Thermal Expansion Coefficient near the Superfluid Transition of ^{4}He in an Aerogel

Melora Larson, Norbert Mulders,[a] and Guenter Ahlers

Department of Physics and Center for Nonlinear Science, University of California, Santa Barbara, California 93106

(Received 23 April 1992)

We measured the isobaric thermal expansion coefficient β_P near the superfluid transition of ^{4}He in an aerogel along six isobars between 3 and 24 bars. The data yield the specific-heat exponents α and α' and amplitude ratio A/A'. Fits by power laws in $t \equiv T/T_c - 1$, where T_c is the transition temperature, produced $\alpha \simeq -0.6$ and $\alpha' \simeq -1.0 \neq \alpha$. Including an analytic background $a \times t$ permitted $\alpha = \alpha' \simeq -0.59$, but required a very large coefficient a and yielded $A'/A \simeq 0$. The exponents are inconsistent with that of ρ_s and hyperscaling in three dimensions.

PACS numbers: 67.40.Kh, 64.60.−i, 67.40.Hf, 67.40.Yv

Recent measurements of the superfluid fraction [1-3] ρ_s/ρ and of the heat capacity [2] C_P near the superfluid transition of ^{4}He contained in aerogel (a porous silica gel of over 90% porosity) have revealed a sharp phase transition, but one for which the usual power-law interpretation of the data yielded a critical behavior which differs dramatically from that of the bulk fluid [4]. The ρ_s data produced [3] a superfluid-density exponent $\zeta = 0.76$, which is significantly greater than the bulk value [4] 0.67. Since ζ should be equal to the correlation-length exponent ν for a system of dimensionality $d = 3$, this value for ζ and the hyperscaling law

$$d\nu = 2 - \alpha \tag{1}$$

implied a specific-heat exponent $\alpha = -0.28$. That prediction corresponds to a sharp cusp in C_P. The shape of the predicted cusp was found to be inconsistent with the C_P measurements at saturated vapor pressure of Wong *et al.* [2]. In the present paper we present measurements of the isobaric thermal expansion coefficient β_P which also give α. Our measurements were made along several isobars for $3 \lesssim P \lesssim 24$ bars, and thus permitted a comparison of the critical singularities of C_P on the two sides of the transition with each other and with that of ρ_s along the full transition line. A fit of our data by power laws in the reduced temperature $t \equiv (T - T_c)/T_c$ gave $\alpha \simeq -0.6$ and $\alpha' \simeq -1.0$, where α and α' are the specific-heat exponents above and below T_c, respectively. This result is in disagreement with the scaling law

$$\alpha = \alpha' . \tag{2}$$

An attempt to recover the scaling prediction Eq. (2) by including an analytic background term in the fit yielded $\alpha = \alpha' \simeq -0.59$. However, this analysis required very large coefficients of the analytic term and gave the unusual result $A'/A \simeq 0$. All of the above exponent values are clearly inconsistent with Eq. (1) and the ρ_s measurements if the dimensionality of the system is taken to be three. Agreement with a modified version of Eq. (1) appropriate to a fractal system can be obtained for a fractal dimension $d = 2.82$, but we do not know of any reason why the system should be fractal.

We used the experimental method of Mueller, Ahlers, and Pobell [5]. A piece of aerogel cut from the same batch as the lightest one ($\rho = 0.140$ g/cm^3) studied by Mulders *et al.* [3,6] almost completely filled the sample volume $V_s \simeq 2.56$ cm^3. A capacitive Straty-Adams pressure gauge [7] was used to control the sample pressure to 0.1 μbar. The sample cell was connected to a volume $V_b \simeq 1.48$ cm^3 containing bulk helium (the "bulk volume") through a 76-μm-diam capillary of 12 cm length. The temperature of the bulk volume was held in the range 2.3-2.6 K, where the thermal expansion coefficient is well known [8] and has no singularities. Both volumes were isolated from the rest of the apparatus by a valve located on the bulk volume. The pressure of the sample volume was controlled by regulating the temperature of the bulk volume. Thus a temperature change ΔT_s of the sample volume resulted in a temperature change ΔT_b of the bulk volume, and the expansion coefficient β_P of the helium in the sample volume was given in terms of that in the bulk volume ($\beta_{P,b}$) by $\beta_P = \lim_{\Delta T_s \to 0}(-\beta_{P,b}\Delta T_b \times N_b/\Delta T_s N_s)$. The mole-number ratio N_b/N_s is simply related to the measured ratio of the two volumes. Because of uncertainties in quantities like the density of ^{4}He in the aerogel and V_s/V_b, systematic errors in β_P might be a few percent. This does not affect the singular behavior of β_P deduced from the data.

Close to the transition, C_P is given [4] in terms of β_P by

$$C_P = \mathcal{A}\beta_P + \mathcal{B} , \tag{3}$$

with $\mathcal{A} = VT(\partial P/\partial T)_t < 0$ and $\mathcal{B} = T(\partial S/\partial T)_t$. Here the derivatives are taken parallel to the transition line. The value of $\mathcal{A}$ does not differ much from that for bulk helium [3], and $\mathcal{B}$ is small. Thus β_P is nearly proportional to $-C_P$. Sufficiently close to T_λ we expect to be able to write

$$\beta_P = (A/\alpha)t^{-\alpha} + B \tag{4a}$$

for $t > 0$ and

$$\beta_P = (A'/\alpha')(-t)^{-\alpha'} + B' \tag{4b}$$

for $t < 0$, with α, α', and A/A' the same as for C_P.

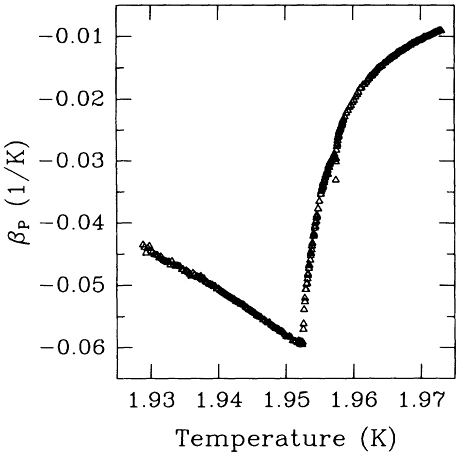

FIG. 1. The thermal expansion coefficient β_P as a function of the temperature T for $P=18$ bars.

Typical data for β_P as a function of T and for $P=18.05$ bars are shown in Fig. 1. The data at other pressures have the same qualitative features, as can be seen in Fig. 2 where they are shown as a function of t. Below T_c, β_P is nearly linear in t. The slope of β_P is much larger above T_c and seems to diverge as the transition is approached. Contrary to the scaling law Eq. (2), this suggests different critical behavior on the two sides of the transition. About 5 mK above T_c there is a small anomaly which is associated with the transition at T_λ of a small amount of bulk helium in the sample cell.

The cusps in β_P shown in Figs. 1 and 2 differ dramatically from the well known "lambda anomaly" which is found in bulk helium. In Fig. 3 we further illustrate the change in the critical behavior due to the presence of the aerogel. We show the expansion coefficient at several pressures on a linear scale as a function of $\log(|t|)$. For each pressure, the upper (lower) set of data is for $T>T_c$ ($T<T_c$). For the bulk system, the data above and below T_λ as a function of $t_b \equiv (T-T_\lambda)/T_\lambda$ would fall on the lines shown in the figure. Above T_c the expansion coefficient is significantly modified by the gel only when the temperature is within about 0.1% of T_c. The situation is very different below T_c, where particularly at the higher pressures the gel influences the thermodynamic properties even when the temperature is several percent below T_c, i.e., even deep in the superfluid phase. In contrast to the behavior above T_c, we see that the influence of the gel at a given value of t below T_c is strongly pressure dependent.

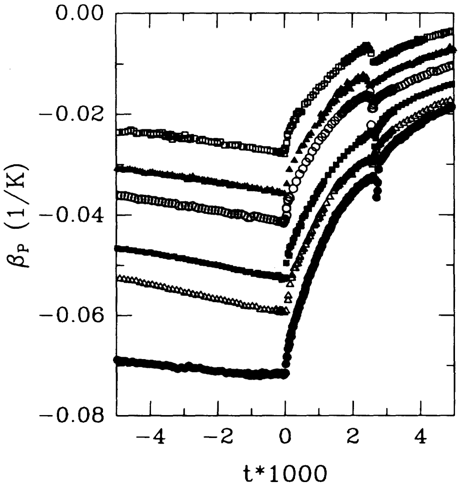

FIG 2. The thermal expansion coefficient β_P as a function of the reduced temperature t for various pressures. From top to bottom, the pressures are 2.7, 5.0, 7.1, 12.1, 18.0, and 24.0 bars.

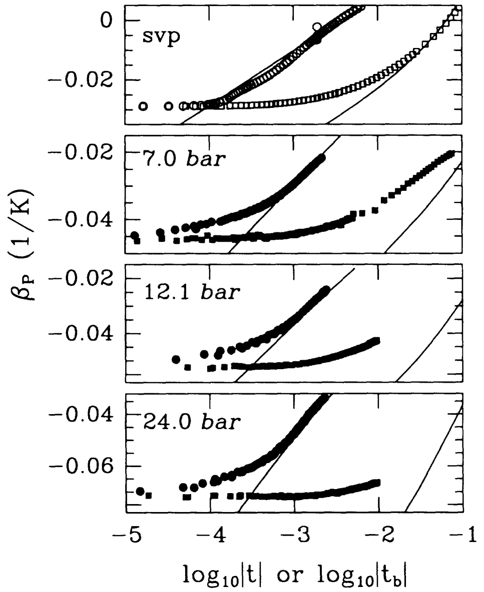

FIG. 3. Data for β_P on a linear scale as a function of the reduced temperature t on a logarithmic scale for various pressures. The data at vapor pressure were obtained from the heat-capacity measurements of Ref. [2] and Eq. (3). At each pressure, the upper (lower) set of data is for $T>T_c$ ($T<T_c$). The solid lines give β_P for bulk helium as a function of $t_b \equiv (T-T_\lambda)/T_\lambda$.

In order to study more quantitatively the critical phenomena near T_c, we fitted the data at each pressure over the range $-0.005 \lesssim t \lesssim 0.002$ by Eqs. (4), allowing α, α', A, A', $B=B'$, and T_c to be adjusted. These fits gave the exponent values shown in Fig. 4. We restricted the fitting range above T_c to avoid the influence of the small bulk anomaly. To verify that our results were not influenced by the bulk helium in the sample cell, we used the known bulk β_P to fit the bulk anomaly at the three highest pressures by adjusting the amount of helium contributing to it. At each pressure, this analysis yielded a bulk volume equal to 3.7% of the total sample volume. We then subtracted this bulk contribution from our data and refitted the "corrected" data by Eqs. (4). The exponents found

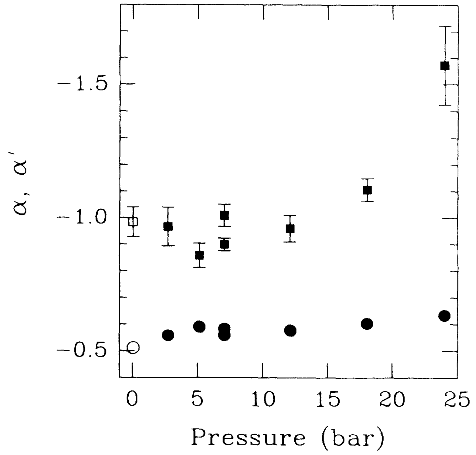

FIG. 4. The exponents α (circles) and α' (squares) as a function of pressure. They were obtained from a fit of Eqs. (4) to the data. Error bars for α are smaller than the symbol size. Solid circles: this work. Open circles: fit to the C_P data of Ref. [2].

for the corrected data were consistent with those shown in Fig. 4. As expected from the qualitative appearance of the data, this analysis yielded $\alpha' < \alpha$, in violation of the scaling law Eq. (2). The values for α and α' were much more negative than the value −0.26 expected from Eq. (1) and the ρ_s measurements [9]. Because of the apparent violation of the scaling laws Eqs. (1) and (2) by the pure-power-law interpretation of the measurements with $t \ll 1$, we will now pursue alternate explanations of the data.

The nearly linear dependence on t below T_c suggests that results consistent with Eq. (2) might be obtained by including an analytic background term in Eq. (4). Such a term would arise from a dependence of the coefficients upon T. Thus we fitted the data with $\alpha = \alpha'$ and $B = B' = B_0(1+at)$, adjusting α, B_0, a, A, A', and T_c. Statistically satisfactory fits were obtained, with the exponent values given in Fig. 5. Including confluent singular terms, as was necessary in the bulk case [5], allowed for the universal value $\alpha = -0.59$. However, the values of a were found to be close to 25 at all pressures. An analytic temperature dependence of B and B' would be expected to lead to values of ~ 1. The amplitude ratio A'/A was found to be nearly zero, as shown in Fig. 6. This is unusual and differs for instance from the n-vector model applicable to many other systems with isotropic interactions, for which $\mathcal{P} \equiv (1 - A/A')/\alpha \simeq 4.2$ for all physically relevant values of n [10,11]. Our aerogel results yield a very large and perhaps infinite value for $\mathcal{P}$. Also, the hyperscaling relation Eq. (1) is still violated by this fit and the ρ_s data. It seems difficult to understand these features of the data for instance within the framework of recently discussed critical points for systems with quenched disorder [12,13].

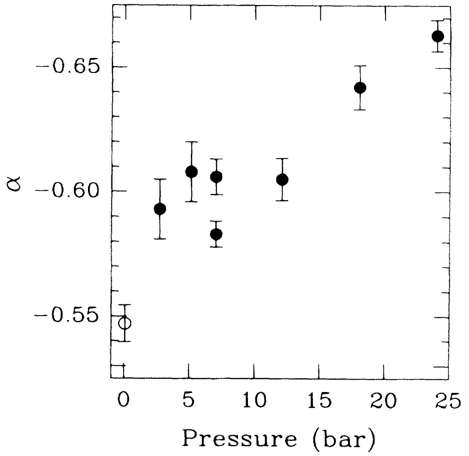

FIG. 5. The exponent $\alpha = \alpha'$ as a function of pressure. The values were obtained from a fit to the data of Eqs. (4) with a temperature-dependent background term.

It has been suggested by dos Santos, Branco, and de Queiroz [14] that hyperscaling might be recovered if it is assumed that the relevant dimensionality of the helium-aerogel system is a surface fractal dimension $d_f < 3$. In that case, $\nu = \zeta/(d_f - 2)$ and Eq. (1) becomes $d_f\zeta/(d_f - 2) = 2 - \alpha$. With these relations the experimental exponent values $\zeta = 0.76$ and $\alpha = -0.59$ yield $d_f = 2.82$. However, we do not know of any reason why the helium-aerogel system should show fractal behavior. In our case the aerogel itself, having been grown in a base solution, is not believed to show self-similar structure over any wide range of distances [15]. In any event, in this interpretation the large values of a and $\mathcal{P}$ would remain to be understood.

So far in the analysis we have allowed the leading critical behavior of the helium-aerogel system to differ from

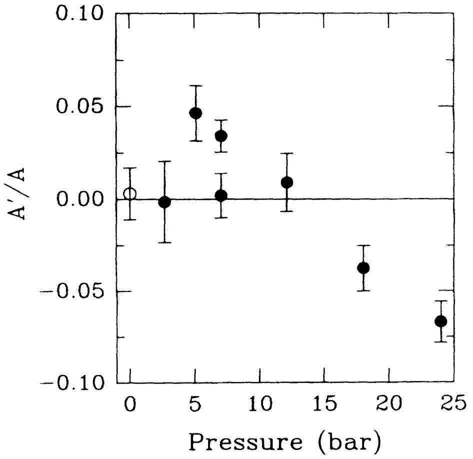

FIG. 6. The amplitude ratio A'/A as a function of pressure for the fits of Fig. 5.

that of bulk helium. An alternate interpretation would suggest that the *leading* critical behavior is unaltered by the presence of the aerogel. The *apparent* change in the universal parameters due to the impurities is then assumed to be caused by slow transients, i.e., by corrections to the asymptotic power law which decay exceptionally slowly with decreasing $|t|$. Such a situation is believed to prevail for instance in the dynamics of the superfluid transition of bulk ^{4}He [16]. It would lead to modified *effective* exponents in the experimentally accessible range of t [17–19]. For the case $\alpha=0$ these transients have been discussed in some detail by Narayan and Fisher [17]. For the superfluid density these authors write $\rho_s \sim |t|^{\zeta_b}[\ln(1/|t|)]^{-\tilde{\zeta}}$, where $\zeta_b=0.671$ is the bulk critical exponent of ρ_s [20]. Previous analysis [3] had indicated the need for including the more conventional confluent singular terms, and thus we fitted the ρ_s data [3] over the range $3\times10^{-5} \lesssim -t \lesssim 0.01$ by

$$\rho_s/\rho = k_0(1+D|t|^{0.5})|t|^{\zeta_b}[\ln(1/|t|)]^{-\tilde{\zeta}}. \tag{5}$$

For $P \lesssim 18$ bars the deviations were essentially random [21] and no larger than those from the fits reported in Ref. [3] which produced $\zeta \simeq 0.76$. The values of $\tilde{\zeta}$ were in the range 1.0 ± 0.1, consistent with the expected universal nature of this parameter [22]. The quality of the fits and the pressure independence of $\tilde{\zeta}$ give credence to the slow-transient interpretation. However, it is difficult to see how this approach will lead to a sufficiently dramatic modification of the heat capacity and the expansion coefficient to explain the *qualitative* difference between the data in Figs. 1 and 3 and the expansion coefficient of pure ^{4}He. In particular, the asymmetry of the deviation from bulk behavior above and below T_c would seem to be difficult to explain.

We conclude that the critical behavior of the helium-aerogel system is far from being understood.

We have benefited from discussions with numerous people, and particularly from those with B. I. Halperin, R. Ferrell, D. S. Fisher, J. D. Reppy, R. R. dos Santos, and G. Wong. This work was supported by National Science Foundation Grant No. DMR 89 18393.

[a] Current address: Department of Physics, University of Exeter, Exeter, EX4 4QL, United Kingdom.

[1] M. Chan, K. Blum, S. Murphy, G. Wong, and J. Reppy, Phys. Rev. Lett. **61**, 1950 (1988).

[2] G. K. S. Wong, P. A. Crowell, H. A. Cho, and J. D. Reppy, Phys. Rev. Lett. **65**, 2410 (1990).

[3] N. Mulders, R. Mehrotra, L. Goldner, and G. Ahlers, Phys. Rev. Lett. **67**, 695 (1991).

[4] See, for example, G. Ahlers, in *The Physics of Liquid and Solid Helium*, edited by K. H. Bennemann and J. B. Ketterson (Wiley, New York, 1976), Pt. I, Chap. 2.

[5] K. H. Mueller, G. Ahlers, and F. Pobell, Phys. Rev. B **14**, 2096 (1976).

[6] The aerogel sample, manufactured by Airglass, Sweden, was provided by Gane Wong of Cornell University.

[7] G. C. Straty and E. D. Adams, Rev. Sci. Instrum. **40**, 1393 (1969).

[8] D. L. Elwell and H. Meyer, Phys. Rev. **164**, 245 (1967).

[9] The inclusion of confluent singular terms (see Ref. [3]) in the data analysis did not significantly influence this situation.

[10] M. Barmatz, P. C. Hohenberg, and A. Kornblit, Phys. Rev. **12**, 1947 (1975).

[11] A. Singsaas and G. Ahlers, Phys. Rev. B **30**, 5103 (1984).

[12] A. Weinrib and B. I. Halperin, Phys. Rev. B **27**, 413 (1983).

[13] J. Machta, Phys. Rev. Lett. **66**, 169 (1991).

[14] R. R. dos Santos, N. S. Branco, and S. L. A. de Queiroz, Europhys. Lett. **13**, 647 (1990).

[15] R. Vacher, T. Woignier, J. Pelous, and E. Courtens, Phys. Rev. B **37**, 6500 (1988).

[16] For a discussion of and references to the dynamics, see, for instance, W. Y. Tam and G. Ahlers, Phys. Rev. B **33**, 183 (1986).

[17] O. Narayan and D. S. Fisher, Phys. Rev. B **42**, 7869 (1990).

[18] J. Machta (to be published).

[19] O. Narayan and D. S. Fisher (private communication).

[20] L. S. Goldner and G. Ahlers, Phys. Rev. B **45**, 13129 (1992).

[21] An equivalent but more complicated equation for the case $\alpha<0$ has been obtained by Narayan and Fisher [19]. A fit of this function to the data leads to similar results.

[22] For $P=24$ and 29 bars $\tilde{\zeta}$ increased to 1.6 and 1.4, respectively; but deviations from the fit were systematic and we do not regard these results as significant.