}{2}s\pi\epsilon_0\Omega_0^2\alpha\cos\theta_k, \tag{11}$$

$\alpha\equiv(1+1/s)e^{-1/s}$, and $\hat{\gamma}\equiv(-\Gamma_-\Gamma_+)^{1/2}$ with $\Gamma_\pm\equiv\Omega_0^2 f(t)-\frac{1}{4}\pm\epsilon_0\Omega_0^2$. In deriving Eq. (11), we have assumed $\epsilon_0<s<1$ to further simplify the analysis. (The present theoretical approach is also applicable to more general cases where, however, the local dispersion relation and the damping rate need to be solved numerically. The details will be given in Ref. [9].) The local dispersion relation, Eq. (10), can be considered as the definition of the WKB ray trajectories in the (θ_k,t) plane. More generally, θ_k can be viewed as the symbol of the differential operator $(-i\partial_t)$ acting on $A(t)$:

$$F(\Omega_0^2 f(t),-i\partial_t)A(t)=0. \tag{12}$$

Thus, $F(\Omega_0^2 f(t),-i\partial_t)$ is a pseudodifferential operator to be interpreted as its series expansion in powers of $(-i\partial_t)$. Equation (12), together with the boundary conditions $|A(t)|\to 0$ as $|t|\to\infty$, defines the most general eigenvalue problem for TAE for arbitrary radial profiles $f(t)$ in the small-shear zero-pressure limit. The calculation can be readily extended to the finite-pressure and large-shear cases. The solution $A(t)$ determines the radial envelope structure. Once Eq. (12) is solved, we have all the ingredients to proceed with the calculation of the damping. Using $\delta\phi_j=A(t)\phi(t,t-j)$ and Eq. (8), the nonresonant part of the Lagrangian can be shown to be

$$\frac{\delta L_{\rm NR}}{2\pi R_0}=2\pi k_\theta^2\frac{2\pi r_0\Delta r_s}{q_0^2R_0^2}\int_{-\infty}^{+\infty}dt\,|A(t)|^2\int_{-\infty}^{+\infty}d\theta\left[\left|\frac{\partial\Phi}{\partial\theta}\right|^2-\Omega_0^2(1+2\epsilon_0\cos\theta)f(t)|\Phi|^2\right][1+s^2(\theta-\theta_k)^2]. \tag{13}$$

For the resonant part, it is possible to show that at a given resonance the main contribution comes from the two poloidal harmonics which are closest to the resonance. The contribution to the resonant Lagrangian coming from the resonance involving the poloidal harmonics l and $l+1$ (which we call resonance "l") is given by

$$\frac{i\delta L_{{\rm R}l}}{2\pi R_0}=k_\theta^2\frac{2\pi r_0\Delta r_s}{q_0^2R_0^2}s^2\int dx\left\{[x^2-\Omega_0^2 f(l)]\left|\frac{\partial}{\partial x}\delta\phi_l\right|^2+[(x-1)^2-\Omega_0^2 f(l)]\left|\frac{\partial}{\partial x}\delta\phi_{l+1}\right|^2-\epsilon_0\Omega_0^2\left(\frac{\partial}{\partial x}\delta\phi_l\frac{\partial}{\partial x}\delta\phi_{l+1}^*+\text{c.c.}\right)\right\}, \tag{14}$$

where $x\equiv t-l$. Obviously, the complete resonant Lagrangian is given by

$$\delta L_{\rm R}=\sum_l\delta L_{{\rm R}l}, \tag{15}$$

where the sum is extended to all resonances. Considering Eq. (6) near the resonance layer, we can show that the corresponding behaviors of $\partial_x\delta\phi_l$ and $\partial_x\delta\phi_{l+1}$ are given by (to lowest order in s)

$$\frac{\partial}{\partial x}\delta\phi_l=\frac{1}{sD}\{C_l\epsilon_0\Omega_0^2+B_l[(x-1)^2-\Omega_0^2 f(l)]\},$$
$$\frac{\partial}{\partial x}\delta\phi_{l+1}=\frac{1}{sD}\{B_l\epsilon_0\Omega_0^2+C_l[x^2-\Omega_0^2 f(l)]\}. \tag{16}$$

In Eqs. (16), B_l and C_l are defined as

$$B_l=[\Omega_0^2 f(l)-\tfrac{1}{4}]A_l-\epsilon_0\Omega_0^2A_{l+1},$$

$$C_l=\epsilon_0\Omega_0^2A_l-[\Omega_0^2 f(l)-\tfrac{1}{4}]A_{l+1},$$

$$D\equiv[x^2-\Omega_0^2 f(l)][(x-1)^2-\Omega_0^2 f(l)]-\epsilon_0^2\Omega_0^4,$$

$A_l\equiv A(t=l)$, and $A(t)$ is the solution of Eq. (12). Carrying out the integration in Eq. (14) as prescribed by causality, we obtain

$$\frac{\delta L_{\rm R}}{2\pi R_0}=-\pi k_\theta^2\frac{2\pi r_0\Delta r_s}{q_0^2R_0^2}\sum_l\lambda_l|\Omega_0^2 f(l)-\tfrac{1}{4}|\left[|A_l|^2+|A_{l+1}|^2-2\frac{\epsilon_0\Omega_0^2}{\Omega_0^2 f(l)-\frac{1}{4}}\text{Re}(A_lA_{l+1}^*)\right][1+O(\delta)], \tag{17}$$

where $\lambda_l=\{[\Omega_0^2 f(l)-\frac{1}{4}]^2-\epsilon_0^2\Omega_0^4\}^{1/2}$ and $\delta=\max(s^2,\epsilon_0\Omega_0^2)$.

Using the variational formulation of the problem, the global dispersion relation for the TAE can be written as

$$\delta L_{\rm NR}+i\delta L_{\rm R}=0. \tag{18}$$

The eigenvalue is $\Omega \equiv \Omega_0 + i\gamma$. Ω_0 is the solution of the eigenvalue problem defined by Eq. (12) and is given by $\Omega_0^2 \simeq \frac{1}{4}[1-\epsilon_0(1-s^2\pi^2/8)]$. To next order, γ is given by

$$\frac{\gamma}{\Omega_0} = -\frac{\delta L_R}{\Omega_0 \partial(\delta L_{NR})/\partial \Omega_0}. \tag{19}$$

Equation (18), with δL_{NR} and δL_R given by Eqs. (13) and (15), is valid for an arbitrary profile function $f(t)$ in the $s<1$ limit.

In order to obtain a quantitative estimate of the magnitude of the damping due to resonant absorption, we consider, in the following, the special case of a linear profile $f(t) = 1 + at$, with

$$|a| \equiv \frac{\Delta r_s}{L_A} = \frac{1}{nq_0' L_A} = \frac{r_0}{L_A}\frac{1}{nq_0 s}, \tag{20}$$

where $L_A^{-1} \equiv |\partial \ln(q^2/v_A^2)/\partial r|$. The damping rate can be calculated analytically [9] and is given by

$$\left(\frac{\gamma}{\Omega_0}\right)_r = -\frac{|a|^{3/2}}{8\pi^2}\left(\frac{\pi}{\epsilon_0 s^2}\right)^{1/2}[e^{-2T}(1+e^{-\mathrm{arccosh}(1/a)})^2 + e^{-2R}(1-e^{-\mathrm{arccosh}(1/a)})^2](1-a^2)^{-1/2}[\mathrm{arccosh}(1/a)]^{-3/2}, \tag{21}$$

where $\Omega_0 \approx \frac{1}{2}$ and

$$T = \frac{\epsilon_0 s^2 \pi^2}{8|a|}\left\{\mathrm{arccosh}\left(\frac{1}{a}\right)\left(1+\frac{a^2}{2}\right) - \frac{3}{2}(1-a^2)^{1/2}\right\}, \tag{22}$$

$$R = \frac{2\epsilon_0}{|a|}\left\{\left[1-\frac{s^2\pi^2}{16}\left(1+\frac{a^2}{2}\right)\right]\mathrm{arccosh}\left(\frac{1}{a}\right) - \left(1-\frac{a^2}{4}\right) + \frac{s^2\pi^2}{32}\left(3+\frac{a^2}{2}\right) + \frac{1}{2}\left[1-\frac{s^2\pi^2}{8}\left(1+\frac{a^2}{2}\right)\right]\ln\left(1+\frac{16}{s^2\pi^2}\right)\right\}. \tag{23}$$

For $s<1$, $(\gamma/\Omega_0)_r$ then reduces to the following characteristic scaling:

$$\left(\frac{\gamma}{\Omega_0}\right)_r \approx -0.0225\frac{\epsilon_0}{s}\left(\frac{R_0}{nq_0 L_A}\right)^{3/2}\frac{\exp[-\frac{1}{4}\pi^2 nq_0 s^3 (L_A/R_0)\mathrm{arccosh}(1/a)]}{(1-a^2)^{1/2}[s\,\mathrm{arccosh}(1/a)]^{3/2}}. \tag{24}$$

The damping rate given by Eq. (21) must be compared with the electron Landau damping $(\gamma/\Omega_0)_L \approx -(\beta_e m_e/m_i)^{1/2}$. For typical ITER and BPX parameters, $(\gamma/\Omega_0)_L$ is approximately 3.7×10^{-3} at $\beta_e \approx 0.05$. On the other hand, for $r_0/L_A \approx 1$, $\epsilon_0 n \approx 1$, $s \approx 0.5$, $n \approx 5$, we have $(\gamma/\Omega_0)_r \approx -0.015$, which is a factor ≈ 5 larger than the electron Landau damping rate. This fact has relevance to the stabilization of high-n modes excited by alpha particles [4,5]. The complete dispersion relation, including circulating alpha-particle excitation and Landau and Alfvén resonant damping, is

$$\left(\frac{\gamma}{\Omega_0}\right) \simeq \beta_\alpha f_\alpha k_\theta \rho_\alpha \frac{R_0}{L_{p_\alpha}} - \left(\frac{\gamma}{\Omega_0}\right)_L - \left(\frac{\gamma}{\Omega_0}\right)_r, \tag{25}$$

with ρ_α being the Larmor radius of the alpha particles, L_{p_α} the scale length of the alpha-particle pressure profile, f_α the fraction of resonant particles, and $k_\theta\rho_\alpha \sim \epsilon_0/s$. With resonant damping effects taken into account, the critical beta value for the alphas to drive the modes unstable is

$$\beta_{\alpha,c} \sim \left(\frac{v_\alpha}{v_A f_\alpha}\right)\left(\frac{\gamma}{\Omega_0}\right)_{r+L} \sim O(10^{-2}, 10^{-1}). \tag{26}$$

The resonant damping mechanism is most effective at small or moderate n, i.e., $n \approx \epsilon_0^{-1}$, due to the fact that the envelope amplitude is exponentially small at the resonance. This causes the poloidal number of spectrum of TAE to be at high n, where the most unstable modes occur [4,5].

This work was supported by the U.S. Department of Energy Contract No. DE-AC02-76-CH3072. Useful discussions with M. N. Rosenbluth, H. L. Berk, and J. W. Van Dam, especially regarding the $|a|^{3/2}$ scaling of Eq. (21), are gratefully acknowledged.

[a] Permanent address: Associazione EURATOM-ENEA sulla Fusione, C.P. 65-00044 Frascati, Rome, Italy.

[1] C. Z. Cheng, Liu Chen, and M. S. Chance, Ann. Phys. (N.Y.) **161**, 21 (1985).
[2] C. Z. Cheng and M. S. Chance, Phys. Fluids **29**, 3695 (1986).
[3] D. J. Sigmar, C. T. Hsu, R. White, and C. Z. Cheng, MIT Plasma Fusion Center Report No. PFC/JA-89-58, 1989 (unpublished).
[4] Liu Chen, in *Theory of Fusion Plasma,* edited by J. Vaclavik, F. Troyon, and E. Sindoni (Centre de Recherche Paul Pascal, Talence, 1988), p. 327.
[5] Liu Chen *et al.*, in *Proceedings of the Twelfth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Nice, France* (IAEA, Vienna, 1989), Vol. 2, p. 77.
[6] G. Y. Fu and J. W. Van Dam, Phys. Fluids B **1**, 1949 (1989).
[7] G. Y. Fu and C. Z. Cheng, Phys. Fluids B **2**, 985 (1990).
[8] F. Zonca and L. Chen, Bull. Am. Phys. Soc. **35**, 2069 (1990). In an independent work, M. N. Rosenbluth *et al.* [following Letter, Phys. Rev. Lett. **68**, 596 (1992)] employing a different theoretical approach, have derived similar results.
[9] F. Zonca and L. Chen (to be published).
[10] J. W. Connor, R. J. Hastie, and J. B. Taylor, Proc. R. Soc. London A **365**, 1 (1979).