Superposition of Coherent States and Squeezing

Recently [1], Janszky and Vinogradov obtained the squeezed vacuum via a one-dimensional Gaussian superposition of coherent states, that is, $|G(x)\rangle = C_G \times \int_{-\infty}^{\infty} G(x)|x\rangle dx$, where $G(x) = \exp(-\gamma^2 x^2)$ and $|x\rangle$ are the coherent states integrated along the real axis. They get for the fluctuations of the two quadratures of the field $\langle(\Delta b_1)^2\rangle = (1+\gamma^2)/4$, $\langle(\Delta b_2)^2\rangle = 1/4(1+\gamma^2)$. In order to obtain the correct expressions we expand the $|G(x)\rangle$ state in terms of the Fock states:

$$|G(x)\rangle = \frac{1}{\sqrt{\cosh r}} \sum_{n=0}^{\infty} (\tanh r)^n \left[\frac{(2n-1)!!}{(2n)!!}\right]^{1/2} |2n\rangle , \quad (1)$$

where $\tanh r = 1/(2\gamma^2+1)$.

This state corresponds to the squeezed vacuum [2] $|G(x)\rangle = |0,\zeta\rangle$ with $\zeta = re^{-i\pi}$. From Eq. (1) it is a simple matter to obtain quadrature fluctuations $\langle(\Delta b_1)^2\rangle = (1+\gamma^2)/4\gamma^2$ and $\langle(\Delta b_2)^2\rangle = \gamma^2/4(1+\gamma^2)$ and therefore the limit $\gamma \to \infty$ does not correspond to strong squeezing but rather to the vacuum. This makes sense since the Gaussian distribution in the limit $\gamma \to \infty$ corresponds to a Dirac delta and therefore $|F(x)\rangle \to |0\rangle$ in this limit. The limit $\gamma \to 0$, however, corresponds to strongly squeezed vacuum. This correction is not a simple scale change, since in the physical model taken by the authors, in the limit $u/\omega \gg 1$ or $\gamma \gg 1$ corresponds to $(\Delta b_1)^2 = (\Delta b_2)^2 = \frac{1}{4}$, that is instead of a large reduction, we get no reduction at all. This can also be seen, by looking at Eq. (22) in Ref. [1] for $\delta = 0$. In the above limit the Gaussian is near a δ function and $|u,t\rangle$ is approximately a coherent state, which is in agreement with the argument given above.

In spite of this error, we feel that the idea of superposing coherent states is appealing. One could, for example, study non-Gaussian distributions and compare the noise reductions with respect to the Gaussian distribution, for the same average photon number.

For example, one could try a distribution $F(x) = C \times \exp(-\gamma^2 |x|^p)$ with varying p. In Fig. 1 we have plotted the difference $\delta = \langle(\Delta b_2)^2\rangle_F - \langle(\Delta b_2)^2\rangle_G$ as a function of the average number of photons:

$$N = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(x)F(x')xx' \exp[-\tfrac{1}{2}(x-x')^2] dx\, dx' , \quad (2)$$

for $p = 1,3,4$. We observe that this difference is everywhere positive and, although the curves become very close to each other for extreme values of N, the minimum fluctuations always correspond to the Gaussian distribution.

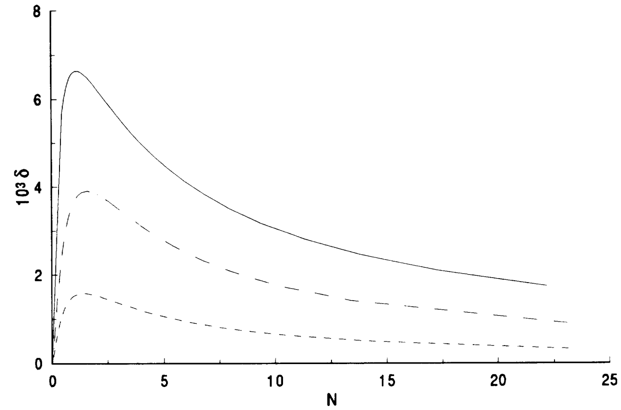

FIG. 1. The fluctuation difference δ vs the average photon number for several values of p, $p=1$ (solid line), $p=3$ (dashed), $p=4$ (dot-dashed).

Alternatively, we also approached this problem via a variational calculation in order to find a distribution $F'(x)$ that will minimize $\langle(\Delta b_2)^2\rangle$, subject to the constraints of normalization and fixed average photon number N. We found Gaussian and non-Gaussian (hypergeometric) distribution functions. Nevertheless, our calculations show that the Gaussian distribution always presents a lower noise level.

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[1] J. Janszky and An. V. Vinogradov, Phys. Rev. Lett. **64**, 2771 (1990).
[2] R. Loudon and P. L. Knight, J. Mod. Opt. **34**, 709 (1987).

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