Upper Bounds on Lepton-Number Violating Meson Decays

Laurence S. Littenberg[(1),(a)] and Robert E. Shrock[(2),(b)]
[(1)]*Physics Department, Brookhaven National Laboratory, Upton, New York 11973*
[(2)]*Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11794-3840*
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From a retroactive data analysis, we obtain the first upper bound on the $|\Delta L|=2$ kaon decay $K^+\to\pi^-\mu^+\mu^+$: $B(K^+\to\pi^-\mu^+\mu^+)<1.5\times10^{-4}$. We also discuss $K^+\to\pi^-\mu^+e^+$ and $|\Delta L|=2$ decays of D and B mesons. We suggest experiments which can improve our limit and discuss the connection with direct searches for heavy neutrinos in $K^+_{\mu2}$ decay.

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The related issues of the conservation of total lepton number, possible neutrino masses, and possible lepton mixing remain of fundamental importance. The decays

$$K^+\to\pi^-l^+l'^+, \quad (1)$$

where l and l' denote e or μ, are of interest in this regard because they violate total lepton number and lepton family number. The $\mu\mu$ mode in (1) is the most interesting here, since it is a probe of lepton-number violation which is not constrained by the stringent limits which have been set on neutrinoless double beta ($0\nu2\beta$) decay of nuclei or the conversion process $\mu^-+(Z,A)\to e^++(Z-2,A)$ [1]. To our knowledge, there has not been any report of an experimental search for, or an upper limit on, this $\mu\mu$ mode.

We have therefore carried out a retroactive analysis of data on $K^\pm$ decays for this purpose. Counter experiments which measured or searched for other decay modes had triggers which, as far as we can tell, would have excluded any events from the decay $K^+\to\pi^-\mu^+\mu^+$. As is well known, bubble-chamber experiments, as usually operated, do not have this drawback. From our examination of bubble-chamber experiments on $K^\pm$ decays, we derive the most stringent limit from the data of a Maryland-Rutgers experiment at Brookhaven National Laboratory which used the 30-in. BNL-Columbia hydrogen bubble chamber [2]. This experiment searched for $K^-\to\pi^+e^-e^-$, utilizing a sample of 65000 K^- decays. We reproduce the spectrum of $K^-\to\pi^+x^-x^-$ events from this experiment in Fig. 1 as a function of $\rho=m_x^2/m_\pi^2$ (see Ref. [2] for details of the data analysis). All (charged) three-prong events with momentum imbalance transverse to the K^- direction of <30 MeV/c were assumed to correspond to decays of the type $K^-\to\pi^+x^-x^-$. The spectrum of Fig. 1 was then calculated on this assumption. In accordance with its aim, this experiment subjected only events having $\rho\simeq m_e^2/m_\pi^2$ to a further kinematic fitting program. A Monte Carlo simulation indicates that the resolution in m_x^2 for the $\pi^+\mu^-\mu^-$ channel is about 15% worse than that for the $\pi^+\pi^-\pi^-$ channel, the resolution of which is evident from Fig. 1. The center of the distribution of possible $\pi^+\mu^-\mu^-$ events is at $\rho=0.572$; integrating the resolution function from $\rho=0.5$ to 0.65, one includes 97.7% of the entire distribution. This interval of the experimental plot includes five events. Assuming Poisson statistics, a sample of 9.27 events will fluctuate down as far as 5 events 10% of the time. Dividing the 9.27 by 0.977, we thus obtain a 90%-C.L. upper limit [3] of 9.5/65000, i.e.,

$$B(K^+\to\pi^-\mu^+\mu^+)<1.5\times10^{-4}. \quad (2)$$

Clearly our direct limit (2) can be greatly improved by a dedicated experiment, which we suggest. Neither of the high-sensitivity Brookhaven K^+ decay experiments E777 [4] and E787 [5] reported any limits on decays of type (1). The proposals for upgrades of these experiments, E865 and E787 (extended), do not mention any search for decays of type (1) [6]. However, with the requisite modifications, they could perform this search. Sensitivities of order 10^{-9} in branching ratio should be possible [7]. Even in advance of dedicated experiments, however, one may observe that in E787, the existing data sample can be analyzed for possible $K^+\to\pi^-\mu^+\mu^+$ events.

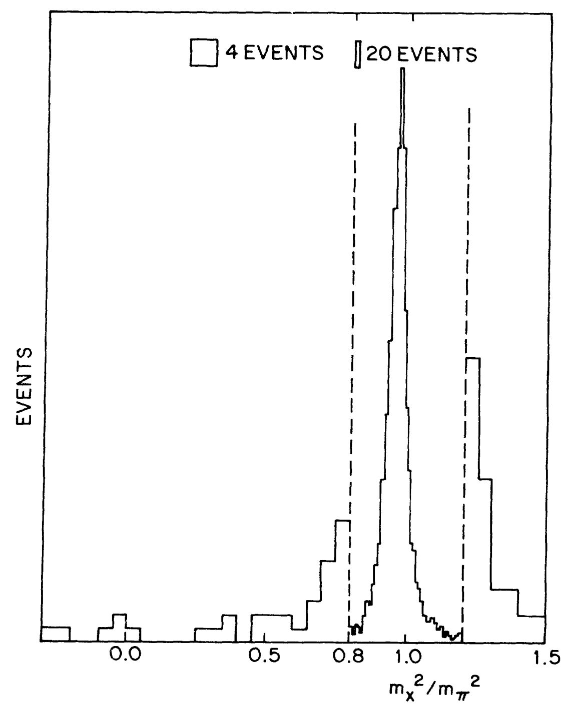

FIG. 1. The event distribution for $K^-\to\pi^+x^-x^-$, as a function of m_x^2/m_π^2 from Ref. [2].

The possible sensitivity with the current data is of order 10^{-8} in branching ratio.

How large might the branching ratios for decays (1) be? In the extension of the standard model to allow general neutrino masses, one includes, in addition to the three left-handed lepton doublets, some number k of neutrino singlets $\chi_{j,R}$, $j=1,\ldots,k$. The neutrino mass terms in the Lagrangian are given by

$$-\mathcal{L}_m=\frac{1}{2}(\bar{\nu}_L\ \bar{\chi}_L^c)\begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}\begin{pmatrix} \nu_R^c \\ \chi_R \end{pmatrix}+\text{H.c.}\,, \qquad (3)$$

where ν_L and χ_R denote the respective three-dimensional and k-dimensional flavor vectors $(\nu_e,\nu_\mu,\nu_\tau)_L^T$ and $(\chi_1,\ldots,\chi_k)_R^T$. M_L and M_R are 3×3 and $k\times k$ symmetric Majorana mass matrices, and M_D is a $3\times k$ Dirac mass matrix. The diagonalization of (3) yields, in general, $3+k$ nondegenerate Majorana neutrino mass eigenstates. There may exist a subset of these corresponding to mass eigenvalues which are degenerate (in magnitude); these would form Dirac neutrinos. Taking into account the respective mixing of neutrino and charged lepton mass eigenstates to form weak eigenstates, we define the lepton mixing matrix via the charged current $J_\lambda=\bar{l}\gamma_\lambda P_L \nu_l$, with $\nu_l=\sum_{i=1}^{3+k}U_{li}\nu_i$ [where $P_{L,R}=(1\mp\gamma_5)/2$].

The diagrams which contribute to the decays (1) to lowest order are shown in Fig. 2. Diagram 2(a) yields, in standard notation,

$$\text{Amp}[2(\text{a})]=2G_F^2 f_K f_\pi (V_{ud}V_{us})^*\sum_j (U_{lj}U_{l'j})^* p_{K,\alpha}p_{\pi,\beta}[L_j^{\alpha\beta}(p_l,p_{l'})-\delta_{ll'}L_j^{\alpha\beta}(p_{l'},p_l)]\,, \qquad (4a)$$

where

$$L_j^{\alpha\beta}(p_l,p_{l'})=m_{\nu_j}[q^2-m_{\nu_j}^2]^{-1}\bar{v}(p_l)\gamma^\alpha\gamma^\beta P_R v^c(p_{l'})\,, \qquad (4b)$$

where q denotes the momentum carried by the virtual ν_j. Graph 2(b) cannot be evaluated so easily, because the hadronic matrix element which occurs,

$$\int d^4x\, d^4y\, e^{i(p_d-p_u)\cdot y}e^{i(p_{\bar{s}}-p_{\bar{u}})\cdot x}\langle\pi^-|[\bar{d}_L(y)\gamma_\beta u_L(y)][\bar{s}_L(x)\gamma_\alpha u_L(x)]|K^+\rangle\,, \qquad (5)$$

cannot be directly expressed in terms of measured quantities, unlike the matrix elements $\langle 0|\bar{s}_L\gamma_\alpha u_L|K^+\rangle$ and $\langle\pi^-|\bar{d}_L\gamma_\beta u_L|0\rangle$ in graph 2(a). The resultant uncertainty in the rate will not be important here. For the moment, let us consider the case where m_{ν_j} is not $\gg 100$ MeV (this is the case of interest for direct searches in $K_{\mu 2}^+$ decay; see below). Then, taking $\langle[q^2-m_{\nu_j}^2]^{-1}\rangle\sim m_K^{-2}$, we estimate roughly that in this extension of the standard model,

$$B(K^+\to\pi^-l^+l'^+)\sim 10^{-(13\pm 2)}r_{ll'}\left|\sum_j U_{lj}U_{l'j}m_{\nu_j}/(100\text{ MeV})\right|^2. \qquad (6)$$

The factor $r_{ll'}$ is a relative phase-space factor with normalization $r_{ee}\equiv 1$.

For the ee mode, one can use the fact that the leptonic part of the amplitude for $K^+\to\pi^-e^+e^+$ is the same as that which enters in the amplitude for neutrinoless double beta decay of nuclei, although the hadronic matrix elements and $\langle q^2\rangle$ values are different. Substituting in (6) the constraint from $0\nu 2\beta$ decay that [1,8] $|\sum_j U_{1j}^2 m_{\nu_j}|\lesssim$ few eV, we estimate that $B(K^+\to\pi^-e^+e^+)\lesssim 10^{-26}$. (For the case where a given $m_{\nu_j}\gg 100$ MeV, the corresponding term from the neutrino propagator in nuclear $0\nu 2\beta$ decay would behave like U_{1j}^2/m_{ν_j} rather than $\sim U_{1j}^2 m_{\nu_j}/[\langle q^2\rangle\simeq(100\text{ MeV})^2]$, and hence Eq. (6) would have a different form. However, since the same leptonic term appears in nuclear $0\nu 2\beta$ decay and in $K^+\to\pi^-e^+e^+$, the upper limit on the latter decay would not change significantly.) For the $\mu\mu$ mode, let us assume, for illustration, that a single ν_j dominates the sum in (6), with a mass $m_{\nu_j}=200$ MeV (see below); then given the limit from searches for neutrino decays [1], $|U_{2j}|^2<3\times 10^{-8}$ (90% C.L.) for this assumed m_{ν_j}, Eq. (6) yields $B(K^+\to\pi^-\mu^+\mu^+)\lesssim 10^{-25}$. With the same input and the 90% C.L. limit [1] $|U_{1j}|^2<1\times 10^{-8}$ for $m_{\nu_j}=200$ MeV, we find $B(K^+\to\pi^-\mu^+e^+)\lesssim 10^{-26}$. In view of these tiny branching ratios, it is not necessary for our present purposes to calculate them more precisely than in our rough estimates.

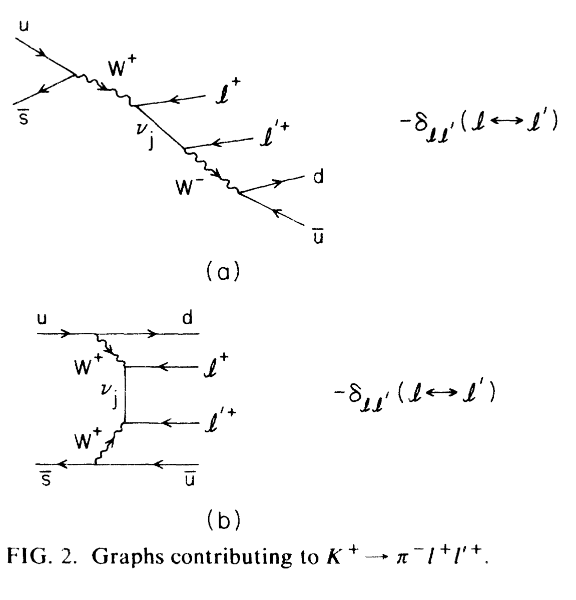

FIG. 2. Graphs contributing to $K^+\to\pi^-l^+l'^+$.

Thus, the theoretical estimates (6) in the simplest extension of the standard model are far below our new limit (2) [and the known experimental limits [1,9] $B(K^+ \to \pi^- e^+ e^+) < 1\times 10^{-8}$ and [10] $B(K^+ \to \pi^- \mu^+ e^+) < 2.8\times 10^{-8}$]. Of course, these estimates depend on the new physics which is assumed [11]. For example, if one considered a model with right-handed weak currents as well as Majorana neutrino masses [e.g., an $SU(2)_L \times SU(2)_R \times U(1)$ model], then the amplitude (4) would not be simply proportional to neutrino masses. In this model or others with new physics it is possible that the branching ratios for the decays (1) could be larger than those given by (6). Moreover, although the theoretical estimates in the extension of the standard model that we considered yield a very small effect, we believe that experimental limits on conservation laws such as total lepton number are of intrinsic and fundamental value. They are analogous to bounds on, e.g., $\mu \to e\gamma$ or $K_L^0 \to \mu^\pm e^\mp$, where the same extension of the standard model again yields branching ratios much smaller than experimental limits, or to bounds on, e.g., electric-charge nonconservation, nonzero photon mass, or *CPT* violation, where the theoretical expectation is that the effect would be exactly zero.

Our limit (2) is also relevant in another context. There are now good bounds on possible massive neutrinos which occur, via lepton mixing, in $K_{\mu 2}^+$ decays [1,12,13] (and in π_{l2}^+ decays [1,12,14,15]). Since measurements of the Z width at the CERN e^+e^- collider LEP show that there are "only three light neutrinos," and since one knows that [1,16] $m(\nu_1) < 9.3$ eV, $m(\nu_2) < 0.27$ MeV, and $m(\nu_3) < 35$ MeV (where ν_i, $i=1,2,3$, are the primary mass eigenstates in the weak eigenstates ν_e, ν_μ, and ν_τ, respectively), one might naively conclude that there is no further interest in experiments to search for heavy admixed neutrinos (where kinematically allowed) in K_{l2}^+ and π_{l2}^+ decays, at least for massive neutrinos heavier than 35 MeV. This naive conclusion is wrong; there is still motivation for such a search. The point is that the LEP experiments really indicate that there are only three weak $T=\frac{1}{2}$, $T_3=\frac{1}{2}$ neutrino eigenstates with dominant mass eigenstates having masses smaller than $m_Z/2$. The actual number $(3+k)$ of mass eigenstates of which these weak eigenstates are composed is unknown. Thus, it is still worthwhile to search for a massive neutrino signal in K_{l2}^+ and π_{l2}^+ decays, $l=\mu$ or e, even (where kinematically allowed) for $m(\nu_j) > 35$ MeV. However, a neutrino with $m(\nu_j) > 35$ MeV would necessarily be a Majorana state, since possible Dirac states are indeed limited to three by the LEP result. Hence one must examine the implications of such a heavy Majorana neutrino, and herein lies the connection with $K^+ \to \pi^- \mu^+ \mu^+$ decay.

Here we concentrate on $K^+ \to \mu^+ \nu_j$ for heavy (and hence Majorana) ν_j [17]. Our reason is that for $m(\nu_j) \gtrsim 50$ MeV, data already collected by BNL E787 may have the sensitivity [18] to probe well below the limits set previously. Given the new LEP results and the resultant implication that a neutrino in this mass range must be Majorana, it is further necessary to check that such a neutrino is not already ruled out by any other data. Such a neutrino would, indeed, contribute to $K^+ \to \pi^- \mu^+ \mu^+$, but substituting the existing limit [1] $|U_{2j}|^2 < 1\times 10^{-7}$ for $m(\nu_j) = 150$ MeV into (6), one finds that the branching ratio is many orders of magnitude smaller than the upper bound (2) which we have derived. Hence, our new limit (2) does not preclude an observable heavy neutrino signal in $K_{\mu 2}$ decay.

We can obtain an upper bound on the mode $K^+ \to \pi^- \mu^+ e^+$ which improves upon the current limit noted above [10] by observing that the leptonic part of the amplitude for this decay is related by crossing to the leptonic part of the amplitude for the conversion process $\mu^- + (Z,A) \to e^+ + (Z-2,A)$. The best bound on the latter process is [1,19] $\sigma(\mu^- \mathrm{Ti} \to e^+ \mathrm{Ca})/\sigma(\mu^- \mathrm{Ti} \to \mathrm{capture}) < 1.7\times 10^{-10}$. Although the hadronic matrix elements are not the same for the $\mu^- \to e^+$ conversion and the $K^+ \to \pi^- \mu^+ e^+$ decay, with reasonable estimates we deduce from this limit the resultant bound

$$B(K^+ \to \pi^- \mu^+ e^+) \lesssim \mathrm{few}\times 10^{-9}. \qquad (7)$$

Of course, this is not a direct limit since it requires a theoretical estimate of the hadronic matrix element as input.

One may also consider decays of the type

$$K_L^0 \to \{\pi^- \pi^- l^+ l'^+ \text{ or } \pi^+ \pi^+ l^- l'^-\}\,, \qquad (8)$$

where again l and l' denote e or μ. These decays do not provide a useful probe for the $\mu\mu$ mode since this mode has almost zero phase space. However, they could be used to search for the μe mode. There have not, to our knowledge, been any experimental searches for this latter decay. In particular, the current high-sensitivity K_L^0 decay experiments E791 at BNL [20] and E137 at KEK [21] concentrated on a search for $K_L^0 \to \mu^\pm e^\mp$ and further measurement of $K_L^0 \to \mu^+ \mu^-$. We suggest that a future K_L^0 decay experiment could carry out a worthwhile search for the μe mode of (8). For example, this might be possible with a requisite modification of the approved upgrade, E871, of E791.

There are also several decays of heavy-flavor mesons analogous to (1) and (2), e.g., the mixing-angle favored decays $D^+ \to K^- l^+ l'^+$, $D^0 \to K^- \pi^- l^+ l'^+$, $D_s^+ \to \pi^- l^+ l'^+$, $B^+ \to D^- l^+ l'^+$, $B_d^0 \to D^- \pi^- l^+ l'^+$, and $B_s^0 \to \{D^- K^- \text{ or } D_s^- \pi^-\} l^+ l'^+$. (Further, there are various mixing-angle suppressed decays, such as $D^+ \to \pi^- l^+ l'^+$, etc.) There are also corresponding decays with 1^- daughter mesons or higher-multiplicity final states, e.g., $D^+ \to \{K^{*-} \text{ or } K^- \pi^0 \text{ or } \bar{K}^0 \pi^- \text{ or } K^- \pi^0 \pi^0\} l^+ l'^+$, etc. To our knowledge, there have not been any searches for, or published bounds on, any of these decay modes. From recent CLEO searches for charm- and b-flavor-changing neutral currents [22,23], we estimate that is should be possible to establish upper limits of $\sim 10^{-3}$–10^{-4} for these $|\Delta L| = 2$ decays.

In conclusion, we have set the first upper limit on the $|\Delta L|=2$ decay $K^+\to\pi^-\mu^+\mu^+$, and an improved limit on $K^+\to\pi^-\mu^+e^+$. The very-high-statistics BNL K decay experiments provide an excellent opportunity to improve these limits (as an auxiliary output of the data analysis). We think that this opportunity should not be missed.

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(a)Electronic address: litt@bnlux1.bnl.gov.

(b)Electronic address: shrock@dirac.physics.sunysb.edu; shrock@sunysbnp.bitnet.

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[7] Judging from the proposals [6], statistical sensitivities quite a bit beyond this level should be attainable, although the possible backgrounds for this search have not been directly addressed.

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