Proposal for an Absolute, Atomic Definition of Mass

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It is proposed that the mass of a particle be defined by its de Broglie frequency, measured as $c^2/\lambdabar\gamma v$, where $\lambdabar$ is the mean de Broglie wavelength of the particle when it has mean speed v and Lorentz factor γ; the masses of systems too large to have a measurable $\lambdabar$ are then to be derived by specifying the usual inertial and additive properties of mass. This avoids the use of an arbitrary macroscopic standard such as the prototype kilogram, and does not even require the choice of a specific particle as a mass standard. Suggestions are made as to how this absolute mass can be realized and measured at the macroscopic level and comments are made on the effect of the new definition on the form of the equations of physics.

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Among the three fundamental quantities of physics —time, length, and mass—time is now defined in terms of a chosen atomic standard frequency (at present, that of the ^{133}Cs ground-state hyperfine transition) and length via a defined value of c, the velocity of light in vacuo, but there is not yet an agreed atomic standard of mass [1,2]. Inertial masses are specified only in a relative manner, the ratio of two masses being defined as the inverse of the ratio of their accelerations when they are subjected to the same force, and so it is necessary to specify an arbitrary standard of mass, namely, that of a certain piece of platinum-iridium alloy, the prototype kilogram, carefully preserved at the Bureau Internationale des Poids et Mesures at Sèvres, France. (In practice [3], one makes use of the principle of equivalence and the assumption of the additivity of mass, and compares gravitational masses.) In this paper we propose an operational definition of mass which is absolute—that is, does not need an arbitrary standard (other than the standards of time and length) —and which, thanks to the achievements of modern metrology, can be accurately related to the conventional SI definition.

In classical mechanics, the fact that the conventional SI mass (hereafter denoted by $\bar{m}$) is defined only relatively implies that classical equations remain unaltered if we multiply all masses—and all quantities proportional to mass, such as kinetic energy and momentum—by an arbitrary factor; that is, $\bar{m}$ has an arbitrary scale and arbitrary dimension M. [If explicit expressions for forces, potential energies, etc., are used, the coupling constants $\bar{e}^2$, G, etc. (see below) must also be scaled by the same factor or its reciprocal.] In quantum mechanics, on the other hand, $\bar{m}$ cannot be arbitrarily scaled in this way; but quantum equations can always be rearranged so that any $\bar{m}$ (or quantity proportional to mass) appears as (or is proportional to) the ratio $\bar{m}/\hbar$—or as the dimensionally more convenient quantity $\bar{m}c^2/\hbar$, which will be referred to as the de Broglie (angular) frequency [4].

It is proposed that the "natural mass" m of a particle be defined [5] as its de Broglie frequency. Operationally, m can be determined (independently of $\bar{m}$) by measuring the mean reduced de Broglie wavelength $\lambdabar = \lambda/2\pi$ of a beam of almost monoenergetic particles with known mean speed v relative to the observer and using the definition

$$m \equiv c^2/\lambdabar\gamma v$$

[where $\gamma = (1 - v^2/c^2)^{-1/2}$]; according to the de Broglie wavelength relation $\lambdabar = \hbar/\bar{m}\gamma v$, the quantity m should be constant and related to $\bar{m}$ by $m = \bar{m}c^2/\hbar$.

This definition of mass has a number of advantages. First, there is no need to specify and preserve an arbitrary macroscopic mass standard, or even to choose a particular kind of particle as the standard. It is an absolute definition of mass, rather than merely a definition of mass ratios, and eliminates the unspecified dimension M of $\bar{m}$ from physical quantities. Finally, the definition is entirely in terms of kinematic quantities—time and length —which can be referred directly to the SI standards and measured with great accuracy. Mass is defined here in terms of frequency, the physical quantity which can be measured with the highest precision—although, of course, the value of m, even for the electron, the lightest accessible material particle, is far too high to be measured directly, and one must make use of submultiples of the frequency, such as are provided by measurements of the de Broglie wavelengths of slow particles.

Any definition must, of course, be checked for consistency and uniqueness. In particular, one must ask whether the above definition is consistent with the two required properties of mass, namely, its inertial property (acceleration inversely proportional to mass for a given force) and its additive property (mass of composite system equal to the sum of masses of constituents minus the mass equivalent of their binding energy). The first property is used in the conventional definition and measurement of mass ratios, and the second is an empirical result which amounts to a combination of the law of conservation of energy with Einstein's mass-energy equivalence relation; both properties are needed here in order to extend the definition of absolute mass to systems whose de Broglie wavelengths cannot be directly measured.

Formally, the consistency of the definition follows from the fact that all we have done is to exploit the arbitrariness in the scale of SI mass $\bar{m}$, that is, to multiply both sides of conventional physics equations by common factors involving c and $\hbar$ and to refer to the frequencies $m_i = \bar{m}_i c^2/\hbar$ that appear as "absolute masses." It is nevertheless very desirable to carry out certain experimental checks. The constancy of $\lambda\gamma v$ for a given particle—that is, the validity of the de Broglie relation—should be tested as accurately as possible. One should, moreover, check experimentally that the ratios of m values obtained from direct de Broglie wavelength measurements on particles and composite systems are indeed equal to the ratios of the corresponding conventional $\bar{m}$ values obtained by assuming the inertial and additive properties of mass (for example, from mass spectrometer or nuclear reaction studies). For composite systems such as ^{3}He, ^{4}He, and H_2 the additivity of de Broglie frequencies has so far been directly checked to an accuracy of about 1% [6].

The required constancy of the ratio $\bar{m}/m$ is tantamount to the assumption that Planck's constant $\hbar$ is the same for all particles; a recent study [7] of the measured values of $\hbar$ derived from various phenomena involving electromagnetic radiation ($\hbar_\gamma$), the electron ($\hbar_e$), and the neutron ($\hbar_n$) concludes that

$$\hbar_e/\hbar_\gamma = 1 + (30 \pm 13) \times 10^{-8},$$

$$\hbar_n/\hbar_\gamma = 1 + (7 \pm 40) \times 10^{-8}$$

(the errors corresponding to 1 standard deviation), so that

$$\hbar_e/\hbar_n = 1 + (23 \pm 42) \times 10^{-8},$$

consistent with a value of $\hbar$ independent of the type of particle.

We now turn to the question of the practical realization of the definition, which in the first instance applies only to elementary particles and other systems whose de Broglie wavelengths can be directly measured; one needs a chain of measurements leading all the way from such light objects to macroscopic objects which can serve as practical secondary standards.

The best starting point for the chain is the neutron, for which corresponding values of mean λ and v can be measured with high accuracy and with relatively little correction for the effects of stray external fields; indeed, one of the main reasons for making this proposal now has been the recent precision measurement [8] of $\lambda\gamma v$ for slow neutrons by Krüger, Nistler, and Weirauch of the Physikalisch-Technische Bundesanstalt (PTB) at Braunschweig, Germany. Working with the high-flux reactor at the Institut Laue-Langevin in Grenoble, they measured $\lambda\gamma v = 6.296\,224\,3(25) \times 10^{-8}$ m^2 s^{-1}, thus determining the absolute mass of the neutron as

$$m_n = 1.427\,451\,02(57) \times 10^{24}\ \mathrm{s}^{-1}\ (0.4\ \mathrm{ppm}),$$

and they anticipate that, with further refinements, the uncertainty can be made even smaller.

From this measurement one can find the masses of other particles and atomic systems by using accurately measured mass ratios and the assumption of the additivity of mass in composite systems, with appropriate corrections for binding energy. High-precision mass spectrometry and nuclear reaction Q-value measurements provide values of atomic masses, relative to the neutron mass, with uncertainties which, in many cases, are of order 0.05 ppm or better [9].

Given accurate measurements of the absolute masses of neutral atoms, one could then construct secondary standards for use in macroscopic mass measurements —crystals of accurately measured volume and lattice spacing for which one can determine the number of atoms, and hence the mass of the crystal by additivity. (The binding energy of an atom of mass number A in a crystal is very small compared with the atomic mass [a few eV compared with $\sim A(10^9$ eV)] and could be ignored in the context of the presently available accuracy of m_n.)

The metrological problems involved in constructing such secondary standards are similar to those encountered in the equivalent task of determining Avogadro's number, and can be tackled in a similar way [10]. That procedure is based on the precise determination [11] of a well-defined length in the angstrom range: the (220) lattice spacing of a silicon crystal with specified isotopic abundances (also used in the m_n measurement [8]), determined so far to 0.2-ppm precision by optical measurement of the shift of x-ray fringes in a Bonse-Hart interferometer [12]. The volume of the crystal can be obtained by interferometric methods—either directly or by comparison of its buoyancy in a fluorocarbon fluid with that of a sphere of accurately measured radius. Both measurements are thus referred to the common scale of optical wavelength.

The use of silicon as a secondary mass standard is, however, complicated by the fact that it contains three stable isotopes whose relative proportions would need to be measured to a precision comparable with that desired for mass. One could, alternatively, use crystals composed of elements containing only one stable isotope, for example, ^{9}Be, ^{23}Na, ^{19}F, ^{27}Al, ^{55}Mn, ^{93}Nb, ^{103}Rh, or ^{197}Au. Their lattice spacings could be related to that of silicon by comparing diffraction angles for selected low-energy gamma rays which (unlike characteristic x rays) are sharp and of known symmetric line shape.

Crystal defects of various types [13] must, of course, be carefully minimized and/or monitored. Surface effects (e.g., from contaminant layers) can be reduced by using a sufficiently large crystal. Edge and screw dislocations are expected to create a long-range shear strain, rather than a dilatation; there is, however, a residual "anharmonic" dilatation [14] (not calculable from classical elasticity) of about one atomic volume per lattice spacing along the dislocation line, so that to keep the net dilatation below 0.01 ppm the single crystal should not have more than about 10^7 dislocations/cm^2. Residual chemical impurities can be allowed for in the mass calculation if one measures their concentrations and knows their locations (substitutional versus interstitial). Frenkel vacancy-interstitial pairs should not cause a significant change in macroscopic crystal density; Schottky vacancies, arising from the migration of atoms from lattice sites to the crystal surface, will do so, but their equilibrium concentration is temperature dependent and their effects can be corrected for by measuring the ratio of crystal size to lattice spacing as a function of temperature and extrapolating to absolute zero. Finally, a given crystal may contain unexpected defects, such as voids, arising from irregularities in its growth; the best check for these is probably the comparison by weighing of the calculated masses of crystals prepared in different ways and, preferably, made of different chemical elements.

One can, alternatively, adopt a method [15] in which the mass of a macroscopic object is related to the mass of the electron—and hence, by the 0.02-ppm Penning trap [16] measurement of the proton-electron mass ratio, to the masses of its atoms—by the comparison of mechanical and electrical work, with the electrical units being defined in terms of fundamental constants via the Josephson and quantized Hall effects.

The construction of acceptable secondary standards will require a considerable effort in metrology and, if the single-crystal method is used, in the characterization of solid-state defects; the problems of their preservation, once the numbers of atoms in them are determined, are similar to those [3] surrounding the prototype kilogram but, it should be emphasized, refer only to secondary standards—the primary definition of mass requires no man-made standard and is permanent, indestructible, and universally accessible.

Several people [2] have proposed a redefinition of the kilogram as the mass of a defined number of specified elementary particles or microscopic systems (e.g., electrons or ^{28}Si atoms); our definition of mass as de Broglie frequency goes further in that, if the present theory is correct, there is no need even to specify a particular microscopic entity as an atomic mass standard. Moreover, its adoption would in principle eliminate all base units except the fundamentally irreducible units of time and length.

If we use m (or the inverse Compton wavelength m/c) instead of $\bar{m}$ in the equations of physics, all coupling constants become dimensionless or have the dimensions of length or time; in particular, there is no need [5] to define a standard of electric charge with its own dimension Q. For example, the classical nonrelativistic equation of motion of particle 1 (position $\mathbf{x}_1$, charge $Z_1\bar{e}$, mass $\bar{m}_1$) under the electrostatic and gravitational forces exerted on it by particle 2 (position $\mathbf{x}_2$, charge $Z_2\bar{e}$, mass $\bar{m}_2$), expressed in terms of SI quantities,

$$\bar{m}_1\frac{d^2\mathbf{x}_1}{dt^2}=\left[\frac{Z_1Z_2\bar{e}^2}{4\pi\varepsilon_0}+G\bar{m}_1\bar{m}_2\right]\frac{\mathbf{x}_1-\mathbf{x}_2}{|\mathbf{x}_1-\mathbf{x}_2|^3},$$

where G is Newton's gravitational constant, becomes

$$\frac{m_1}{c}\frac{d^2\mathbf{x}_1}{d(ct)^2}=\left[Z_1Z_2\alpha+\gamma_P^2\frac{m_1}{c}\frac{m_2}{c}\right]\frac{\mathbf{x}_1-\mathbf{x}_2}{|\mathbf{x}_1-\mathbf{x}_2|^3},$$

where $\alpha\equiv\bar{e}^2/4\pi\varepsilon_0\hbar c\approx 1/137.036$ is a dimensionless measure of the strength of electromagnetic coupling and $\gamma_P\equiv(G\hbar/c^3)^{1/2}\approx 1.616\times10^{-35}$ m is the Planck length. From the present viewpoint, it is α, rather than $\bar{e}$, which is measured in electromagnetic processes and γ_P, rather than G, in gravitational experiments; one can define a "charge quantum" $e\equiv\sqrt{4\pi\alpha}$ $[=\bar{e}/(\varepsilon_0\hbar c)^{1/2}]$, but this is dimensionless and does not require the definition of some arbitrary unit of charge, except of course as a convenient secondary standard for use when masses are expressed in terms of the kilogram.

The use of absolute mass and coupling constants such as α and γ_P, instead of SI mass and charge, also eliminates Planck's constant $\hbar$ from all the equations of quantum physics—not by choosing new units of mass, length, and time such that $\hbar=1$, but by keeping the SI units of time and length and recognizing that $\hbar$ is simply a conversion factor connecting absolute mass with SI mass and, from the present viewpoint, is just a property of the standard kilogram. If and when the realization of absolute mass is considerably improved beyond its presently achievable $\sim$1-ppm precision, the prototype kilogram itself would become a secondary standard which could be calibrated in terms of absolute mass [on the basis of the current best value [17] of $\hbar$, 1 kg corresponds to $8.5224585(51)\times10^{50}\,\mathrm{s}^{-1}$].

Finally, we remark that these considerations give rise to some intriguing questions. What physical interpretation is to be given to the de Broglie frequency we have taken as our fundamental definition of mass? And, if the use of this definition leads to the disappearance of $\hbar$ from quantum equations, what is the role of the quantization procedure, whose "scale" is given by $\hbar$? Such questions lie beyond the scope of the present Letter, but we refer to one possible approach [5,18] to them in terms of a picture of particles as extended objects, to be discussed further in a separate paper [19].

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