Double Peaks in the Dissipation of Vibrating Superconductors

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The two distinct peaks observed as a function of temperature in the dissipation of vibrating superconducting slabs in a tilted magnetic field are explained by thermally activated flux diffusion across the thickness and width (or length) of the slab. These conspicuous peaks thus do not necessarily indicate multiple phase transitions of the flux-line lattice as proposed previously. Anisotropic mobility of the flux lines may increase or decrease the separation of the peaks.

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Recently two distinct peaks in the dissipated power as a function of temperature T have been observed in superconductors performing tilt vibrations [1–4] or put into a magnetic ac field [5,6]. The double peak was observed only when the applied dc field B (ranging from 0.1 to 10 T) was not along one of the symmetry axes of the specimen; elsewhere one of the peaks disappeared. This conspicuous phenomenon was reported for the high-T_c superconductors (HTSC) Y-Ba-Cu-O [6] and Bi-Sr-Ca-Cu-O [1,5], for an organic superconductor [2], for Nb and $NbSe_2$ [3], and for artificially layered Mo-Ge films [4]. The dissipation peaks in Refs. [1,4–6] were interpreted as evidence for multiple phase transitions like melting of the flux-line lattice (FLL), melting, decoupling, or depinning of the two-dimensional lattice of pancake vortices [7] in layered HTSC, or the formation of vortex kinks [8]. These explanations relate the peaks to material properties of the superconductor, in particular to its pronounced anisotropy or layered structure.

In this paper I show that even for *isotropic* superconductors containing a FLL without any phase transition a double peak of *geometric* origin is expected when the specimen is a slab or disk. Here the two peaks correspond to flux motion along the thickness and width of the superconductor, respectively, as already suggested in [2]. In the general case the observed double peak is the combined effect of both the *specimen shape* and the *anisotropy* or layered structure.

In order to quantify this statement I consider a superconducting slab with length $l \gg$ width $w \gg$ thickness $d \gg$ penetration depth λ_{ab}, filling the space $|x| \leq d/2$, $|y| \leq w/2$, $|z| \leq l/2$. Various relaxation times τ (i.e., penetration times for magnetic perturbations) result then for various diffusion modes of the FLL along the thickness, width, or length of the slab, e.g., $\tau = d^2/\pi^2 D$, $wd/\pi^2 D$, or $l^2 d/w\pi^2 D$. When the temperature T or applied field B is swept, the flux diffusivity $D(T,B)$ changes and a dissipation peak occurs whenever one of the values $1/\tau$ coincides with the circular frequency ω of the tilt vibration or of the applied ac field, provided the corresponding diffusion mode is excited in the given geometry. If B is along x, y, or z, then only one diffusion mode is excited and only *one* peak occurs.

In HTSC the observed dissipation peaks can be quite sharp since, due to thermally activated depinning, the flux diffusivity depends strongly on T and typically also on B. The peaks typically occur in the region of thermally assisted flux flow (TAFF) [9] where the resistivity $\rho = \rho_{\text{TAFF}}(T,B)$ is Ohmic and exhibits Arrhenius behavior, $\rho/\mu_0 \propto D \propto 1/\tau \propto \exp[-U(T,B)/k_B T]$. At lower T, in the region of strong pinning and slow flux creep, the resistivity is highly nonlinear and the dissipation at small amplitudes is very weak. As T is increased, some vortices depin and the dissipation reaches a maximum when $\tau(T,B) \approx 1/\omega$. Above the peak, the dissipation decreases again since the vortices are almost completely depinned and usual flux flow (FF) occurs [10,11]. In general, the dissipation is linear only in the flux-flow regime or at extremely small amplitudes; near and below the peak, depinning processes cause additional hysteretic damping and the dissipation becomes nonlinear [12,13] and yields an amplitude-dependent attenuation. In conventional (low-T_c) superconductors a similar peak occurs at constant T when B approaches the upper critical field B_{c2} where the pinning strength drops to zero [12].

The thermally assisted diffusivity $D(T,B)$ is related to the TAFF resistivity $\rho(T,B)$ and the TAFF viscosity (drag) of the FLL per unit volume $\eta = \eta_{\text{TAFF}}(T,B)$ by $D = \rho/\mu_0 = B^2/\mu_0\eta$. For anisotropic superconductors D and η depend on the orientation $\hat{\mathbf{B}}$ of the flux lines and on the direction $\hat{\mathbf{v}}$ of flux motion, which is always perpendicular to $\hat{\mathbf{B}}$. One has $D_{\hat{\mathbf{v}}}^{\hat{\mathbf{B}}} = B^2/\mu_0\eta_{\hat{\mathbf{v}}}^{\hat{\mathbf{B}}}$.

The theory of superconductors performing flexural vibration (cantilevered reed [12,14]) or tilt vibration (specimen glued on a silicon [13] or polymer [2] tongue) was developed in [12,14,15]. For a review see [16]. In the following it is assumed that the dc magnetization is negligible; i.e., the internal field B approximately equals the applied field. Periodic tilting of a specimen by a small angle $\phi(t) = \phi_0 \cos\omega t$ in a dc field B is equivalent to applying an ac field $B_{\text{ac}}(t) = B\phi(t)$ oriented perpendicular to B and to the rotational axis. For a slab with $B_{\text{ac}}(t) = B_s \cos\omega t$ parallel to its surface in the TAFF and FF regions, this ac field (or the vortex tilting) penetrates the specimen by *linear diffusion* [9–11,15–18] according to $\dot{\mathbf{B}}(\mathbf{r},t) = D\nabla^2\mathbf{B}$, causing a time and space averaged linear dissipation per unit volume

$$P = (\omega B_s^2/2\mu_0)|\mu''(\omega)|. \quad (1)$$

Here μ'' is the imaginary part of the ac susceptibility $\mu(\omega) = \mu' + i\mu'' = \tanh(u)/u$ [15] or explicitly [9,17]

$$\mu = \frac{(\sinh v + \sin v) - i(\sinh v - \sin v)}{v(\cosh v + \cos v)}, \quad (2)$$

with $u = v/(1-i)$, $v = (\omega d^2/2D)^{1/2} = d/\delta$, and δ the skin depth. This result applies to both the penetration of compressional and tilt waves of the FLL [18]. The dissipation (1) and (2) has a maximum as a function of v or D at $v = v_{\max} = 2.2542$, where $\mu'' = \mu''_{\max} = 0.417\,23$. The damping peak occurs thus when $d/2\delta = 0.887 \approx 1$ and $\omega\tau = 0.97 \approx 1$ with $\tau = d^2/\pi^2 D$; this is expected intuitively.

Results for six possible geometries and modes are summarized in Table I. Each of these geometries was realized in at least one of the experiments [1–6]: (1) $\mathbf{B}\|\mathbf{x}$ (perpendicular field), (2) $\mathbf{B}\|\mathbf{z}$ (longitudinal field), and (3) $\mathbf{B} = (B\cos\theta, 0, B\sin\theta)$ (oblique field). In cases 1–3 the periodic rotation (tilt vibration) is about the y axis (along the width w). Three more cases are obtained when the slab rotates about the z axis (along the length l): (4) $\mathbf{B}\|\mathbf{x}$ (perpendicular field), (5) $\mathbf{B}\|\mathbf{y}$ (field along the slab width), and (6) $\mathbf{B} = (B\cos\theta, B\sin\theta, 0)$. Cases with B not perpendicular to the rotation axis are not considered here. If B is exactly parallel to the rotation axis, no dissipation should be observed at all [13].

In each of the oblique-field cases 3 and 6 *three* diffusion modes occur, corresponding to field penetration along x, y, and z. Cases 1,2 and 4,5 follow as special cases from cases 3 and 6, respectively. In Table I the symbol D denotes the in general *anisotropic* flux diffusivity $D_{\hat{\mathbf{v}}}^{\hat{\mathbf{B}}}$, with $\hat{\mathbf{B}}$ and $\hat{\mathbf{v}}$ given by columns 3 and 7; explicit expressions for the anisotropic $D_{\hat{\mathbf{v}}}^{\hat{\mathbf{B}}}$ are listed in the last column. The relaxation times and dissipation peaks for the various modes were obtained by the following arguments.

In case 1 (Fig. 1, middle) the ac field $B_{\text{ac}} = B_{\text{ac},z} = \phi B$ penetrates according to the diffusion equation $\dot{B}_z = D\,\partial^2 B_z/\partial x^2$. Thus Eqs. (1) and (2) apply and one gets for the maximum dissipation $P_{\max} = P_1 = 0.417\,23\langle\phi^2\rangle\omega B^2/2\mu_0$ and for the diffusivity where this maximum occurs $D_{\max} = D_1 = (\omega d^2/2)(1/2.254)^2 = 0.0984\omega d^2$, where $\langle\phi^2\rangle$ is the oscillating tilt angle squared and averaged over time (and in the case of flexural vibrations also over the specimen). The relaxation time for the fundamental diffusion mode with wavelength $2d$ or $k = \pi/d$ is $\tau_1 = 1/k^2 D = d^2/\pi^2 D$.

In case 2 (Fig. 1, top) the ac field $B_{\text{ac}} = B_{\text{ac},x} = \phi B$ is *perpendicular* to the slab and thus causes large demagnetizing effects. At large ω, the ac field is completely shielded from the slab's interior and, therefore, has to "flow" around the slab. The magnetic energy of the resulting inhomogeneous ac field outside the slab is [12] $(\pi w^2/4)B_{\text{ac}}^2/2\mu_0 = \frac{1}{2}\phi^2 T_l$ per unit length along z, where $T_l = (\pi w^2/4)B^2/\mu_0$ is a force which tends to realign the FLs along $\mathbf{B}\|\mathbf{z}$. Denoting the FL displacements $\|x$ by $u(z,t)$ we get an elastic force density $T_l\,\partial^2 u/\partial z^2$ which is counteracted by the viscous force density $-\eta d w \dot{u}(z,t)$. In this mode, $\eta = \eta_x^z$ is the viscosity of a FLL oriented along z and moving along x. The tilting of the FLs by an angle ϕ starts at the ends of the slab at $z = \pm l/2$ and diffuses into the middle, $z = 0$, according to $\dot{u} = D_{\text{eff}}\,\partial^2 u/\partial z^2$ with an effective "diffusivity" $D_{\text{eff}} = T_l/\eta d w = \pi w B^2/4\eta d\mu_0 = (\pi w/4d)D \gg D$ and a relaxation time $\tau_2 = l^2/\pi^2 D_{\text{eff}} = 4l^2 d/\pi^3 w D$, where $D = D_x^z$. The position $D_{\max} = D_2$ and height $P_{\max} = P_2$ of the dissipation peak is thus again obtained from Eqs. (1) and (2). As compared to mode 1, the resulting $P_2 = 0.417\,23\langle\phi^2\rangle(\omega B^2/2\mu_0)\pi w/4d$ is enhanced, and

TABLE I. Characteristics and results for various geometries and diffusion modes of a vibrating superconducting slab with dimensions $|z| < l/2$, $|y| < w/2$, $|x| < d/2$, and $l \gg w \gg d$ in a dc magnetic field B. $\hat{\mathbf{B}} = 100$ means $\hat{\mathbf{B}}_x = 1$, $\hat{\mathbf{B}}_y = 0$, $\hat{\mathbf{B}}_z = 0$, etc.; $S = \sin\theta$, $C = \cos\theta$, θ is the angle between B and the slab normal $\hat{\mathbf{x}}$; $P_1 = 0.417\langle\phi^2\rangle\omega B^2/2\mu_0$, $P_2 = (\pi w/4d)P_1$.

Case or mode	Axis of rotation	$\hat{\mathbf{B}}$	$\hat{\mathbf{B}}_{\text{ac}}$	Penetrating ac field: Oriented along	Penetrating ac field: Moving along	Flux-line velocity $\hat{\mathbf{v}}$	Relaxation time × diffusivity $\tau\pi^2 D$	Maximum dissipation $P_{\max}$	D/D_a^c for $\mathbf{c}\|\mathbf{x}$
1	y	100	001	z	x	001	d^2	P_1	1
2	y	001	100	x	z	100	$4l^2d/\pi w$	P_2	$1/\Gamma$
3.1	y	$C0S$	$S0C$	z	x	$S0C$	d^2	P_1C^2	ϵ_θ
3.2	y	$C0S$	$S0C$	x	z	$S0C$	$4l^2d/\pi w$	P_2S^2	ϵ_θ
3.3	y	$C0S$	$S0C$	x	y	010	$\approx wd/C^2$	$\approx P_2S^2$	$1/\epsilon_\theta$
4	z	100	010	y	x	010	d^2	P_1	1
5	z	010	100	x	y	100	$\approx wd$	$\approx P_2$	$1/\Gamma$
6.1	z	$CS0$	$SC0$	y	x	$SC0$	d^2	P_1C^2	ϵ_θ
6.2	z	$CS0$	$SC0$	x	y	$SC0$	$\approx wd$	$\approx P_2S^2$	ϵ_θ
6.3	z	$CS0$	$SC0$	x	z	001	$4l^2d/\pi w$	P_2S^2	$1/\epsilon_\theta$

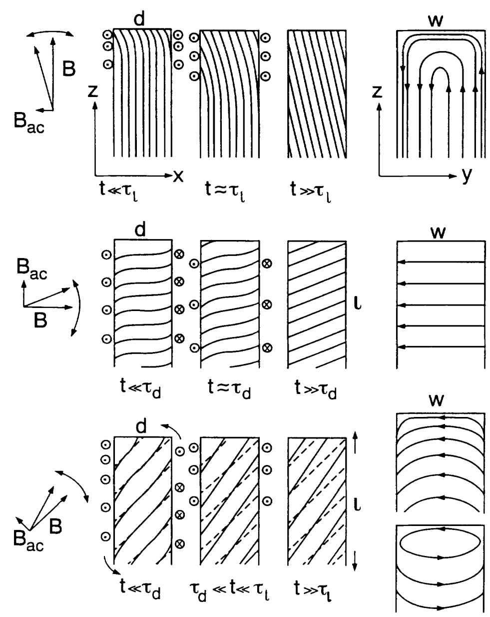

FIG. 1. Visualization of two relaxation modes occurring in a superconducting slab which performs tilt vibrations in a tilted dc field B. Depicted is the case where the tilt of B (about the y axis) is suddenly increased by a small angle ϕ. The surface shielding currents occurring immediately after this tilt are shown as circles with dots and crosses. The surface current pattern on both flat sides is depicted at the right. The figures at the left show how the deformation of the flux-line lattice proceeds. Top: $\mathbf{B}\|\mathbf{z}$, longitudinal mode (mode 2 of Table I). The flux-line tilt by an angle ϕ diffuses from the ends ($z = \pm l/2$) to the middle of the slab in a time $\tau_l = \tau_2 \approx l^2 d/\pi^2 wD$. Middle: $\mathbf{B}\|\mathbf{x}$, perpendicular mode (mode 1 of Table I). The short flux lines curve and straighten again in a time $\tau_d = \tau_1 = d^2/\pi^2 D$. Bottom: B tilted away from x by an angle θ, which is suddenly increased, $\theta \to \theta + \phi$. In a first, fast relaxation mode the flux lines curve and straighten across the slab thickness to realign along B (mode 3.1, like mode 1). This small additional tilt increases the flux density, which thus has to expand in a second, slower mode, either along z (mode 3.2, depicted here) or along y (mode 3.3, typically faster). The dashed lines indicate flux lines before relaxation.

$D_2 = 0.0984\omega l^2(4d/\pi w)$ is reduced by a factor $\pi w/4d$ due to surface shielding currents.

In the oblique-field case 3 (Fig. 1, bottom) *both modes 1 and 2 occur*: When the slab is tilted by an additional angle $\phi \ll 1$ the (already tilted) flux lines first form an S curve and then stretch until they are again parallel to the applied field; this tilt is equivalent to the penetration of $B_{\mathrm{ac},z}$ with a short relaxation time $\tau_{3.1} = \tau_1 = d^2/\pi^2 D$ (mode 3.1). Such a tilt slightly increases (or decreases) the flux density; the FLL will thus expand (or contract) in a second, slower mode, which means a diffusion of $B_{\mathrm{ac},x}$. This second relaxation can occur along the slab *length* l (mode 3.2) in a time $\tau_{3.2} = \tau_2 = 4l^2 d/\pi^3 w$. The corresponding linear dissipation peaks are reduced by factors $\cos^2\theta$ and $\sin^2\theta$, since $B_{\mathrm{ac},z} = \phi B\cos\theta$, thus $P_{3.1} = P_1\cos^2\theta$, and $B_{\mathrm{ac},x} = \phi B\sin\theta$, thus $P_{3.2} = P_2\sin^2\theta$.

However, the relaxation of $B_{\mathrm{ac},x}$ can also occur along the *width* w of the slab. This *novel mode* 3.3 has no equivalent in the "pure" case 1 or 2 where there is either no driving force (if $\theta = \pi/2$) or no component $B_{\mathrm{ac},x}$ (if $\theta = 0$). For a large range of angles $|\cos\theta| > w/l$, mode 3.3 will be faster than mode 3.2 and thus causes the observed dissipation peak. (After $B_{\mathrm{ac},x}$ has penetrated along y it need no longer penetrate along z.) One thus gets at lower temperature T a peak of height $P_{3.1}$ when $\tau_{3.1} \approx 1/\omega$, and at higher T a peak of height $P_{3.3}$ when $\tau_{3.3} \approx 1/\omega$. Only in almost longitudinal geometry, for $|\cos\theta| < w/l$, the second peak is caused by mode 3.2, with height $P_{3.2} \approx P_{3.3}$ and position $1/\omega \approx \tau_{3.2}$. At the crossover $|\cos\theta| = w/l$, the heights and positions of the peaks 3.2 and 3.3 coincide.

The theory of mode 3.3 (and of 6.2 below) is more difficult than the theory of the remaining, diffusive modes. One has to solve the problem of the penetration of a transversal magnetic field into an infinite slab of width w, thickness d, and resistivity ρ, described by the integral equation (29) of Ref. [15]. Here I *estimate* the relaxation time $\tau_{3.3}$ and dissipation $P(\omega) \approx 2P_{3.3}/(1+\omega^2\tau_{3.3}^2)$ by considering a sudden increase of the perpendicular field component $B_x \to B_x + B_{1x}$ and assuming a compressional motion of the FLL with velocity $v \propto |y|$ held constant over a time interval $\tau_{3.3}$ required for full penetration. From geometry one gets $\langle v^2\rangle = \delta^2/3\tau_{3.3}^2$, where $\delta = (w/2)B_{1x}/B_x$ is the flux-line displacement $\|x$ at the slab edge $|y| = w/2$. Equating the total dissipated energy $U_{\mathrm{diss}} = \eta\langle v^2\rangle\tau_{3.3}lwd$ to the energy stored in the stray field before this relaxes, $U_{\mathrm{field}} = (B_1^2/\mu_0)l\pi w^2/4$ [12,15], and noting that $B_x = B\cos\theta$ and $B^2/\mu_0\eta = D$, one obtains $\tau_{3.3} = wd/3\pi D\cos^2\theta$, in agreement with the estimate of [15], which gave, for $\theta = 0$, $\tau_{3.3} = cwd/\pi^2 D$ with $c \approx 1$. Assuming further an approximate Debye susceptibility $\mu(\omega) \approx 1/(1+i\omega\tau)$, which gives $|\mu''_{\max}| = |\mu''(\omega = 1/\tau)| = 1/2$, and an enhancement of the magnetic energy in (1) by the "stray-field factor" $\pi w/4d$ [12,15], we get for the maximum dissipation $P_{3.3} = \langle\phi^2\rangle\omega B^2\sin^2\theta/4\mu_0 \approx P_2\sin^2\theta$.

Finally, I consider the modes where the slab rotates about its long axis (z axis). In the perpendicular-field case 4 ($\mathbf{B}\|\mathbf{x}$) one has the same τ and $P_{\max}$ as in case 1. The parallel-field case 5 ($\mathbf{B}\|\mathbf{y}$) now corresponds to the more complicated mode 3.3 above; one has $\tau_5 \approx wd/\pi^2 D$ and $P_5 \approx P_2$. In the oblique-field case 6 there are again three modes: In the rapid mode 6.1, $B_{\mathrm{ac},y}$ diffuses along the thickness d, like in mode 3.1, with $\tau_{6.1} = \tau_{3.1} = \tau_1 = d^2/\pi^2 D$. In the slower mode 6.2, $B_{\mathrm{ac},x}$ diffuses along the width w, like in mode 5. I find $\tau_{6.2} = \tau_5 \approx wd/\pi^2 D$ for all θ, and $P_{6.2} = P_5\sin^2\theta$. Finally, mode 6.3 with $B_{\mathrm{ac},x}$ diffusing along the length l is always much slower than mode 6.2, $\tau_{6.3} = (l/w)^2\tau_{6.2}$, and is thus irrelevant. The observed double peak in this geometry thus corresponds to modes 6.1 and 6.2.

The above discussion assumes isotropic flux diffusivity. If the anisotropic diffusivities relevant in mode 3.2 or mode 6.3 were larger by a factor l^2/w^2 than the diffusivities of the competing (and typically faster) modes 3.3 and 6.2, then the longitudinal modes 3.2 and 6.3 would be observed rather than the transverse modes 3.3 and 6.2. However, in experiments on HTSC so far c-oriented slabs were used; i.e., the crystalline c axis was along x. In this situation the flux-flow [19] and TAFF diffusivities of the short flux lines (D_y^x and D_z^x) are much larger than those of the long flux lines moving in the c direction (D_x^z and D_x^y). Therefore, if in c-oriented HTSC the anisotropy of D is accounted for, the *rapid* modes (along the thickness) become even faster, and the *slow* modes (along width or length) slower; the double peak thus splits wider than in the isotropic case.

Explicit expressions for the anisotropic TAFF diffusivity may be obtained from the highly useful scaling prescription [20]. This concept works for uniaxially anisotropic London or Ginzburg-Landau (GL) superconductors if length scales $> \lambda_c$ are unimportant or if B=const may be assumed. Any anisotropic property of the HTSC, even with pinning, is then obtained by a simple transformation from the corresponding property of the isotropic superconductor. This idea also gives the flux-flow and TAFF viscosities. If B forms an angle θ with the c axis one finds $\eta_1(\theta) = \eta/\epsilon_\theta$ if the flux lines move in the B-c plane, and $\eta_2(\theta) = \eta\epsilon_\theta$ if the flux lines move perpendicular to the B-c plane [21]. Here $\eta = \eta_b^c = \eta_a^c$ (for flux flow or TAFF), $\epsilon_\theta^2 = \Gamma^{-2}\sin^2\theta + \cos^2\theta$, and $\Gamma = \lambda_c/\lambda_{ab} = (m_c/m_{ab})^{1/2} \gg 1$ is the anisotropy ratio. Thus, $\eta_b^a:\eta_b^c:\eta_c^a = 1:\Gamma:\Gamma^2$ or $D_b^a:D_b^c:D_c^a = 1:1/\Gamma:1/\Gamma^2$. This result differs slightly from the flux-flow anisotropy of [19], $\eta_b^a:\eta_b^c:\eta_c^a = 1:4\Gamma:3\Gamma^2$, because in the time-dependent GL theory implicitly used in [20] the anisotropies of the resistivity and mass are not independent as assumed in [19]. The anisotropic flux diffusivities of the various modes are listed in the last column of Table I for slabs with $\mathbf{x}\|\mathbf{c}$.

In conclusion, the relaxation times and linear dissipation peaks were obtained for the flux-motion modes occurring in slabs of isotropic or anisotropic superconductors which vibrate or are put into a weak ac field. More work remains to be done if the layered structure of the HTSC or the amplitude dependence become important. These problems are presently under investigation.

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