Peltier Coefficient and Thermal Conductance of a Quantum Point Contact

L. W. Molenkamp, Th. Gravier,(a) H. van Houten, O. J. A. Buijk, and M. A. A. Mabesoone
Philips Research Laboratories, 5600 JA Eindhoven, The Netherlands

C. T. Foxon(b)
Philips Research Laboratories, Redhill, Surrey, RH1 5HA, England
(Received 12 February 1992)

We report the observation of quantum size effects in the thermal conductance and Peltier coefficient of a quantum point contact. Our experimental method involves a novel usage of quantum point contacts as local electron gas thermometers.

PACS numbers: 73.50.Lw, 72.20.Pa, 73.40.Kp

An electrical current I through a conductor is accompanied by a thermal current Q. These currents are driven by the differences in voltage (ΔV) and temperature (ΔT) across the conductor. In the regime of linear response, this is conventionally expressed as [1]

$$-\Delta V = RI + S\Delta T\,, \tag{1}$$

$$Q = \Pi I - \kappa \Delta T\,. \tag{2}$$

Here R is the electrical resistance of the conductor, S the thermopower, Π the Peltier coefficient, and κ the thermal conductance. The cross coefficients S and Π are related by the Kelvin-Onsager relation

$$\Pi = ST\,. \tag{3}$$

In recent years, epitaxial growth and nanofabrication techniques have made it possible to study the regime of quantum ballistic transport [2]. Ideally, in this regime, transport is governed by collisions of charge carriers with the boundaries of the conductor, rather than with impurities or defects. In addition, the lateral size of the conductor can be made comparable to the Fermi wavelength, so that it acts as an electron waveguide.

An experimental realization of such an electron waveguide is the quantum point contact (QPC), an electrostatically defined narrow constriction in the two-dimensional electron gas (2DEG) of an (Al,Ga)As heterostructure. Electron transport through the point contact is ballistic since the mean free path is much larger than the size of the constriction. In the wider regions on either side of the QPC the transport is diffusive. These regions have a very small resistance (compared to that of the QPC) and may be regarded as electron gas reservoirs [2]. The most striking manifestation of quantum ballistic transport is the quantization in units of $2e^2/h$ of the electrical conductance of a QPC [3, 4]. This quantization is observed as a series of plateaus in a plot of the conductance versus the voltage on the gate electrodes defining the constriction; the conductance quantum $2e^2/h$ corresponds to a perfectly transmitted one-dimensional (1D) subband or transverse mode in the QPC.

Theoretically, it has been argued that quantum size effects in a QPC due to the 1D subband structure should occur as well for the thermal conductance, the thermopower, and the Peltier coefficient [1, 5–8]. Indeed, quantum oscillations in the thermopower, which line up with the steps between the plateaus in the conductance, have recently been observed experimentally [9].

The conductance and thermopower of a QPC can both be measured, on applying the appropriate current or temperature difference, simply with a voltmeter. A measurement of the *Peltier coefficient* and the *thermal conductance* requires, in addition, a means of detecting the heat flow Q through the constriction. In this Letter, we present a method by which this is possible. We infer Q from the resulting change in the local temperature δT of the electron gas on one or both sides of the QPC. These temperature changes are detected by measuring the thermovoltage V_{thermo} across additional QPC's situated in those areas. This novel technique has allowed us to observe, for the first time, quantum size effects in Π and κ, as we will show below.

The nanostructures used in our experiments are defined electrostatically in a 2DEG of mobility $\mu = 1.3 \times 10^6$ cm^2/V s and sheet electron concentration $n_s = 3.3 \times 10^{11}$ cm^{-2} in a (Al,Ga)As heterojunction structure. A schematic layout of the samples is shown in Fig. 1(a). The basic feature of the sample geometry is a long channel of width $W_{\text{ch}} = 4$ μm and length $L_{\text{ch}} = 40$ μm . In the center of the channel, a QPC is defined using gates B and E. This is the QPC for which we wish to determine κ and Π. On one edge of the channel, two more QPC's are defined (at a separation d from the central QPC), using gates A,B, and B,C, respectively. These extra point contacts are the ones used as thermometers. Their electrical conductance is adjusted in between the $N = 1$ and $N = 2$ plateaus (where $G \approx 1.5 \times 2e^2/h$), so that the thermopower $S = -(k_B \ln 2)/1.5e \approx -40$ μV/K has its maximum value [6, 7, 9] . The temperature difference δT over the QPC then follows from the thermovoltage V_{thermo} through $\delta T = -V_{\text{thermo}}/S$.

We have fabricated three different types of structures, having $d = 1.5$, 5, and 10 μm. The relatively wide gaps (ca. 2 μm) between gates D and E and E and F on the

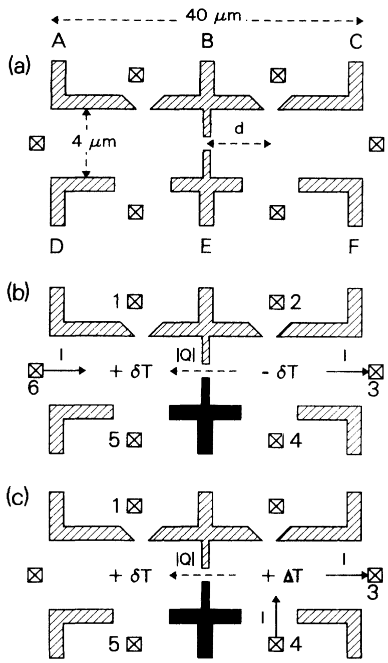

FIG. 1. (a) Schematic of the samples used in the experiments. Hatched areas are gates, and crosses indicate Ohmic contacts. Total length of the channel is 40 μm, and $d = 1.5$, 5, or 10 μm. (b),(c) Illustrations of the experimental configurations used to study, respectively, the Peltier coefficient and the thermal conductance of the central QPC (BE).

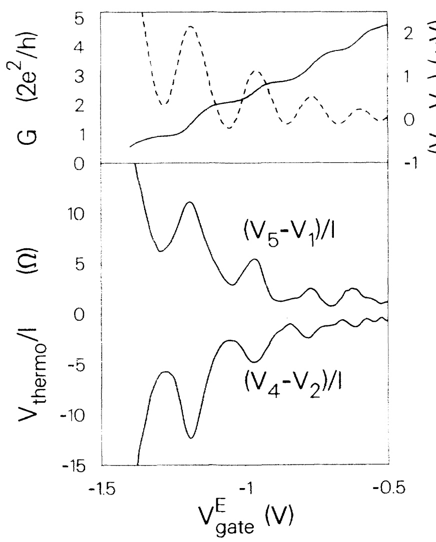

FIG. 2. The bottom panel shows the thermoelectric voltages measured to the left ($V_5 - V_1$) and right ($V_4 - V_2$) of QPC BE, divided by the current ($I \sim 0.2$ μA), as a function of the voltage on gate E (sample with $d = 5$ μm). For comparison, conductance quantization (solid line) and thermovoltage oscillations (dashed line) of QPC BE are plotted in the top panel. In the bottom panel, a positive voltage implies an increase in electron temperature in the channel. The oscillations demonstrate the quantum size effect in the Peltier coefficient Π of QPC BE.

bottom edge of the channel serve to define voltage probes, used as reference in the thermovoltage measurements. We have verified that only negligibly small thermovoltages develop across these gaps, and that the resistance of the probes is not significantly influenced by scanning gate E. The samples are kept in a cryostat at 1.8 K, and all transport measurements are done using low-frequency lock-in techniques, mostly at zero magnetic field. (We have performed some additional experiments at low magnetic fields to verify that the signals reported below are not sensitive to electrons traveling ballistically between the central and thermometer point contacts.)

For a measurement of the Peltier coefficient of the central QPC (BE), we use Ohmic contacts 6 and 3 as current source and drain, respectively, and scan the voltage on gate E [black in Fig. 1(b)]. As is also indicated in this figure, the electrical current I is accompanied by a heat current $Q = \Pi I$. This is the Peltier effect. The heat flow leads to an increase, by an amount δT, of the electron temperature in the channel to the left of QPC BE, and a decrease of equal magnitude to the right of QPC BE. We deduce the electron temperature changes on either side of QPC BE from a measurement of the thermovoltages $V_5 - V_1$ and $V_4 - V_2$. Since the thermopower $S < 0$, these voltages are *positive* if the electron gas temperature in the channel exceeds that in the region behind the QPC's, where the electron gas is assumed to be in equilibrium with the lattice. Since the temperature changes due to the Peltier effect are linear in the applied current [see Eq. (4) below], we detect only the component of the thermovoltages at the fundamental frequency of the ac current. This eliminates contributions due to an increase in the electron gas temperature from dissipation, which is quadratic in I, and positive throughout the channel.

The results of such an experiment, obtained for a $d = 5$ μm sample at $I \simeq 0.2$ μA, are plotted versus gate voltage $V^{\rm E}_{\rm gate}$ in the bottom panel in Fig. 2. The experiment was done under voltage bias; to compensate for the resulting $V^{\rm E}_{\rm gate}$ dependence of the current I we have divided the measured thermovoltages by I. Oscillations in both $(V_5 - V_1)/I$ and $(V_4 - V_2)/I$ are clearly visible. The oscillation maxima are aligned with the steps between conductance plateaus, shown for comparison in the top panel of Fig. 2 (solid line). The amplitude of the oscillations increases for more negative gate voltages, as expected theoretically [7]. The Peltier signals $(V_5 - V_1)/I$ and $(V_4 - V_2)/I$ are of opposite sign, consistent with heating occurring in the left, and cooling in the right part of the channel (for the current direction used). We have verified that both signals are independent of I, up to currents larger by at least 1 order of magnitude (not shown) [10]. This implies that the experiments were done well within the linear transport regime. The oscillatory shape of the curves can be derived theoretically

[7]. It is more instructive, however, to directly compare the Peltier signals with experimental data on the oscillatory thermovoltage ($V_3 - V_6$) of QPC BE (obtained by passing an ac current of 1 μA from Ohmic contacts 2 to 4), shown as the dashed line in the top panel of Fig. 2. The thermovoltage oscillations are due to maxima in the thermopower at gate voltages where a subband in the QPC is depopulated, as was established in a previous paper [9]. The presence of similar oscillations in the Peltier and thermovoltage signals is a direct consequence of the Kelvin-Onsager relation $\Pi = ST$ [Eq. (3)].

To confirm our interpretation of the data in Fig. 2 as a Peltier effect it is thus only necessary to estimate the magnitude of Π. For a given current I, the temperature rise (in the left part of the channel) and decrease (in the right part) of absolute magnitude δT follows from the measured thermovoltages $V_5 - V_1$ and $V_4 - V_2$, using the estimated value for the thermopower $S \approx -40$ μV/K of the thermometer QPC's AB and BC. We can also relate δT to the current I using a heat balance:

$$\frac{c_v A \delta T}{\tau_{\text{e-p}}} \approx Q = \Pi I - \kappa \delta T \,, \tag{4}$$

with $c_v = (\pi^2/3)(k_B T/E_F) n_s k_B$ the heat capacity of the 2DEG per unit area, and A a measure for the area of the 2DEG which has the nonequilibrium electron temperature $T + \delta T$ to $T - \delta T$ (we assume δT to be uniform within this area, and zero beyond it). We use the electron-phonon relaxation time $\tau_{\text{e-p}} \sim 10^{-10}$ s deduced from our previous thermopower experiments [9], and estimate the area A as the product of the channel width and the diffusion length $A = (D\tau_{\text{e-p}})^{1/2} W_{\text{ch}} \sim 50$ μm^2 ($D = 1.5$ m^2/s in this material). Neglecting the second term on the right-hand side of Eq. (4), we thus find for the first Peltier maximum to the right of the $N = 1$ conductance plateau $\Pi = c_v A V_{\text{thermo}}/S\tau_{\text{e-p}} I \approx -130$ μV [11]. This value is within a factor of 2 from the theoretical value for an ideal QPC [6–9], which is $\Pi = ST = -(k_B T \ln 2)/(N + \frac{1}{2})e \approx -70$ μV at the lattice temperature of $T = 1.8$ K. The predicted $1/(N + \frac{1}{2})$ dependence of the maxima does agree with the decrease in amplitude with $V^{\text{E}}_{\text{gate}}$ observed experimentally.

We now turn to the thermal conductance. We use a sample of a similar layout [cf. Fig. 1(c)], but pass an ac current I through the right part of the channel only (using Ohmic contacts 3 and 4). Through current heating, the electron gas temperature in this part of the channel is increased by ΔT (which is proportional to I^2 [9, 12]). The temperature difference ΔT across the central QPC (BE) gives rise to a heat flow Q through this point contact. This causes a small temperature rise $\delta T \ll \Delta T$ in the electron gas in the left part of the channel, which we detect by measuring the thermovoltage $V_5 - V_1$, as before. This time, however, the measurement is performed by means of a lock-in amplifier tuned to the second harmonic of the frequency of the ac current I (since in this experiment $\delta T \propto Q \propto \Delta T \propto I^2 R$).

Figure 3 shows plots of the measured thermovoltage $V_5 - V_1$ as a function of the voltage on the gate E, for a $d = 1.5$ μm sample, and constant heating currents of (curve a) 1.0, (b) 1.3, and (c) 1.5 μA. The thermovoltage scales with I^2, as expected. Sequences of plateaus in the thermovoltage are clearly visible, and each plateau lines up with a quantized conductance plateau of the point contact (dotted curve) [13]. Since the measured thermovoltage is directly proportional to δT, which in turn is proportional to the heat flow Q through the point contact, this result demonstrates the occurrence of a quantum size effect in the thermal conductance $\kappa = -Q/\Delta T$ of a QPC. A theoretical reason for expecting a quantum size effect in κ similar to that in G is the Wiedemann-Franz relation,

$$\kappa \approx L_0 T G \,, \tag{5}$$

where $L_0 \equiv k_B^2 \pi^2 / 3e^2$ is the Lorenz number. Our experiment confirms qualitatively that the Wiedemann-Franz relation holds for a QPC. In contrast to the Kelvin-Onsager relation (3), the Wiedemann-Franz relation is an approximate one, and it is violated when the transmission probability $t(E)$ of a conductor varies strongly with energy (on the scale of $k_B T$). Theoretically [7, 8], small deviations from Eq. (5) have been predicted for an ideal QPC [which has a true step-function transmission probability $t(E)$], at very low temperatures. These deviations become negligible at higher temperatures, or when one uses an expression for $t(E)$ which is derived from a more realistic saddle-point potential model for the QPC.

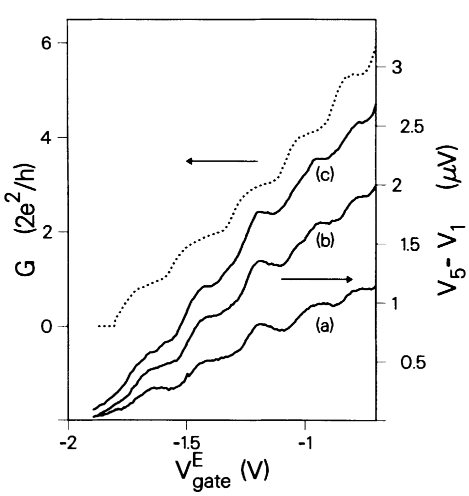

FIG. 3. Electrical conductance (dotted line) of QPC BE and thermovoltage $V_5 - V_1$ as a function of the voltage on gate E (sample with $d = 1.5$ μm). An ac current I is passed between Ohmic contacts 3 and 4. The thermovoltage is detected at the second harmonic frequency of I. Results are shown for (curve a) $I = 1.0$ μA, (b) 1.3 μA, and (c) 1.5 μA. The plateaus demonstrate the quantum size effect in the thermal conductance κ of QPC BE.

To estimate κ from the results in Fig. 3, we again use a heat balance for a region of area A to the left of QPC BE,

$$\frac{c_v A\delta T}{\tau_{\text{e-p}}} \approx Q = -\kappa\Delta T\,. \tag{6}$$

Now consider the $N = 5$ plateau of curve a in Fig. 3, where $V_5 - V_1 \approx 1.1$ μV, implying that $\delta T = -(V_5 - V_1)/S \approx 27$ mK. From an independent thermovoltage measurement across QPC BC we estimate that $\Delta T \approx 0.8$ K for a heating current of 1 μA. Inserting the same values for A, c_v, and $\tau_{\text{e-p}}$ as used above for our estimate of Π, we arrive at an experimental value for the thermal conductance of $\kappa \approx c_v A\delta T/\Delta T\tau_{\text{e-p}} = 3.3\times10^{-11}$ W/K. The Wiedemann-Franz relation (5), using $G = N(2e^2/h)$, implies $\kappa = 1.7\times10^{-11}$ W/K (for the $N = 5$ plateau). Thus the estimate based on our experimental result is off by a factor of 2. All approximations considered, this seems quite satisfactory.

In summary, we have observed pronounced quantum size effects in the Peltier coefficient Π and the thermal conductance κ of a QPC. These effects are due to the 1D subband structure in the QPC. Our results are consistent with the Kelvin-Onsager and Wiedemann-Franz relations, which relate Π to the thermopower S, and κ to the electrical conductance G. Our experiments demonstrate the use of QPC's as sensitive local 2DEG thermometers. This method promises to be useful in other studies of nonequilibrium phenomena in 2DEG's.

The authors would like to thank B. W. Alphenaar, C. W. J. Beenakker, and A. A. M. Staring for valuable discussions, and M. J. P. Brugmans for preliminary measurements of the thermal conductance. This research was partly funded by the European Strategic Programme for Research and Development in Information Technology under Basic Research Action Project No. 3133.

[a] Permanent address: Ecole de Physique de Grenoble, Université Joseph Fourier, 38041 Grenoble CEDEX, France.
[b] Present address: Department of Physics, University of Nottingham, Nottingham NG7 2RD, United Kingdom.

[1] P. N. Butcher, J. Phys. Condens. Matter **2**, 4869 (1990).
[2] A comprehensive review of quantum transport in semiconductor nanostructures is C. W. J. Beenakker and H. van Houten, in *Solid State Physics*, edited by H. Ehrenreich and D. Turnbull (Academic, New York, 1991), Vol. 44, p. 1.
[3] B. J. van Wees, H. van Houten, C. W. J. Beenakker, J. G. Williamson, L. P. Kouwenhoven, D. van der Marel, and C. T. Foxon, Phys. Rev. Lett. **60**, 848 (1988).
[4] D. A. Wharam, T. J. Thornton, R. Newbury, M. Pepper, H. Ahmed, J. E. F. Frost, D. G. Hasko, D. C. Peacock, D. A. Ritchie, and G. A. C. Jones, J. Phys. C **21**, L209 (1988).
[5] U. Sivan and Y. Imry, Phys. Rev. B **33**, 551 (1986).
[6] P. Streda, J. Phys. Condens. Matter **1**, 1025 (1989).
[7] A more extensive discussion of the theory can be found in H. van Houten, L. W. Molenkamp, C. W. J. Beenakker, and C. T. Foxon, in Proceedings of the Seventh International Conference on Hot Carriers in Semiconductors, Nara, 1991 [Semicond. Sci. Technol (to be published)].
[8] C. Proetto, Phys. Rev. B **44**, 9096 (1991); Solid State Commun. **80**, 909 (1991).
[9] L. W. Molenkamp, H. van Houten, C. W. J. Beenakker, R. Eppenga, and C. T. Foxon, Phys. Rev. Lett. **65**, 1052 (1990); in *Condensed Systems of Low Dimensionality*, edited by J. Beeby, NATO Advanced Study Institutes, Ser. B, Vol. 253 (Plenum, New York, 1991), p. 335.
[10] Measurements of Peltier heating and cooling in samples having d= 1.5 or 10 μm for a comparable ac current $I\sim 0.2$ μA yield oscillating thermovoltages of very similar magnitude as found in the $d = 5$ μm sample (Fig. 2). This indicates that for this current level the temperature difference δT is uniform on both sides of the central QPC.
[11] We may neglect $\kappa\delta T$ with respect to ΠI in Eq. (4), as follows: From Fig. 2 we find that $\delta T \approx 40$ mK; using the Wiedemann-Franz relation (5) for evaluating κ we arrive at $\kappa\delta T \approx 2.5\times10^{-13}$ W, which is much smaller than $\Pi I \approx 2\times10^{-11}$ W.
[12] B. L. Gallagher, T. Galloway, P. Beton, J. P. Oxley, S. P. Beaumont, S. Thoms, and C. D. W. Wilkinson, Phys. Rev. Lett. **64**, 2058 (1990).
[13] In the thermal conductance experiments, the thermovoltages measured for a given ac current I in $d = 5$ or 10 μm samples are considerably smaller than in a $d = 1.5$ μm sample (Fig. 3). In order to obtain a reasonable signal-to-noise ratio in experiments on these samples, one needs to increase the ac current to levels where thermal smearing obscures the quantum size effect.