Sawtooth Oscillations with the Central Safety Factor, q_0, below Unity

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Essential experimental features of sawtooth crashes are not explained by existing theories, especially the fact that q_0 remains well below unity throughout the sawtooth cycle. Here, a model consistent with experimental results is presented. The model considers the sawtooth crash as involving full reconnection of the magnetic field lines, leaving $q_0 < 1$. It leads to stronger plasma redistribution than the previous reconnection model; however, $q(r)$ may remain almost unchanged. States with q_0 well below unity are shown to be possible results of different types of sawtooth crashes.

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The physics of sawtooth oscillations in a tokamak plasma is mainly determined by the picture of the sawtooth crash (collapse) together with the precursor and postcursor (successor) phenomena. Experimental results indicate that there are at least two distinct types of typical sawtooth crashes [1,2]. One of them starts by forming a cold plasma crescent enclosing a hot central plasma region. Until now the best description of this type of crash is given by the model suggested by Kadomtsev [3] and modified later in Refs. [4–6]. The second type of crash is characterized by the formation of a hot crescent enclosing a cold plasma center. The most relevant model for this case has been proposed by Wesson [7].

However, the mentioned models [3–7] are not consistent with recent experimental observations. In particular, the basic assumption of the Wesson model [7] that $1-q_0 \cong 10^{-2}$–10^{-3} is in conflict with the experimental values of q_0, $q_0 \cong 0.6$–0.8, which seems to be a characteristic feature of many experiments (Tokapole-II [8], Textor [9], JET [10]). In addition, the theory of Wesson does not provide any description of the transition to the final stage of the sawtooth crash and of the subsequent evolution to an axisymmetric magnetic configuration. The only existing theory describing a transition to an axisymmetric state is the field-line-reconnection theory of Kadomtsev [3], which leads to $q(r) \geq 1$ at the end of the transition process. However, this prediction does not agree with the very small observed change of q_0 during sawtooth crashes, $\Delta q_0 \cong (1$–$2)\%$ [8–10]. (On the other hand, note that the presently used diagnostic equipment may not be able to resolve the fast drops of q_0 connected with the large second derivative of the neoclassical conductivity at $r=0$ [4].) The complete reconnection picture of Ref. [3] is also in contradiction with observations of the "snake phenomenon" (i.e., the very localized density perturbation with $m=n=1$ surviving for many sawtooth periods) [11] since this phenomenon indicates that the $q=1$ surface persists throughout the sawtooth cycle.

In the present work, a new model is proposed which offers a possible explanation of both types of sawtooth crashes. The main feature of our model is that the crashes are considered to involve two kinds of plasma motions occurring simultaneously: (i) a rigid shift of the plasma core and (ii) a quasi-interchange flow in the vicinity of the $q=1$ surface. Thus the two types of crashes discussed above represent limit cases of the plasma motion. When the motion is primarily of kind (i), the first type of sawtooth crash appears and therefore it will be referred to as "shift dominated." When the main plasma motion is of kind (ii) the second type of crash occurs which will be called "interchange dominated."

The present model is based on the ideal MHD conservation laws, cf. Ref. [3]. We show that these laws allow a transition of the plasma from a state with $q_0 < 1$ to a new one which is also characterized by $q_0 < 1$ but with a reduced value of r_s, r_s being the radius of the $q=1$ surface. Figures 1 and 2 show possible changes in topology of the magnetic field during such a transition for the interchange-dominated and shift-dominated crashes, respectively. According to these figures the full reconnection of the magnetic field lines leads to $q_0 < 1$ provided that two (rather than one) current layers arise during some stage of the plasma motion.

The model is in agreement with the following experimental facts: (i) q_0 remains well below unity throughout the sawtooth cycle; (ii) the shear in the region close to r_s is very small; (iii) sawtooth crashes only weakly affect q_0 and the shear at r_s; (iv) the change of r_s caused by the crashes is relatively small; (v) a crash results in strong plasma redistribution; (vi) the energy flux across the $q=1$ surface is small before the final stage of the interchange-dominated crash, whereas it is large in the case of the shift-dominated crash; (vii) the interchange- and shift-dominated crashes are characterized by the inertial time scale and the time scale of reconnection, respectively. However, it should be pointed out that at present there are no numerical simulations of the MHD dynamics during sawteeth supporting our model.

Let us now examine the change in the magnetic field configuration due to the sawtooth crash. Using helical

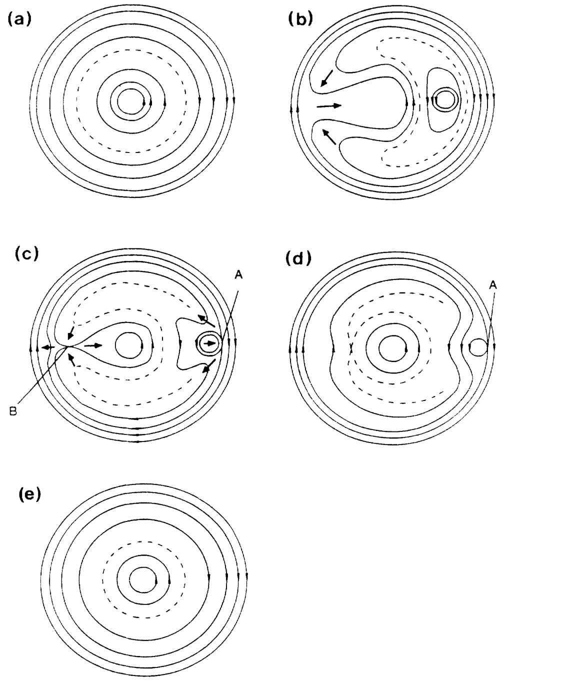

FIG. 1. Evolution of $\mathbf{B}_*$ during an interchange-dominated crash with successors, consistent with the experimentally observed evolution of the plasma temperature [13] (see also [10]): (a) the initial axisymmetric state with a hot core; (b) shift of the narrow plasma core and formation of a large cold bubble; (c) formation of a cold core with $q<1$ enclosed by a hot crescent; (d) formation of the $q=1$ surface with reduced radius; (e) the final axisymmetric state with a cold core and $q_0<1$. Notations: dashed line represents the $q=1$ surface ($B_*=0$); clockwise-directed field lines, $q>1$; counterclockwise directed lines, $q<1$; A and B, reconnection layers; bold arrows indicate direction of plasma motion.

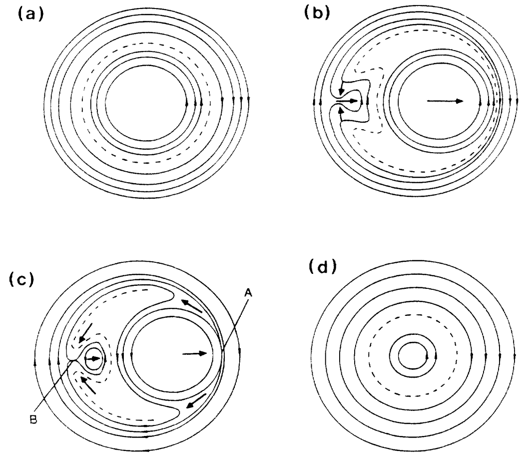

FIG. 2. Evolution of B_* during a shift-dominated crash (cf. tomographic reconstruction of the x-ray emission in JET, Fig. 5 of Ref. [1]): (a) the initial state; (b) shift of the large hot plasma core and formation of a small cold bubble; (c) the hot core is enclosed by a cold crescent which is perturbed by a colder island with $q<1$; (d) the final axisymmetric state with a cold plasma core and $q_0<1$. Notations are the same as in Fig. 1.

coordinates r, $\omega \equiv \theta - \varphi$, and φ (r, θ, and φ are the radial, poloidal, and toroidal coordinates, respectively) we introduce the magnetic flux function $\psi(r,\omega)$ as (cf. [3,12])

$$\psi(r,\omega) = R\int_0^r dr\, r\mathbf{e}^2\cdot\mathbf{B}_* = B_T\int_0^r dr\, r\left[\frac{B_p(r,\omega)}{\varepsilon B_T} - 1\right]. \tag{1}$$

Here

$$\mathbf{B}_* = \mathbf{B} - \mathbf{B}_H = \mathbf{B}_p\left[1 - \frac{\varepsilon B_T}{B_p(r,\omega)}\right] + \mathbf{B}_r\,, \tag{2}$$

$\mathbf{B}_p$, $\mathbf{B}_T$, $\mathbf{B}_H$, and $\mathbf{B}_r$ are the poloidal, toroidal, helical, and radial components of the magnetic field, respectively, R is the large radius of the torus, and $\mathbf{e}^2 = \nabla\omega$ is the contravariant base vector. It follows from Eqs. (1) and (2), that in the absence of MHD perturbations, $B_* = B_p(1-q)$ and $\psi(r)$ has a maximum at $r=r_s$. We assume that this is the case both before and after the sawtooth crash (with possible precursors and successors). We also assume that the magnetic field is frozen into the plasma everywhere except in the narrow current layers where the reconnection process is localized. Then the magnetic flux $d\psi$ and the volume dV of the connecting layers are approximately conserved. This makes it possible to find the relation between the flux $\psi(r)$ [and also $q(r)$] before and after the crash. By such a procedure it was concluded in Ref. [3] that reconnection results in $q(r) \geq 1$. Here, we will show that reconnection may also lead to other possible states. Indeed, the conservation laws for magnetic flux and volume may be written as follows (we assume that $dq/dr \geq 0$ before the crash):

$$\psi^-(r_1^-) = \psi^-(r_2^-) = \psi^+(r_1^+) = \psi^+(r_2^+)\,, \tag{3}$$

$$\frac{d\psi^-(r_1^-)}{\partial r_1^-}dr_1^- = \frac{\partial\psi^-(r_2^-)}{\partial r_2^-}dr_2^- = \frac{\partial\psi^+(r_1^+)}{\partial r_1^+}dr_1^+ = \frac{\partial\psi^+(r_2^+)}{\partial r_2^+}dr_2^+\,, \tag{4}$$

$$dV_1^- + dV_2^- = dV_1^+ + dV_2^+\,, \tag{5}$$

where $dV = 2\pi Rr\,dr$, the subscripts 1 and 2 label values at $r<r_s$ and $r>r_s$, respectively, and a minus (plus) denotes values before (after) the crash. Equations (3)-(5) yield Kadomtsev's result only in the special case $dV_1^+=0$. However, in general the system of equations (1)-(4) also possesses other solutions which may explain a variety of observed types of sawtooth crashes. Guided by experimental observations [8-10], let us consider sawtooth oscillations with $q_0<1$ throughout the sawtooth cycle. We assume that the region r_2^- contributes to the formation of the whole plasma mixing region, whereas the region r_1^- takes part only in producing the region r_2^+. This assumption corresponds to the evolution of the magnetic field, which is shown in Figs. 1 and 2. Consequently, we introduce a flux surface function $W(\psi)$ which determines the fraction of the volume dV_2^- producing the cold plasma core with $q<1$. Then to restrict the class of possible solutions of Eqs. (3)-(5) to those corresponding to the described physical picture, we split Eq. (5) as follows:

$$dV_1^+ = W dV_2^- , \tag{6}$$

$$dV_2^+ = (1-W) dV_2^- + dV_1^- , \tag{7}$$

which also may be written as

$$\frac{dx_1^+}{d\psi} = -W\frac{dx_2^-}{d\psi} , \tag{8}$$

$$\frac{dx_2^+}{d\psi} = (1-W)\frac{dx_2^-}{d\psi} - \frac{dx_1^-}{d\psi} , \tag{9}$$

where $x = r^2/(r_s^-)^2$. Note that in writing Eqs. (8) and (9) we have taken into account that $d\psi/dx_1 > 0$ and $d\psi/dx_2 < 0$. It is clear that when $W=0$, we regain the Kadomtsev reconnection scenario leading to $q(r) \geq 1$ after the crash.

In order to demonstrate our model, we consider crashes with a very small change of q_0, a situation which corresponds to experimental observations [8-10]. For simplicity we take $q_0^+ = q_0^-$, i.e.,

$$\left.\frac{d\psi(x_1^+)}{dx_1^+}\right|_{x_1^+=0} = \left.\frac{d\psi(x_1^-)}{dx_1^-}\right|_{x_1^-=0} \tag{10}$$

and assume

$$W = \begin{cases} \text{const}, & x_2^- < x_{20}^- , \\ 0, & x_2^- \geq x_{20}^- , \end{cases}$$

where x_{20}^- is a parameter, $\psi^+(x_1^+=0) = \psi^-(x_{20}^-)$. Equations (10) and (8) then enable us to find the set of equations relating W to the observable variables (r_s^+ and q_0) as

$$1-\mu^-(x_{20}^-) = [\mu^-(x_1^-=0)-1]W, \quad x_{20}^- = 1 + x_s^+/W , \tag{11}$$

where $\mu \equiv q^{-1}$ and $x_s^+ = (r_s^+/r_s^-)^2$. The system of equations (1), (3), (4), (8), (9), and (11) has a unique solution which determines the only possible profile of $q(r)$ resulting from the crash, provided $q(r)$ before the crash as well as the radius of the $q=1$ surface after the crash are known.

It can be seen in Figs. 1 and 2 that during the transition from an initial axisymmetric state in Figs. 1(a) and 2(a) to the state in Figs. 1(b) and 2(b) the plasma core is shifting almost rigidly, whereas in the region close to r_s the quasi-interchange flow is taking place. Note that the distinct feature of Figs. 1(b) and 2(b) consists in different sizes of the plasma core and the cold plasma bubble, which reflects the different characters of the shift-dominated and interchange-dominated crashes. The discussed process seems to be possible when q_0 is well below unity and the shear in the region close to r_s is very small, i.e., when $\mu(r)$ has a "head and shoulder" profile. Such a profile of $\mu(r)$ before the crash may be approximated by [12]

$$\mu(x^-) = \begin{cases} \mu_0 \dfrac{x_* - x_1^-}{x_*} + \mu_* \dfrac{x_1^-}{x_*}, & x_1^- < x_* , \\ \mu_* \dfrac{1-x_1^-}{1-x_*} + \dfrac{x_1^- - x_*}{1-x_*}, & x_* < x_1^- < 1 , \\ \dfrac{a^2 - x_2^-}{a^2-1} + \mu_a \dfrac{x_2^- - 1}{a^2-1}, & x_* < x_2^- , \end{cases} \tag{12}$$

where $a = a/r_s^-$ and $\mu_* - 1 \cong 10^{-2}$. One may expect that, depending on the width of the region with small shear, $1-x_*$, different types of crashes occur: Rigid-dominated crashes take place when x_* is close to unity, whereas interchange-dominated crashes are possible when x_* is well below unity.

In order to find an explicit solution we take as an example $q_0^- = q_0^+ = 0.8$, $q_a = 3$, $q_*^- = 0.995$, $r_s^+ = r_* = r_s^-/\sqrt{2}$, and $r_s^- = a/3$. These parameters are similar to

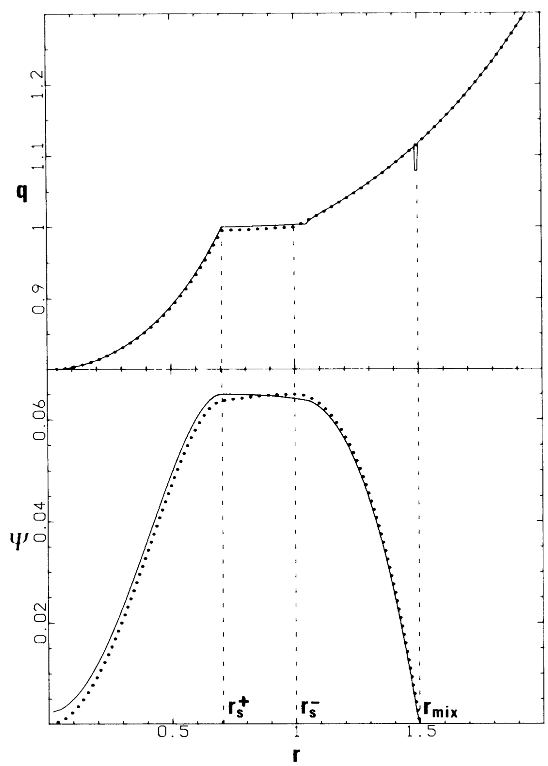

FIG. 3. Safety factor $q(r)$ and flux $\psi(r)$ before the crash (points) and in the final axisymmetric state (solid lines) in the case of $q(r)$ given by Eq. (12) and $q_0^- = q_0^+ = 0.8$, $q_a = 3$, $q_*^- = 0.995$, $r_s^+ = r_* = r_s^-/\sqrt{2}$, $r_s^- = a/3$; r_{mix} is the mixing radius; r is normalized by r_s^- and ψ by $B_T(r_s^-)^2/2$.

those of JET experiments [10,11]. The corresponding results are shown in Fig. 3 from which it is seen that the $q(r)$ profile before the crash and after the successors is almost unchanged. Nevertheless, it follows from Figs. 1 and 2 that as a consequence of the sawtooth crash, the plasma is strongly redistributed, the core being colder and the periphery hotter as compared to the result of the Kadomtsev reconnection process [3].

In conclusion, we have shown that the ideal MHD conservation laws are consistent with observed different sawtooth crashes, in particular, with those which leave q_0 well below unity [the $q(r)$ profile may even be almost unchanged] and result in strong plasma redistribution. During the transition to a new state with $q_0 < 1$, the magnetic field structure is changing in the whole region $r < r_{\rm mix}$ ($r_{\rm mix}$ is the mixing radius), and thus, unlike in the generally accepted opinion, full reconnection of the field lines does not inevitably lead to a state with $q(r) \geq 1$. It should be emphasized that both the shift-dominated and the interchange-dominated crashes may result in $q_0 < 1$. We have performed a simplified analysis (not presented here), in cylindrical geometry, showing that the change of the poloidal magnetic energy, δQ, is negative $[-\varepsilon_s^2(\mu_0^- - 1)^2 Q < \delta Q < 0$, Q being the total magnetic energy, $\varepsilon_s = r_s/R]$, and that the work due to the plasma pressure, δA, is positive ($\delta A \approx \varepsilon_s^2 B Q$, $\beta = 8\pi p/B^2$) provided $\partial p/\partial r < 0$. Consequently, the plasma entropy increases, supporting the accessibility of the state with $q_0^+ < 1$ within the considered model.

Finally, we note that the proposed model is supported by (i) experimental data concerning $q(r)$ [8–11] and x-ray emission [13], (ii) energetics calculations, (iii) the plasma flow patterns at the linear stage of the $m = 1$ instability for the head and shoulder $\mu(r)$ profile [14], and (iv) the numerical simulation of Ref. [15] showing that a plasma with a head and shoulder $\mu(r)$ profile can be stable or unstable depending on the pressure profile. Nevertheless, to reach a decisive conclusion whether our model adequately describes sawtooth oscillations requires further investigations. These should include numerical simulations as well as theoretical analysis of the full sawtooth cycle, taking into account the real geometry, plasma pressure, etc.

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