Skyrmions and the Nuclear Force

Niels R. Walet and R. D. Amado
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104-6396

Atsushi Hosaka
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3
(Received 21 January 1992)

The derivation of the nucleon-nucleon force from the Skyrme model is reexamined. Starting from previous results for the potential energy of quasistatic solutions, we show that a calculation using the Born-Oppenheimer approximation, properly taking into account the mixing of nucleon resonances, leads to substantial central attraction. We obtain a potential that is in qualitative agreement with phenomenological potentials.

PACS numbers: 21.30.+y, 11.10.Lm, 11.40.Fy, 13.75.Cs

The Skyrme Lagrangian, which describes a nonlinear field theory of interacting pions, has proven to be a successful effective theory of QCD in the long wavelength or nonperturbative regime and in the limit of a large number of colors (N_C) [1]. It possesses a topologically conserved charge B that is identified with baryon number. The $B=1$ soliton solutions yield an appealing picture of the nucleon and its excited states, but since the theory is a large-N_C approximation, its accuracy for real nucleon observables is typically $\sim 1/N_C \sim 30\%$ (improvements can be made by adding some finite N_C corrections to the calculations). Until very recently attempts to apply the Skyrme model to the nucleon-nucleon interaction ($B=2$) have not met with great success [2,3]. Though all $B=2$ approaches give a one-pion-exchange potential at large distances and a short-range repulsion, the intermediate-range central attraction has been elusive. Recent work by Walhout and Wambach [4] appears to show that careful treatment of the nonlinear nature of the Skyrme equations yields $B=2$ configurations with some central attraction at the classical level, or, equivalently, to the leading order in $1/N_C$. It is not immediately obvious, however, that this attraction survives the $1/N_C$ corrections, which corrections could go either way.

In this Letter we argue that the method used in [4] and most previous work to project the nucleon-nucleon interaction from the $B=2$ Skyrmion configurations has overlooked distortions of the Skyrmion fields at intermediate distances. These distortions coupled with some finite-N_C corrections yield considerably more midrange attraction, and potentials in qualitative agreement with experiment. This suggests that the attraction may be a robust property of the Skyrme model that survives at finite N_C. Thus the Skyrme approach gives promise of yielding a correct, detailed description of the nucleon-nucleon interaction and of providing a vital link in the quest to "derive" nuclear physics from QCD.

There are only two stable time-independent Skyrmion configurations for $B=2$. One consists of two $B=1$ Skyrmions infinitely separated. The other is a compact bound state of toroidal baryon density in which the identity of the two individual Skyrmions is totally lost [5]. To obtain an interaction energy one must describe intermediate Skyrmion configurations at finite separation. This corresponds to the definition of an adiabatic collective manifold [6], called the unstable manifold by Manton [7]. Each point on this manifold represents a configuration that is stabilized by imposing a constraint. It has been argued [7] that this manifold is at least six dimensional, with three coordinates corresponding to the relative separation of the two Skyrmions and three coordinates describing the relative orientation. The dimensionality makes it virtually impossible to calculate the potential energy over the whole manifold.

Fortunately studies of the $B=2$ system with the product ansatz [2,3,8], with the Atiyah-Manton ansatz [9,10], and with direct numerical integration of the equations [4] have shown that much information about the $B=2$ energy as a function of separation R is contained in a particular choice of three paths, corresponding to three definite choices of the relative orientation, or equivalently to a special symmetry of the pion field. This allows us to calculate an approximate form for the interaction energy of two Skyrmions which can be given by the expression (the form used in Ref. [4] is fully equivalent)

$$v(R)=v_1(R)+v_2(R)W+v_3(R)Z\,, \tag{1}$$

which corresponds to an expansion up to first order in terms of the two scalar operators W and Z. It is shown in [8] that these collective operators can be written as

$$W=T^{\alpha}_{pi}T^{\beta}_{pi}\,, \tag{2}$$

$$Z=T^{\alpha}_{pi}T^{\beta}_{pj}[3\hat{R}_i\hat{R}_j-\delta_{ij}]\,, \tag{3}$$

where T^{γ}_{pi} is a one-body operator with isospin and spin one, carried by the p,i labels, where $\gamma=\alpha,\beta$ labels each $B=1$ Skyrmion and $\hat{\mathbf{R}}$ is a unit vector along the direction of separation [11].

One of the important consequences of the introduction of these operators is that algebraic methods [8,12] can be used to obtain their *explicit* finite-N_C realization. In this way we can include some (arguably the most important)

finite-N_C corrections in the calculation. This includes the well-known amplification of the nucleon-nucleon matrix element of T by the factor $(N_C+2)/N_C$.

Since calculations in the product ansatz have shown that higher-order corrections to (1) are small [8], we take it to be exact. The problem we address here is how to obtain a nucleon-nucleon interaction which includes some finite-N_C effects, from (1). To date most workers have simply sandwiched (1) between states containing two nucleons for each value of R and called that the interaction. This is definitely not the nucleon-nucleon interaction determined phenomenologically from scattering phase shifts, where we can only require that the state contains two nucleons at large separation. For shorter distances when the hadrons are interacting they can be (virtually) whatever the dynamics requires, for example, Δ's. We know [10] that the baryon density is strongly deformed for moderate R—a deformation that finally results in the toroidal shape with its 70 MeV of binding. That configuration corresponds to a great deal of mixing between all states of the individual Skyrmions. This admixture will lead to additional attraction at intermediate distances. The purpose of this Letter is to estimate the effect of this mixing on the N-N interaction for $N_C=3$ starting with (1). The need for state mixing has been realized before [13] in the context of the product ansatz. But that ansatz is a poor starting point. Furthermore, as we state below, state mixing only makes sense for finite N_C. At the same time there are algebraic finite-N_C corrections to matrix elements that are difficult to include in the formalism of [13]. For these reasons, these early attempts to introduce state mixing did not find sufficient central attraction. We should remark here that a very different formalism, based on a study of fluctuations around the product ansatz [14], has been used to show that some attraction exists in two-pion range. Unfortunately the expansion underlying this approach fails in the region that is most crucial to central attraction, when $1<R<2$ fm.

For large N_C, one can distinguish two energy scales or reciprocally two time scales. The slower time scale is associated with the motion in the collective manifold, i.e., R and the orientation. The other time scale corresponds to the almost instantaneous response of the pion field to changes in R and the relative isospin orientation. We cannot separate the time scales for the two sets of adiabatic modes, as can be seen in the highly correlated doughnut. For N_C equal to 3 the situation changes. The R motion is typically much slower than the rotational motion which leads to the separation of the nucleon and Δ states. We thus have three energy scales, and we can calculate a Born-Oppenheimer potential for the R motion, which constitutes the slowest degree of freedom.

All of the effects of quick response of the pion field at each R are already in Eq. (1). It is the effect of the rotational states we want to include. As a guide, we begin by studying it perturbatively. Let us call the full potential between the two nucleons, including the effects of the rotational excited states, V. In terms of the Skyrmion interaction v of (1) and to second order, V is given by

$$V(R)=\langle NN|v(R)|NN\rangle + {\sum_s}' \frac{\langle NN|v(R)|s\rangle\langle s|v(R)|NN\rangle}{E_{NN}(R)-E_s(R)} . \quad (4)$$

(Here E_{NN} is the two-nucleon energy and E_s is the energy of the relevant excited state.) The first term on the right-hand side is the direct nucleon-nucleon projection of v and is the term that has appeared in the literature. The second term is the correction due to rotational or excited states. It is clear from the energy denominator that the second term is attractive. For $N_C=3$ the states $|s\rangle$ that can enter are $|N\Delta\rangle$, $|\Delta N\rangle$, and $|\Delta\Delta\rangle$. Except for a centrifugal term we add below, the excitation energy is assumed to be independent of R, and thus can be expressed in terms of the N-Δ energy difference, 300 MeV.

To evaluate (4) it is convenient to introduce states $|NNLSJT\rangle$ labeled by the conserved (total) isospin T and angular momentum J, as well as the NN channel orbital angular momentum L and spin S. The model space splits into two parts that are either symmetric or antisymmetric under interchange of the particles [15]. We select the antisymmetric part where the NN states satisfy the standard selection rule $L+S+T$ is odd. To account for the effects of centripetal repulsion at small R, we add the centrifugal energy, $\hbar^2L(L+1)/2MR^2$, to channel energies. We take M to be the reduced nucleon mass in all channels.

For each total isospin, $T=0,1$ we write V in the $|NNLSJT\rangle$ space as

$$\langle NNLSJT|V_c^T+V_s^T\boldsymbol{\sigma}^1\cdot\boldsymbol{\sigma}^2+V_t^T\sigma_i^1\sigma_j^2(3\hat{R}_i\hat{R}_j-\delta_{ij})|NNL'SJT\rangle . \quad (5)$$

It is the V_c^T, V_s^T, and V_t^T that we study. We have extracted the necessary values of v_1,v_2,v_3 from the work of Walhout and Wambach [4]. This is an uncertain process for large R since the potentials are quite small in this region and are calculated in [4] on a lattice by first calculating the total interaction energy of the $B=2$ system and then subtracting the rest energy of two free Skyrmions. In Fig. 1 we show the central potentials V_c^T calculated from Eq. (4), first from the first term on the right-hand side of (4) only (this is the result of Ref. [4]) and then with the second-order corrections. Note we only show the result for intermediate R, $1<R<2$ fm. The effects of mixing are quite large and attractive, as we expect. We do not plot the other potentials since there is only a minor change relative to the results obtained by [4]. We see

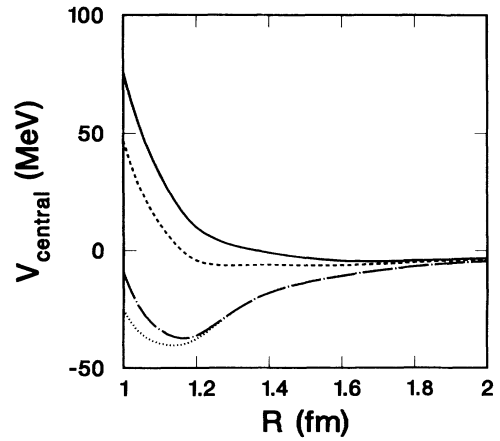

FIG. 1. The central potential V_c^T as a function of R. The solid line gives the result of [4], identical for $T=0,1$. The dashed line and the dotted line represent the perturbative result, for $T=0,1$, respectively. The dash-dotted line represents the result of the diagonalization for $T=1$. The $T=0$ diagonalization result is almost indistinguishable from the perturbative one.

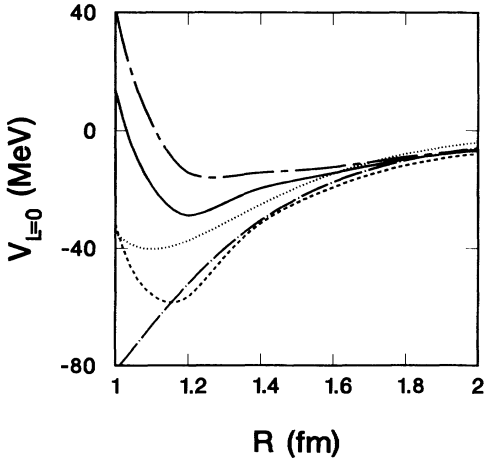

FIG. 2. The potential in the $^1S_0$ ($T=1$) and $^3S_1$ ($T=0$) channels. The solid line ($T=0$) and the dashed line ($T=1$) are our results obtained through diagonalization. The dotted and dash-dotted lines gives the same channels ($T=0$ and $T=1$, respectively) for the Reid soft-core potential [18]. The long-short-dashed line gives the same result for the nucleons-only calculation of [4], which is identical for $T=0,1$.

that the correction is of shorter range than the leading term. This is as we expect since, in the language of meson exchange, it requires more exchanges. We also see that though the effects of the perturbative corrections are modest for large R they become large for intermediate R and the perturbative approach becomes suspect. We must then turn to a more complete treatment.

To calculate the full effect of the mixing on the NN interaction we follow a procedure familiar from the Born-Oppenheimer approach to molecules. We diagonalize the interaction in the NN, $N\Delta$, ΔN, $\Delta\Delta$ space at each R in the $|LSJT\rangle$ representation. We follow a state from large R where it is made up purely of nucleons, to small R where we have mixing. The energy eigenvalue of this state as a function of R gives the adiabatic potential curve familiar from molecular physics, that is, the interaction energy in that channel. Among the many coupled $LSJT$ channels we choose a bare minimum of lowest J that allow us to extract the V_i^T. The result of this calculation is also plotted in Fig. 1. We see that for large R the result is identical with the perturbative result, as it should be. At intermediate R the perturbative and exact diagonalization answers are different in the $T=1$ channel, but the effect of mixing the Δ states is still large. Clearly the mixing leads to considerably increased attraction.

We wish to compare the Skyrme-based potential with a realistic nucleon-nucleon interaction. One cannot relate our result to modern effective potentials, such as the Bonn [16] and Paris [17] potentials, since their central parts contain explicit momentum-dependent terms. For that reason we compare our potentials to the Reid soft-core (RSC) potential [18]. We form V_c^T and V_s^T from the RSC in exactly the same channels as used to extract the Skyrme-based potential, and we find that the V_c pieces in the RSC are somewhat less attractive than in our calculation. However, if we compare the more physical S-channel ($L=0$) potentials, see Fig. 2, we find that the potentials obtained from Skyrmions with channel mixing and the RSC look remarkably similar, while those obtained without mixing do not resemble the RSC potential. At the largest value of R the details of the comparison should not be taken too seriously, because of the uncertainties in the underlying $v_i(R)$ at large R. Finally the tensor interaction in our calculation (which is almost identical to the one found in [4]) looks remarkably similar to the tensor interaction in the RSC.

Here we have shown that the Skyrme model can give strong midrange attraction that is in qualitative agreement with phenomenological potentials. This is in sharp contrast to early results based on the product ansatz. Two improvements play a key role in obtaining the attraction. First one must pay attention to the nonlinear nature of the Skyrme Lagrangian, as shown in Refs. [4,10], but second one must also include configuration mixing at intermediate distances as we have stressed here. This mixing is not easily formulated in the large-N_C limit, but is easily included for $N_C=3$, and hence brings with it some other finite-N_C effects. The combined effect of the careful treatment of the nonlinear equations and the configuration mixing is to give *substantial* central midrange attraction for the NN system that is in qualitative *agreement* with the data. To go from this work to a theory that can be confronted with experiment in detail is a difficult challenge. There are R-dependent corrections to inertial parameters to include, there are dynamical quantum corrections, there are nonadiabatic effects that

are particularly important at small R, and there are other mesons to include in the Skyrme Lagrangian. All these are under study. The success so far encourages us to proceed and to suggest that the results obtained so far will be robust under these refinements.

In summary we have shown that including the effects of channel coupling (Δ mixing) in the projection of the nucleon-nucleon interaction from recent studies of the potential in baryon-number-two Skyrmion systems, substantially increases the strength of the midrange central attraction, bringing it into qualitative agreement with experiment. To obtain this result we introduce some quantum corrections (equivalently corrections for the finite number of colors). Many more sophisticated quantum corrections remain to be made. But our results show that a nonperturbative approach to the problem of obtaining the "static" nucleon-nucleon interaction from QCD based on the Skyrme approach has great promise not only of phenomenological success, but also of being expressed in the language of meson exchange that has been the *lingua franca* of that interaction for 45 years.

This work was partially supported by the U.S. Department of Energy and National Science Foundation and the Natural Sciences and Engineering Research Council of Canada.

[1] T. H. R. Skyrme, Nucl. Phys. **31**, 556 (1962); I. Zahed and G. E. Brown, Phys. Rep. **142**, 1 (1986); *Chiral Solitons,* edited by K. F. Liu (World Scientific, Singapore, 1987).

[2] A. Jackson, A. D. Jackson, and V. Pasquier, Nucl. Phys. **A432**, 567 (1985).

[3] R. Vinh Mau, M. Lacombe, B. Loiseau, W. N. Nottingham, and P. Lisboa, Phys. Lett. **150B**, 259 (1985).

[4] T. S. Walhout and J. Wambach, Phys. Rev. Lett. **67**, 314 (1991).

[5] V. B. Kopeliovic and B. E. Stern, Pis'ma Zh. Eksp. Teor. Fiz. **45**, 165 (1987) [JETP Lett. **45**, 203 (1987)]; J. J. M. Verbaarschot, T. S. Walhout, J. Wambach, and H. W. Wyld, Nucl. Phys. **A468**, 520 (1987); N. S. Manton, Phys. Lett. B **192**, 177 (1988); J. J. M. Verbaarschot, Phys. Lett. B **195**, 235 (1987).

[6] A. Klein, N. R. Walet, and G. Do Dang, Ann. Phys. (N.Y.) **208**, 90 (1991), and references therein.

[7] N. S. Manton, Phys. Rev. Lett. **60**, 1916 (1988).

[8] M. Oka, R. Bijker, A. Bulgac, and R. D. Amado, Phys. Rev. C **36**, 1727 (1987).

[9] M. F. Atiyah and N. S. Manton, Phys. Lett. B **222**, 438 (1989); N. S. Manton, in *Geometry of Low-Dimensional Manifolds: Proceedings of the Durham Symposium,* edited by S. K. Donaldson and C. B. Thomas (Cambridge Univ. Press, Cambridge, 1990).

[10] A. Hosaka, M. Oka, and R. D. Amado, Nucl. Phys. **A530**, 507 (1991).

[11] The expectation value of T_{pi} between states containing nucleons only equals that of the product $\tau_p\sigma_i$, as discussed in [8].

[12] R. D. Amado, R. Bijker, and M. Oka, Phys. Rev. Lett. **58**, 654 (1987).

[13] A. Depace, H. Müther, and A. Faessler, Phys. Lett. B **188**, 307 (1987); G. Kälbermann and J. M. Eisenberg, J. Phys. G **15**, 157 (1989).

[14] H. Yamagishi and I. Zahed, Phys. Rev. D **43**, 891 (1991); V. Thorsson and I. Zahed, Phys. Rev. D **45**, 965 (1992); T. Otofuji, Y. Kondo, S. Saito, and R. Seki, Phys. Rev. D **45**, 2528 (1992).

[15] This corresponds to assuming either symmetry or antisymmetry between the individual Skyrmions. In the semiclassical quantization of the doughnut a similar problem is encountered, in terms of the Finkelstein-Rubinstein constraints [E. Braatten and L. Carson, Phys. Rev. D **38**, 3525 (1988)].

[16] C. Holinde, R. Machleidt, and C. Elster, Phys. Rep. **149**, 1 (1987).

[17] M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Cote, D. Pires, and R. DeTourreil, Phys. Rev. C **21**, 861 (1980).

[18] R. V. Reid, Jr., Ann. Phys. (N.Y.) **50**, 411 (1968).