Minimal Electroweak Model for Monopole Annihilation

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We construct the minimal (most economical in fields) extension of the standard model implementing the Langacker-Pi mechanism for reducing the grand unified theory (GUT) monopole cosmic density to an allowed level. The model contains just a single charged scalar field in addition to the standard Higgs doublet, and is easily embeddable in any GUT. We identify the region of parameter space where monopoles annihilate in the high temperature early Universe. A particularly alluring possibility is that the demise of monopoles at the electroweak scale is in fact the origin of the Universe's net baryon number.

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Our knowledge of terrestrial particle physics has made a profound impact on our understanding of the early history of the Universe. At critical temperatures, phase transitions occur, and symmetries restore or break. If a detailed balance of reactions is lost, particle decay and annihilation out of thermal equilibrium can generate entropy. Phase transitions may also have topological implications. The formation or destabilization of monopoles, cosmic strings, and/or more exotic topological objects may occur when the Universe passes through a phase transition. In particular, if the $SU_L(2)\times U_Y(1)$ standard model (SM) is the low-energy effective theory of a grand unified theory (GUT), then the phase transition at which the U(1) subgroup appears is a source of an embarrassing overabundance of monopoles. However, if the U(1) breaks at a later phase transition, the monopole-antimonopole annihilation rate may be sufficiently rapid to eliminate the "GUT monopole problem" [1]. At a still lower temperature another change of phase must occur to restore electromagnetic $U_{EM}(1)$; this implies that the epoch of broken U(1) is temporary. In this Letter we construct a minimal model which solves the monopole problem via a temporarily broken U(1) phase. Remarkably, the minimal model is just the minimal SM plus a charged Higgs singlet. We will discuss the cosmology of this model and its embedding in GUTs, and comment on its phenomenology.

The present value of the monopole-to-entropy ratio r_M can be bounded in several ways [2]. The most certain bound comes from requiring that the monopole mass density does not overclose the Universe. If the U(1) subgroup first appears at the GUT phase transition, one obtains a monopole mass $M\sim 10^{16}$ GeV, which translates into a bound of $r_M \leq 10^{-24}$. More stringent bounds come from more speculative arguments. Observational limits on x-ray luminosity from neutron stars translate into the bounds $r_M \leq 10^{-32}\times(1\text{ mb})/\sigma_{\Delta B}$ if monopoles do not cluster on galaxies, and $r_M \leq 10^{-37}\times(1\text{ mb})/\sigma_{\Delta B}$ if they do. Here $\sigma_{\Delta B}$ is the cross section for monopole catalyzed baryon decay, predicted to be ~ 1 mb in GUT models [3]. When the symmetry breaks to a phase with no U(1) factors, the vacuum expels magnetic flux and the monopoles and the antimonopoles quickly begin to annihilate as they find themselves at the opposite ends of strings with enormous tension. Using causality arguments, Weinberg [4] argued that the monopole-to-entropy ratio surviving the broken U(1) epoch is $r_M \geq 10^{-2}(T_3/M_{Pl})^2$, where T_3 is the (final, in our model) phase-transition temperature where $U_{EM}(1)$ is restored, and M_{Pl} is the Planck mass. Thus the mass density bound is satisfied with $T_3 \leq 10^8$ GeV. If $\sigma_{\Delta B}$ is of typical strong interaction size, then the first bound derived from the neutron-star x-ray luminosity limit is satisfied if $T_3 \leq 10^4$ GeV, and the second bound is satisfied if $T_3 \leq 30$ GeV. It was subsequently shown by Vilenkin [5] that monopoles annihilate even faster than the rates inferred from naive causality. Long strings are expected to have a "Brownian" shape and relativistic transverse velocities. They are cut to shorter bits when they intercommute. It is also possible that longitudinal motion simply breaks the long strings. At each cut or break a new monopole-antimonopole pair appears. Through this mechanism, long strings quickly become exponentially suppressed, leaving shorter strings and a concomitantly shorter annihilation time for the monopole and antimonopole at the string ends. Thus it appears that U(1) restoration at a *weak* scale temperature (100 GeV–1 TeV) is sufficient to satisfy even the most stringent monopole bound. Note that the duration of the broken U(1) epoch is not so important; since the Universe is decelerating, or equivalently, since $t\sim 1/T^2$, almost all annihilation occurs just above the restoration temperature T_3. We now turn to our field-theoretic derivation of the broken U(1) epoch.

Previously we studied some generic features of phase transitions in the early Universe [6,7], in the context of a simple $U_A(1)\times U_B(1)$ model with a complex scalar field in each of the U(1)'s. Various symmetry-breaking patterns emerged, depending on the parameters of the model. We argued that many qualitative features of the $U_A(1)\times U_B(1)$ model should transfer to a phenomenologically realistic $SU_L(2)\times U_Y(1)$ model. In this work we consider an $SU_L(2)\times U_Y(1)$ model with a standard Higgs doublet (Φ) and a charged scalar singlet (U). Although the Φ has both $SU_L(2)$ and $U_Y(1)$ charges, the Higgs potential has the same general form as that of Ref. [6]. At zero temperature,

$$V_0(\Phi,U)=-\mu_2^2\Phi^*\Phi-\mu_1^2U^*U+\lambda_2(\Phi^*\Phi)^2 +\lambda_1(U^*U)^2+2\chi(U^*U)(\Phi^*\Phi). \quad (1)$$

The subscripts 1 or 2 on parameters refer to the singlet and doublet nature of U and Φ under the $SU_L(2)$ subgroup. The parameters μ_1^2 and μ_2^2 are chosen to be positive. In order to discuss cosmology, we also need the one-loop finite temperature correction to the above potential, which is [8]

$$\Delta V=T^2(\beta_2\Phi^*\Phi+\beta_1U^*U), \quad (2)$$

where

$$\beta_2=\frac{1}{48}\left[8(3\lambda_2+\chi)+3(3g^2+g'^2)+4\sum_f N_C G_{2f}^*G_{2f}\right] \quad (3)$$

and

$$\beta_1=\frac{1}{48}\left[16(\lambda_1+\chi)+3g'^2Y_U^2+4\sum_f N_C G_{1f}^*G_{1f}\right]. \quad (4)$$

Here g and g' are the standard $SU_L(2)$ and $U_Y(1)$ gauge coupling constants, respectively; Y_U is the hypercharge of the U particle; G_{1f} and G_{2f} are the Yukawa couplings between the Higgs singlet and doublet, and the matter fermions; and N_C is 3 for quarks and 1 for leptons. $V_T=V_0+\Delta V$ is bounded from below if $\lambda_1>0$, $\lambda_2>0$, and $\chi>-(\lambda_1\lambda_2)^{1/2}$. We make the simplifying but unnecessary assumption that the full symmetry of the theory [here $SU_L(2)\times U_Y(1)$] is restored at high temperature. This requires $\beta_1>0$ and $\beta_2>0$. Spontaneous symmetry breaking is signaled when T^2 drops below either μ_2^2/β_2 or μ_1^2/β_1.

In Ref. [6] we showed that an epoch of temporary but totally broken symmetry occurs in the region of parameter space delimited by

$$\frac{\beta_1}{\beta_2}<r \text{ and } \lambda_2 r<\chi<\min\left[\frac{\lambda_1}{r},(\lambda_1\lambda_2)^{1/2}\right], \quad (5)$$

where $r=\mu_1^2/\mu_2^2$ and λ_1, λ_2, μ_1^2, μ_2^2, β_1, and β_2 are all positive. Given any point in this parameter region, the cosmological history of the SM contains four different vacuum phases.

$0\le T\le T_3$: SU(2)×U(1) is broken to $U_{EM}(1)$ in the usual way, i.e., $\langle\Phi\rangle$ is nonzero and $\langle U\rangle$ is zero.

$T_3\le T\le T_2$: SU(2)×U(1) is completely broken; both $\langle\Phi\rangle$ and $\langle U\rangle$ are nonzero.

$T_2\le T\le T_1$: U(1) is broken but SU(2) is not; $\langle U\rangle$ is nonzero but $\langle\Phi\rangle$ is zero.

$T_1\le T$: SU(2)×U(1) symmetry is completely restored; $\langle\Phi\rangle$ and $\langle U\rangle$ are zero.

The absence of an Abelian symmetry in the interval $T_3\le T\le T_1$ reveals this temperature range as the epoch of monopole annihilation. The three critical temperatures are simple functions of the parameters $\{\mu_1^2,\mu_2^2,\lambda_1,\lambda_2,\chi,\beta_1,\beta_2\}$ [6]. It may be of interest to note here that in the $T\in[T_3,T_2]$ epoch, both diagonal generators T^3 and Y are broken, there is no conserved charge, and even electric-charge nonconserving processes may take place.

In Fig. 1 we show the vacuum expectation values (VEVs) v_1 $(=\sqrt{2}\langle U\rangle)$ and v_2 $(=\sqrt{2}\langle\Phi\rangle)$ and the physical Higgs boson masses, M_H and M_U, as a function of cosmic temperature, for a typical set of parameters satisfying Eq. (5). We have neglected all Yukawa couplings except the one coupling Φ to the top quark, $G_{2t}=m_{top}/(175\text{ GeV})$; we have taken $m_{top}=100$ GeV. The gauge couplings have standard values $g=0.65$ and $g'=0.34$. Assigned values are $\lambda_2=0.184$, $\lambda_1=0.444$, $M_H(T=0)=\sqrt{2}\mu_2=150$ GeV, $\mu_1=116$ GeV, $\chi=0.263$, and $Q_U=Y_U/2=1$. The derived values are $\beta_2=0.304$, $\beta_1=0.264$, and $M_U(T=0)=51.4$ GeV. A consequence of the constraints in Eq. (5) is the inequality T_2

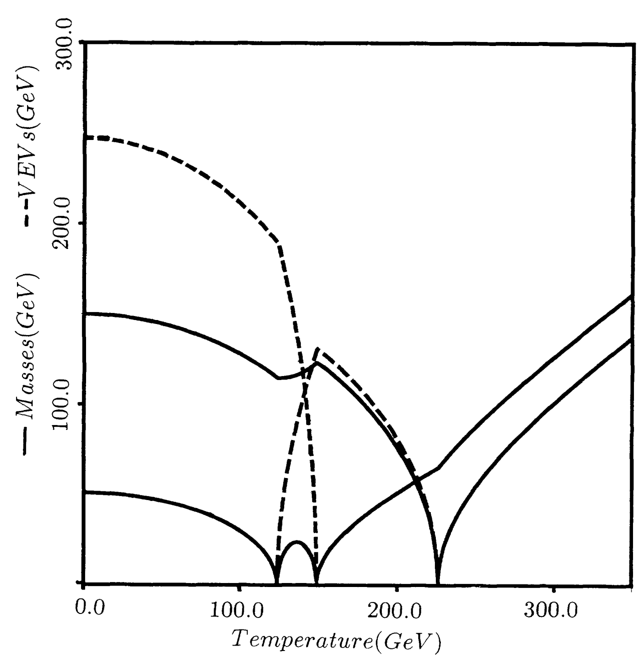

FIG. 1. The dashed lines are the doublet and singlet VEVs v_2 and v_1 as a function of cosmic temperature. $SU_L(2)$ is broken below T_2, and $U_Y(1)$ is broken in the interval $[T_3,T_1]$. Monopoles annihilate during the broken $U_Y(1)$ phase. The turn-on of v_2 drives v_1 to zero at low temperature, restoring electromagnetic U(1) as an exact symmetry. The solid lines are the temperature-dependent Higgs boson masses.

$< M_H(T=0)/(2\beta_2)^{1/2} < T_1$. Since $(2\beta_2)^{1/2}$ is ~ 1, this inequality implies that $M_H(T=0)$ is an indicator of the temperature scale where $U_Y(1)$ was broken and monopoles were annihilated.

Because monopole annihilation occurs very rapidly in the interval $[T_3, T_1]$, this interval need not be large. However, if the interval is small, one might worry that corrections from the two- (and higher-) loop effective potential might in fact remove the epoch completely. The temperature interval in Fig. 1 is large enough that such higher-loop effects are of no concern here. We have stepped through the parameter space of Eq. (5) to test the stability of the interval $[T_3, T_1]$. We found that relatively light M_U mass is preferred—the interval shrinks with increasing M_U— and raising M_H is of little value. Thus a charged scalar with mass $M_U \leq 100$ GeV seems to be a loose prediction of this model. On the other hand, raising the top mass to 200 GeV roughly doubles the value of β_2, thereby increasing the parameter space of Eq. (5).

The model presented here easily fits into grand unified theories. Consider first the unified SU(5) group. The only SU(5) Higgs irreducible representations (irreps) with a charged $SU_C(3) \times SU_L(2)$ singlet U that couple to matter fermions are a $\mathbf{10}_H$ with $Y_U = \pm 2$ and a $\mathbf{50}_H$ with $Y_U = \pm 4$. The operators coupling the scalars to the fermions are $5_F 5_F \overline{\mathbf{10}}_H$ and $\overline{\mathbf{10}}_F \overline{\mathbf{10}}_F \mathbf{50}_H$. Each operator violates the conventional fermion number by two units; we may, however, define an unconventional conserved fermion number by assigning the U scalar a fermion number equal to 2. Fermi statistics and group theory require the Yukawa couplings for the $\overline{\mathbf{10}}_H$ $(\mathbf{50}_H)$ to be antisymmetric (symmetric) in family space. Furthermore, hypercharge (or equivalently, electric charge) assignments are such that only couplings to leptons are allowed. An example of the $5_F 5_F \overline{\mathbf{10}}_H$ operator [9] with nonzero coupling is $e^- \nu_\mu U^+$ and an example of $\overline{\mathbf{10}}_F \overline{\mathbf{10}}_F \mathbf{50}_H$ is $e^+ e^+ U^{--}$. It is also an option to decouple the U particle from matter altogether, by assigning it to an irrep having $|Y| \neq 2,4$, or by imposing the discrete symmetry $U \to -U$. This gives a much-longer-lived U particle.

These results generalize easily to O(10) and E_6 grand unified theories, with fermions in the $\mathbf{16}_F$'s and $\mathbf{27}_F$'s, respectively. If the U particle couples directly to leptons, then it must reside in a Higgs irrep that when reduced to SU(5) contains a $\mathbf{10}_H$ or a $\mathbf{50}_H$. In O(10) the only charged singlets coupling to fermions are in a $\overline{\mathbf{10}}$ from a **120** or in a $\overline{\mathbf{10}}$ or a **50** from a $\overline{\mathbf{126}}$. In E_6 a similar analysis leads to a charged singlet in an O(10) **120** from a **351** or in an O(10) $\overline{\mathbf{126}}$ from a **351**′. Some larger irreps of SU(5) containing an $SU_C(3) \times SU_L(2)$ charged Higgs singlet are listed in Table I. The next smallest irrep beyond the $\mathbf{50}_H$ is the $\mathbf{175}_H$ of Dynkin label (1101) or the $\mathbf{175}''_H$ of Dynkin label (0300). Since the **175**, **175**″, and higher irreps do not appear in the reduction of $\mathbf{16}_F \times \mathbf{16}_F$ of O(10) or $\overline{\mathbf{27}}_F \times \overline{\mathbf{27}}_F$ of E_6, the charged Higgs singlets in these irreps do not couple to the standard fermions.

TABLE I. Charged $SU_C(3) \times SU_L(2)$ Higgs singlets in SU(5).

Dynkin label	Irreps	Electric charge
(0100)	**10**	1
(0020)	**50**	−2
(1101)	**175**	1
(0300)	**175**″	3
(0210)	**315**	1
(0040)	**490**	−4
(1021)	**720**	−2
(0130)	**980**	−2

At low energy after the electroweak breaking, U can couple to the Z via the electroweak mixing. Thus it can be pair produced from the Z if kinematically allowed. If the U particle does not couple to the fermions, it will be quasistable. The decay rate of $Z \to UU^*$ is

$$\Gamma(Z \to UU^*) = \tfrac{1}{4} \alpha_{EM} Q_U^2 \tan^2\theta_W \times M_Z (1 - 4M_U^2/M_Z^2)^{3/2} \theta(M_Z - 2M_U) . \quad (6)$$

For small M_U, $\Gamma(Z \to UU^*) \sim 0.257 Q_U^2 \Gamma(Z \to \nu\bar{\nu})$. We conclude that either $M_U \geq 45$ GeV, or the measured Z width places an upper bound on its hypercharge (which is not *a priori* fixed by the theory, though the relation $Q_U = Y_U/2$ still holds).

In summary, we have demonstrated by explicit construction that the standard model plus a single charged $SU_L(2)$ singlet scalar field can undergo an early Universe phase of broken $U_Y(1)$ and $U_{EM}(1)$. During this epoch, the Universe is superconducting, and magnetic flux is confined to strings, where tension pulls the end-point monopole and antimonopole together. Annihilation occurs and the GUT monopole problem is solved. This solution was originally implemented by Langacker and Pi [1], who introduced three Higgs doublets into the standard model. Any model with more than one charged scalar field is potentially a solution to the monopole problem. Our implementation is the minimal solution. We have shown how our charged singlet fits naturally into the grand unified theories which give rise to GUT monopoles. There is another sense in which the existence of the charged singlet may be natural. In the SM, for each left-handed fermion doublet there is a charged right-handed singlet (except for the neutrino). The charged scalar singlet could be the supersymmetric partner of a right-handed charged fermion [10]. If the field is the quark partner, color would also be temporarily broken in the early epoch.

Finally, we speculate on mechanisms by which monopole and antimonopole annihilation might in fact provide the observed net excess of baryon number in the Universe. The observationally inferred value of n_B/s is $\sim 10^{-10}$, where n_B is the net baryon density and s is the entropy density. Monopole annihilation will certainly increase s; the monopole will annihilate into lighter species, and the strings connecting monopoles will form cusps which radiate energy [11]. However, the monopole annihilation may be responsible for, or at least contribute to, n_B. We list three possible mechanisms: (i) GUT-mass monopole annihilation at the electroweak scale $T \sim 1$ TeV is an out-of-equilibrium reaction, and so the GUT fields interior to the monopole can produce net n_B if the GUT theory is B and CP violating [12]. (ii) The monopole is an extended configuration of scalar and gauge fields, as is the sphaleron solution [13] connecting degenerate vacua having different baryon number. The transition amplitude from the extended monopole-antimonopole solution to the sphaleron should be estimated to see if it is non-negligible. (iii) String cutting/breaking produces a very large monopole density prior to annihilation, and annihilation in turn produces a very large light particle density. The forward scattering of the latter on the former is thought to be an efficient mechanism (termed "monopole catalysis") of B production [8].

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