Measurement-Induced Diffraction and Interference of Atoms

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The position of an atom passing through a standing light wave is localized by making a quadrature phase measurement on the light field. This localization can be thought of as the creation of a virtual slit (or slits) for the atom by the field measurement. Diffraction and interference behavior may be observed in the far field.

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The resolving power of a Heisenberg microscope can be no better than the wavelength of the scattered light it detects. We propose a scheme in which a measurement of a light field will localize the position of an atom to much less than the wavelength of the light [1]. This localization can be thought of as the creation of a virtual slit (or slits) for the atom by the field measurement. Whether diffraction can be observed from such a virtual slit was suggested by Popper [2] as a crucial test of the Copenhagen interpretation of quantum mechanics. In this Letter we present an analysis which predicts both diffraction from a single virtual slit and interference from two such slits.

When a two-level atom is passed through a standing-wave mode in an optical cavity the interaction with the field depends on the position of the atom. By making a quadrature phase measurement on the field it is possible to localize the position of the atom very precisely within a wavelength of the light in the cavity. Depending on the initial position distribution of the atom, a field measurement may localize the atom into one or more "slits."

The use of a similar arrangement has been proposed to measure the photon number of the field, either by measuring the atomic phase [3] or by measuring the atomic deflection [4].

We first demonstrate that by varying the phase of the field quadrature measured we can vary the width of a single slit, and accordingly vary the width of the far-field diffraction pattern. Of course this slit is not a real physical slit which the atom passes through, but a virtual slit created by our knowledge from the field measurement of where the atom is. Popper [2] proposed a scheme which he claimed would create such a virtual slit of adjustable width, and suggested that such a scheme would test whether knowledge alone is sufficient to create uncertainty (as is contended under the Copenhagen interpretation), or whether scattering of a particle depends on the physical presence of a slit. Popper's proposed experiment does not, however, provide such a test, due to other uncertainties inherent in the scheme which Popper had not included in his analysis [5]. We claim that our position localization scheme can provide an experimentally realizable test to resolve Popper's original question, namely, whether knowledge alone is sufficient to create uncertainty.

Secondly, we show that particular field measurements can localize the single atom into two or more spatially distinct regions, and interference can occur from such virtual slits.

Using the rotating-wave approximation, the Hamiltonian for the system is

$$H=\hbar\omega_a a^\dagger a+\hbar\omega_0\sigma_z+p^2/2m + \hbar(ga^\dagger\sigma^-+g^*\sigma^+a)\cos(kx+\phi)\,. \quad (1)$$

Here a and $a^\dagger$ are the annihilation and creation operators for the cavity field, and σ_z, σ^-, and σ^+ are the internal atomic operators. ω_a and k are the frequency and wave number of the cavity mode. ω_0 is the atomic transition frequency, which is detuned from the cavity mode frequency by an amount $\Delta=\omega_0-\omega_a$. g is the atom-field coupling constant (equal to the one-photon Rabi frequency).

We assume that the transverse motion of the atom during its passage through the standing wave is negligible (the Raman-Nath approximation). We are then justified in neglecting the $p^2/2m$ term in the Hamiltonian.

For very large detuning compared to the Rabi frequency the atom has a negligible probability of making a transition between the ground and excited states. Hence if the atom is initially in its ground state, the population of the excited state will always be small, and spontaneous emission can be neglected. The interaction is manifested as virtual transitions between the ground and excited states, in which the atom absorbs a photon from the field and then reemits it immediately by stimulated emission. Every absorption or emission of a photon gives the atom a momentum kick $\hbar k$. If the emitted photon is in the same direction as the absorbed photon the total momentum change of the atom will be zero. However, if the photon is emitted into the reflected wave traveling in the opposite direction the total momentum change of the atom will be $2\hbar k$. The effective interaction Hamiltonian for large detuning is [4]

$$V_i=2\hbar(|g|^2/\Delta)\sigma_z a^\dagger a\cos^2(kx+\phi)\,. \quad (2)$$

The cavity mode is assumed to be initially in a coherent state $|\alpha\rangle$, which satisfies the relation $a|\alpha\rangle=\alpha|\alpha\rangle$. The atom is initially in the ground state with transverse spread in position $\phi(x)$. This implies phase coherence

across the width of the distribution, so the atom must be cooled before entering the cavity. We can write the initial state of the system as

$$|\psi(0)\rangle = \int dx\, \phi(x)|\alpha,x,0\rangle\,, \tag{3}$$

where the state is labeled by field amplitude, atomic position, and internal atomic state, respectively. In a frame rotating at the cavity frequency, the state of the system after an interaction time t in the field is

$$\begin{aligned}|\psi(t)\rangle &= \int dx\, \phi(x) \exp[-i(V_i+\hbar\Delta\sigma_z)t/\hbar]|\alpha,x,0\rangle \\ &= e^{i\Delta t/2}\int dx\, \phi(x)|\alpha\exp[i(|g|^2t/\Delta)\cos^2(kx+\phi)],x,0\rangle\,.\end{aligned} \tag{4}$$

The effect of the interaction is to change the phase of the coherent field state by an amount which depends on the vacuum light shift of the atomic ground state and on the position of the atom. Thus the system is left in an entangled state of the atom and field. A measurement of the field quadrature $X_\theta = ae^{-i\theta} + a^\dagger e^{i\theta}$ will then localize the atom. This measurement can be made using a homodyne detector [6]. To find the atomic state after the field measurement we project the field state onto an eigenstate $|X_\theta\rangle$ of the quadrature phase and evaluate the resulting inner product [7]:

$$\begin{aligned}|\psi(t)\rangle_{\text{atom}} &= N\int dx\, \phi(x)\langle X_\theta|\alpha\exp[i(|g|^2t/\Delta)\cos^2(kx+\phi)]\rangle|x,0\rangle \\ &= N\int dx\, \phi(x)\frac{1}{(2\pi)^{1/4}}\exp\{-[(\alpha_1 - X_\theta/2)^2 + i\alpha_2(\alpha_1 - X_\theta)]\}|x,0\rangle\,,\end{aligned} \tag{5}$$

where

$$\alpha_1 + i\alpha_2 \equiv \alpha\exp\{i[(|g|^2t/\Delta)\cos^2(kx+\phi) - \theta]\} \tag{6}$$

and N is a normalization factor.

In order to observe the correlation between the atomic position and the phase of the field, the transit time of each atom through the cavity must be much shorter than the cavity lifetime, which in turn must be much shorter than the time interval between successive atoms.

The best localization is obtained with $|g|^2t/\Delta = \pi$ and a high field intensity. However, if the scheme is implemented at optical wavelengths the Raman-Nath condition imposes a severe restriction on the interaction time and the field strength. The additional requirement of a low transition probability between internal atomic states leads to the following condition on the atom-field coupling constant [8]:

$$|g| \gg 2\langle n\rangle^{3/2}(\pi^2\eta\hbar/\lambda^2 m)\,. \tag{7}$$

η is a proportionality factor characterizing the momentum uncertainty of the atom after the interaction, and is independent of the cavity frequency and approximately independent of the field strength. Assuming $|g|^2t/\Delta = \pi$, we can obtain significant localization if the mean number of photons in the field is greater than about 8. For optical transitions the required value for $|g|$ is extremely high (of the order of 10^8 Hz). Such high values have recently been obtained by Kimble *et al.* [9] using a very short cavity of high finesse. Vacuum level shifts of the required order (but at smaller detunings than are needed for our scheme) have been observed by Heinzen and Feld [10].

Figure 1(a) shows the initial position distribution of the atom (iii) and the near-field distributions resulting from field measurements $X_0 = 0$ (i) and $X_{\pi/2} = 2\alpha$ (ii). In this graph, and in all the subsequent graphs, we have used $|g|^2t/\Delta = \pi$ with $\alpha = \sqrt{8}$. By varying the phase of the field

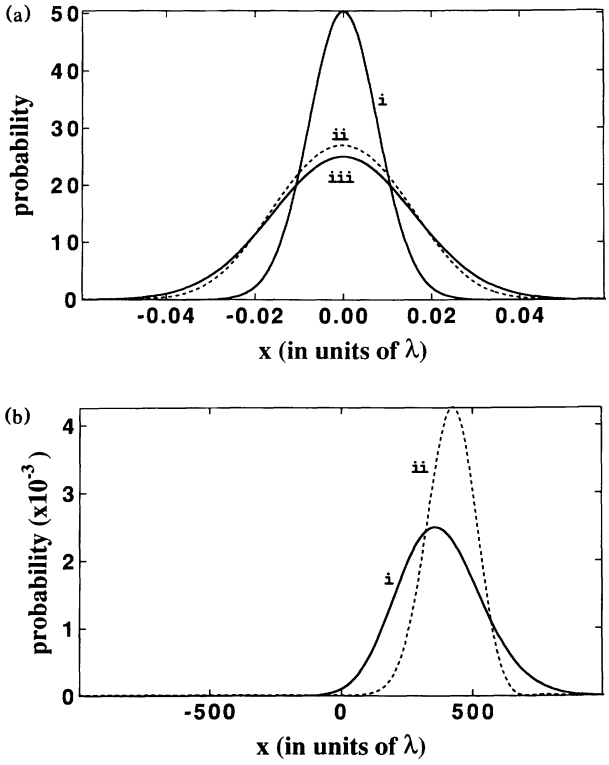

FIG. 1. (a) The near-field position distribution of the atom before the field measurement (iii), after a field measurement yielding the value $X_0 = 0$ (i), and after a field measurement yielding the value $X_{\pi/2} = 2\alpha$ (ii). The position distribution of the atom before it enters the cavity is taken to be Gaussian with standard deviation $\sigma = 0.1\lambda/2\pi$ centered midway between a node and an antinode of the field ($\phi = -\pi/4$). (b) The far-field position distribution for the same parameters after the measurement of X_0 (i), and after the measurement of $X_{\pi/2}$ (ii).

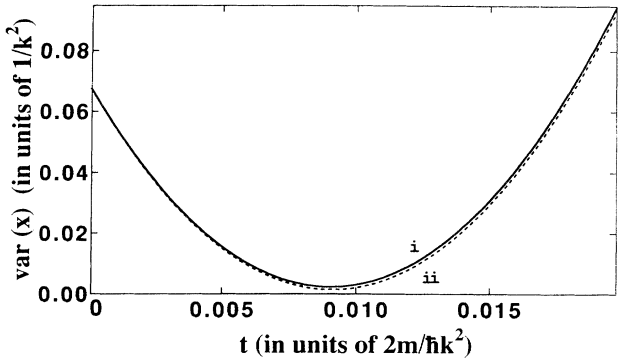

FIG. 2. Curve i, the position variance as a function of time for the contractive state produced by our position localization scheme. It is compared with the ideal focusing achieved by a "twisted coherent state" with the same momentum variance and initial position variance (ii). Our contractive state (i) is produced by making a field measurement $X_0 = -2\alpha$ after the atom has crossed the cavity. The position distribution of the atom before it enters the cavity is taken to be Gaussian with standard deviation $\sigma = 0.5\lambda/2\pi$ centered at an antinode of the field ($\phi = 0$).

quadrature measured we vary the degree of localization and effectively create an atomic slit of adjustable width.

Figure 1(b) shows the far-field distribution of the atom after the measurement of X_0 (i) and after the measurement of $X_{\pi/2}$ (ii). It is clear that the wide virtual slit has produced a narrower diffraction pattern than the narrow virtual slit. This is exactly what we would expect if the atom passed through a real physical slit. In fact, as the width of the virtual slit is reduced, by varying the phase of the field quadrature from $\pi/2$ to 0, the width of the diffraction pattern increases smoothly. The product $\Delta x \Delta p$ is close to the uncertainty limit, but there is some excess noise due to a small degree of nonlinearity in the phase change across the atomic wave front. A linear component in the phase change across the virtual slit produces a deflection of the atomic beam, which is responsible for the asymmetry of the far-field distribution.

If the measurement is such that the atom is localized at an antinode of the field, then the phase change across the atomic wave front is approximately parabolic, and the atom is focused. That a measurement can produce focusing is contrary to the intuitive notion that momentum uncertainty introduced by a position measurement should cause the position distribution to spread out with time. The requirement for focusing is that the measurement leave the system in a state with a negative correlation between its position and momentum. States whose position distribution contracts under free evolution (termed "contractive states") have been described by Yuen [11] and Ozawa [12]. Figure 2 compares the focusing of a contractive state produced by our position localization scheme with the theoretical maximum focusing, achieved by Yuen's "twisted coherent state."

If the initial position distribution of the atom is wider

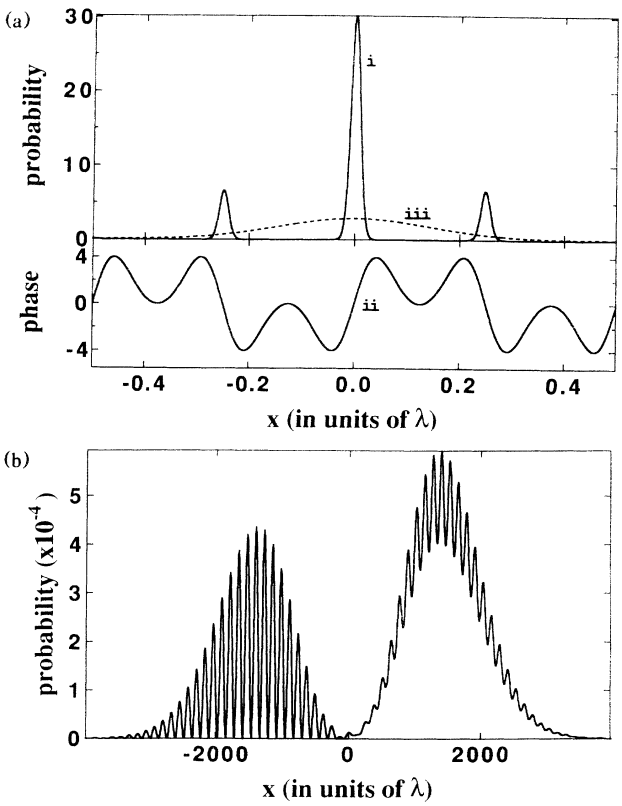

FIG. 3. (a) The near-field position distribution of the atom with a broader initial distribution, before the field measurement (iii), and after the value $X_0 = 0$ has been measured for the field (i). Curve ii shows the phase (in radians) of the atomic wave front after the measurement. The position distribution of the atom before it enters the cavity is assumed to be Gaussian with standard deviation $\sigma = 0.9\lambda/2\pi$ centered midway between a node and an antinode of the field ($\phi = -\pi/4$). (b) The far-field position distribution for the same parameters after the field measurement.

than that used in Fig. 1 then a measurement of $X_0 = 0$ will localize the atom as indicated by curve i in Fig. 3(a). The position distribution now has two smaller peaks located at $x = \lambda/4$ and $-\lambda/4$ on either side of the central peak. The initial Gaussian distribution of the atom is indicated by curve iii, and the phase of the atomic wave front after the field measurement by curve ii. The phase shift across each of the three peaks is approximately linear. However, the phase change across the central peak is positive and the phase change across each of the two side peaks is negative. Hence the central atomic beam is deflected to the right and the two side beams are deflected to the left. Here the term "atomic beam" is used loosely because the three "beams" constitute the wave packet of a single atom. Figure 3(b) shows the far-field position distribution of the atom. The left half of the distribution shows complete interference between the atomic beams from the side peaks. The right half of the distribution is the diffraction pattern of the central

peak. The partial interference in the right half of the distribution is due to the presence of very small peaks located at $x=\lambda/2$ and $-\lambda/2$ in curve i of Fig. 3(a).

Diffraction of atoms moving through a standing light wave has been observed without a field measurement [13]. The standing-wave light field acts as a diffraction grating which produces a periodic phase modulation of the atomic wave front. However, by ignoring the field information the atom is not localized in the near field.

If the initial position distribution of the atom is centered at an antinode of the field and the value $X_0=0$ is measured after the interaction then the near-field position distribution of the atom will exhibit two slits, with opposite phase change across each slit. The two atomic beams converge, and interference can be observed in the near field where they cross. Alternatively a second cavity in antiphase with the first can be placed immediately after the first cavity to act as an atomic lens. If the value $X_{0\,(\text{second cavity})}=-X_{0\,(\text{first cavity})}$ is measured the phase change across the atomic wave front is eliminated and interference from the two adjacent slits can be observed in the far field.

We have shown that a quadrature phase measurement on the field can localize the position of the atom very precisely within a wavelength of the cavity field. However, a single field measurement cannot determine "which wave" the atom went through, so if the initial position distribution is spread over many wavelengths the distribution after a field measurement will contain correspondingly many peaks. If the atom passes through a second cavity tuned to a slightly different frequency immediately after exiting the first cavity, one or more of these peaks can be selected out by a quadrature phase measurement of the field in the second cavity.

Rydberg atoms also satisfy the requirements for this position measurement scheme. Because Rydberg atoms have huge electric dipole matrix elements, coupling constants as high as 500 kHz are possible. Rydberg atoms also have very long spontaneous emission times (of the order of 10^{-2} s for circular atoms), so long atom-field interactions can be achieved. The wavelengths for Rydberg transitions fall in the millimeter domain, and suitable cavities can be constructed with lifetimes even longer than the atomic lifetime. Because of the long wavelengths of the Rydberg transitions, the Raman-Nath condition is easily satisfied. However, because the wavelengths are much larger than a reasonable atomic de Broglie wavelength we cannot expect any phase coherence over the initial atomic distribution, which is assumed to be spread over a distance of the order of a wavelength of the cavity mode. In this case the scheme described above behaves as a classical position measurement of the atom rather than a quantum localization, and there is no interference in the far field.

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