Spatiotemporal Dynamics of Lasers in the Presence of an Imperfect O(2) Symmetry

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We show theoretically and experimentally that an infinitesimal imperfection in the symmetry of a system has a macroscopic influence on the stability of spatiotemporal patterns originated in a symmetry breaking process. This result makes it possible to discuss differences between experiments and models in which such corrections have not been taken into account. We show that the stability of the solutions arising from the first symmetry breaking transition in the Maxwell-Bloch equations is valid for every laser with radial symmetry.

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In this paper we analyze the symmetry breaking process that leads to the formation of spatiotemporal patterns in optical systems. While this subject received some attention during the 1960s [1], it has recently been possible to return to the problem with mathematical tools that were not available at that time [2,3]. Nonlinear dynamical theory and bifurcation theory applied to symmetry groups may be used to explain the transition from simple to complex patterns. This approach is valid for any dynamical system possessing the same symmetry.

The formation of patterns and their dynamical evolution in nonlinear optics has been the subject of several papers during the last years [4-9]. It has been shown experimentally that a spontaneous symmetry breaking process takes place in lasers as the Fresnel number is increased adiabatically [5]. Similar experimental results have been obtained in He-Ne lasers [6], and in photorefractive oscillators [7]. Most of these papers do not address the question of the transition from simple to complicated patterns but just the facts that optical systems display complex structures, and/or that it is possible to characterize them in terms of statistical functions [7], the presence of defects [8,9], or a modal decomposition of the electromagnetic field [4,6,9,10]. A different strategy was taken in Ref. [5] using a group theoretical approach where we provided an interpretation of the transition from a completely symmetric state to one in which all symmetries of the original O(2) group are broken. Although symmetry considerations are independent of particular models, the question arises of how perfect the symmetry of the system has to be to justify an application of such methods to the interpretation of experimental results.

Here we study the transition to spatiotemporal complexity in lasers with "imperfect symmetry," comparing experimental with theoretical results. We show that slight imperfections will not affect the general spatial configuration of the possible solutions, but that they can strongly affect their stability, limiting the validity of numerical results obtained from symmetric models. We also prove that for laser models the stability of the patterns that break the cylindrical symmetry is independent of the parameters and therefore valid for any kind of laser system. By considering imperfect symmetry we are able to predict the bifurcation sequence as it is observed experimentally. To our knowledge a dynamical study of the type presented here does not exist for any experimental physical system.

In our experiment we have used two different CO_2 lasers, one with a cylindrical tube and one with a so-called annulus configuration. In the latter case we allow the interaction between the electromagnetic field and the atoms of the active medium to take place only in a narrow range of values of the radial coordinate. In this way the angular coordinate θ is the only spatial variable of the system and the boundary conditions are periodic. This configuration simplifies the comparison with theoretical results because the symmetry breaking process departs from an initially vanishing electric field.

Both experimental setups are in principle equivariant to rotations around the optical axis [actually O(2) symmetry group]. The first symmetry breaking bifurcation of this group yields two possible solutions: traveling waves (TW) or standing waves (SW) in the azimuthal direction. Then, the field E is described by

$$E=(z_1e^{il\theta}+z_2e^{-il\theta})e^{i\omega t}, \quad (1)$$

where l is the wave number, and the complex functions z_1 and z_2 may depend upon the radial coordinate, and must satisfy the condition

$$dz_k/dt=(\lambda+i\omega)z_k+\sum_j M_{kj}z_k|z_j|^2, \quad (2)$$

where $\lambda+i\omega$, the eigenvalues, and M_{kj}, the coupling coefficients, depend in principle on the parameters of the laser.

Taking "all possible versions" of the well-known Maxwell-Bloch equations [11] we compute the coefficients M_{kj} and the result takes the general form

$$M_{11}=M_{22}=-A,\quad M_{12}=M_{21}=-2A, \quad (3)$$

where A is a positive quantity that depends on the relaxation rates, the detuning, and the gain of the laser. After substituting (3) into (2) we get

$$\begin{aligned} dz_1/dt&=(\lambda+i\omega)z_1-A(|z_1|^2+2|z_2|^2)z_1,\\ dz_2/dt&=(\lambda+i\omega)z_2-A(|z_2|^2+2|z_1|^2)z_2. \end{aligned} \quad (4)$$

If we write $z_k=\rho_k e^{i\phi_k}$, the equations for the amplitudes ρ_k decouple from the equations for the phases ϕ_k. The dynamics of the phases is trivial, and the dynamics of the amplitudes is summarized in Fig. 1. For $\lambda<0$ there is only one fixed stable point, the trivial solution. For $\lambda>0$

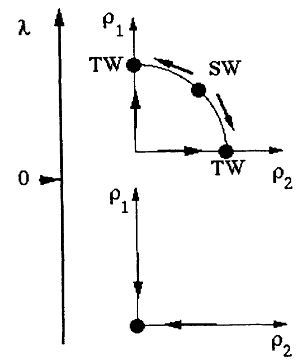

FIG. 1. Phase portrait showing the stable and unstable fixed points as a function of the control parameter λ.

the trivial solution becomes unstable, and three fixed points appear: a saddle $\{\rho_1,\rho_2\}=\{[\lambda/3A]^{1/2},[\lambda/3A]^{1/2}\}$, corresponding to a standing wave, and a pair of attractors: $\{\rho_1,\rho_2\}=\{[\lambda/A]^{1/2},0\}$ and $(\rho_1,\rho_2)=\{0,[\lambda/A]^{1/2}\}$. The two attractors correspond to traveling waves. Therefore the process of symmetry breaking in every homogeneously broadened laser with cylindrical symmetry must begin with the appearance of a TW. It is worthwhile to note that the inclusion of higher-order terms in the normal form reduction does not stabilize the standing waves. Nevertheless, experimental evidence in Ref. [5] and in this paper shows preference for SW solutions.

These discrepancies lead us to study a Z_2 equivariant system (symmetric under reflection), highly degenerate towards an O(2) symmetry. This would be appropriate if a slight asymmetry is considered in any of the laser parameters (e.g., cavity losses or excitation current). In this case Eqs. (4) become [12]

$$\begin{aligned} dz_1/dt &= (\lambda+i\omega)z_1 - A(|z_1|^2+2|z_2|^2)z_1+\varepsilon z_2\,, \\ dz_2/dt &= (\lambda+i\omega)z_2 - A(|z_2|^2+2|z_1|^2)z_2+\varepsilon z_1\,, \end{aligned} \tag{5}$$

where $\varepsilon=\rho_\varepsilon e^{i\alpha}$ is the symmetry breaking parameter which is assumed to be much smaller than A in modulus. In Eqs. (5) it is no longer possible to decouple amplitudes and phases; yet, the effective dimension of the system is three and not four, as the equation for $\phi_1+\phi_2$ decouples from the equation for ρ_1, ρ_2, and $\theta=\phi_1-\phi_2$.

For $\alpha=0$ and $\varepsilon\neq0$ the TW solutions no longer exist, and for values of the control parameter λ in the range $-\rho_\varepsilon<\lambda<2\rho_\varepsilon$ the SW is the stable solution. When λ grows greater than $2\rho_\varepsilon$ a pitchfork bifurcation takes place, the SW solution becomes unstable, and a new pair of attractors appear. They are a superposition of a SW and a TW, both with the same optical frequency ω. These results are summarized in Fig. 2.

In the general case of $\alpha\neq0$, the stability of the SW solution is easily obtained from a linear stability analysis. If $\alpha=\pi/4$, the SW solution is destabilized by a secondary Hopf bifurcation. This new solution with two frequencies is called a modulated wave (MW) [12].

Experimentally we study the influence of slight asymmetries with an annulus laser. We begin our studies with the annulus laser since it has the advantage of quenching cylindrically symmetric distributions of the laser intensity I; then the symmetry breaking bifurcation has to occur from the trivial solution ($I=0$) and the temporal dynamics of the intensity becomes very simple. The spatial distribution of the average intensity is observed with infrared image plates while the temporal behavior is recorded with two HgCdTe detectors. One of the detectors is mounted on a rotating table to allow the measurement of the relative phase of the intensity oscillations at different places in the pattern. The degree of imperfection in the O(2) symmetry is controlled by laterally displacing an intracavity iris. When this diaphragm is centered we have the maximum possible symmetry in our system.

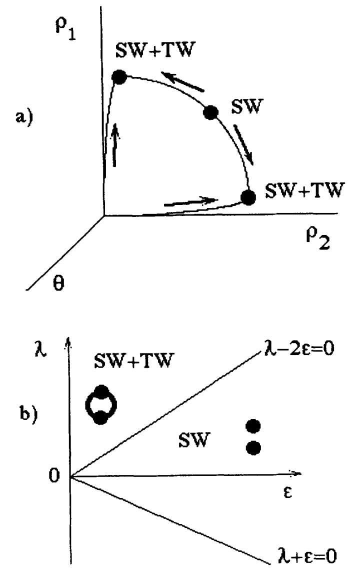

FIG. 2. (a) As Fig. 1 with $\varepsilon\neq0$ and $\lambda>2\varepsilon$. (b) Region in parameter space where the SW and SW+TW are stable.

The bifurcation parameter used in this experiment is the gain of the CO_2 medium which increases as the excitation current increases. For currents of the order of 5 mA the laser is below threshold and the output intensity is zero. As the current increases above 5 mA, the laser intensity increases and its spatial distribution consists of several maxima in the azimuthal direction at a given radial position [Fig. 3(a)]. The detectors show no evidence of intensity oscillations, and the intensity vanishes between consecutive peaks. This configuration corresponds obviously to a standing wave of the electromagnetic field in the azimuthal direction oscillating at a single optical frequency and with wave number $l=7$. It can be associated with the SW solution of the theoretical results in the region in which $\lambda<2\rho_\varepsilon$. Changing adiabatically the control parameter leads to a secondary bifurcation. The spatial intensity distribution [Fig. 3(b)] shows a continuous ring superimposed on the previous peaks. The intensity is still time independent but it does not vanish between the

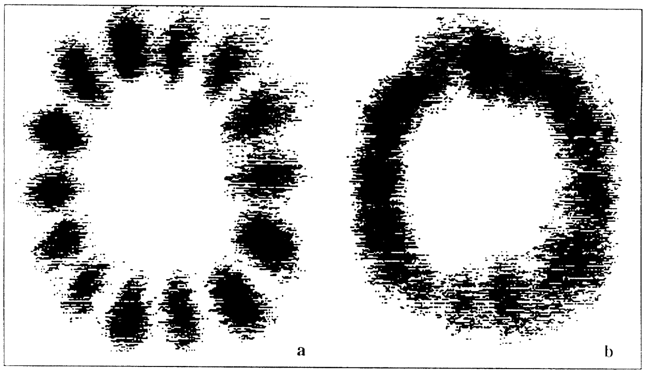

FIG. 3. Spatial intensity distribution of the laser: (a) At low excitation current 14 maxima with the same radial coordinate and equally spaced in the azimuthal direction are observed. (b) At high excitation current a ring appears superimposed to the SW. The intensity is time independent evidencing that the SW and TW superimposed have the same frequency.

peaks. This pattern can only be interpreted as the superposition of a standing wave and a traveling wave both oscillating at the same optical frequency. This observation is evidence that the imperfections are playing a fundamental role on the pattern formation because this solution does not exist in the case of a perfect O(2) symmetry where a superposition of a SW+TW is always followed by a secondary Hopf bifurcation that gives rise to oscillations in the intensity. A further increase in the excitation current generates a more homogeneous pattern in the spatial distribution of the intensity which is still time independent. This corresponds to a predominance of the TW over the SW solution. However, the presence of both is determined by the difference in intensity as a function of the angular coordinate.

If this pattern is perturbed, long transients (of the order of seconds) appear. During these periods of time intensity oscillations are present, and two particular points are distinctively observed in the spatial distribution of the time averaged intensity [Fig. 4(a)]. The time behavior is periodic, and the amplitude of the oscillations vanishes at those particular points in space characterized by a minimum of the intensity [Fig. 4(b)]. As the position of one of the detectors is rotated along the ring, a gradual change of the phase of the oscillations is observed. These measurements indicate the presence of two waves traveling in opposite directions departing from a single point and ending at another point on the opposite side of the pattern. This may be explained by the formation of a transient source and sink of waves [13]. However, after the transient the order is reestablished and just one of the two traveling waves survives as the asymptotic solution superimposed to a standing wave.

By carefully centering the iris we decrease the amount of imperfection to the O(2) symmetry. The qualitative behavior of the system is very similar with the decreasing region of the stability of the SW solution. For high current values (14 mA) we observe stable periodic oscillations in the output intensity. This solution can be associated with the MW described above.

Since the theoretical results do not make use of the particular annular cross section of our laser, they are valid for any configuration with cylindrical symmetry, in particular for a laser in which the central region remains active. Because of this we tested the scenario with such a laser and we obtained results qualitatively identical to those of the annulus. Differences arise due to the presence of radially symmetric solutions from which the SW and SW+TW bifurcate. This difference causes oscillations in the intensity due to beating between the patterns, and the average intensity does not provide by itself an unambiguous method for the identification of the patterns. However, the presence of radially symmetric solutions is an indication of the high degeneracy the system

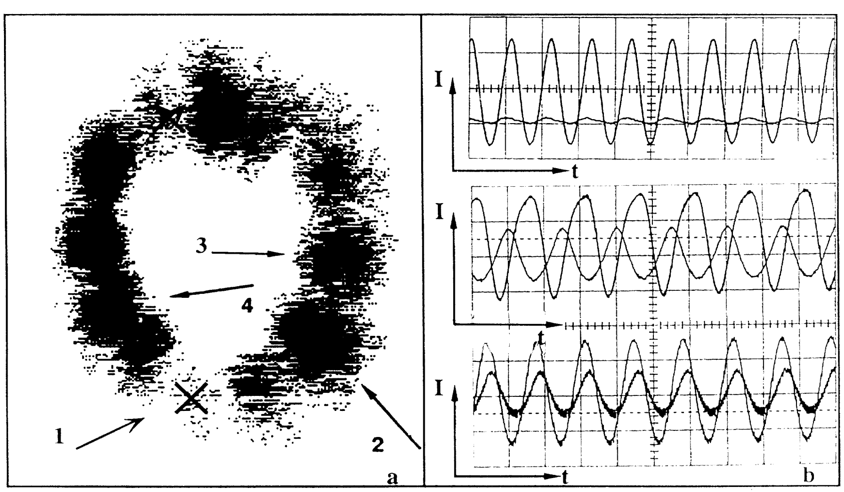

FIG. 4. (a) Intensity distribution during transients. (b) From top to bottom: Transient oscillations of the intensity. The high amplitude signal was taken in point 3 of (a). The low amplitude signal corresponds to points 1 (source of waves), 2, and 4 of (a), respectively.

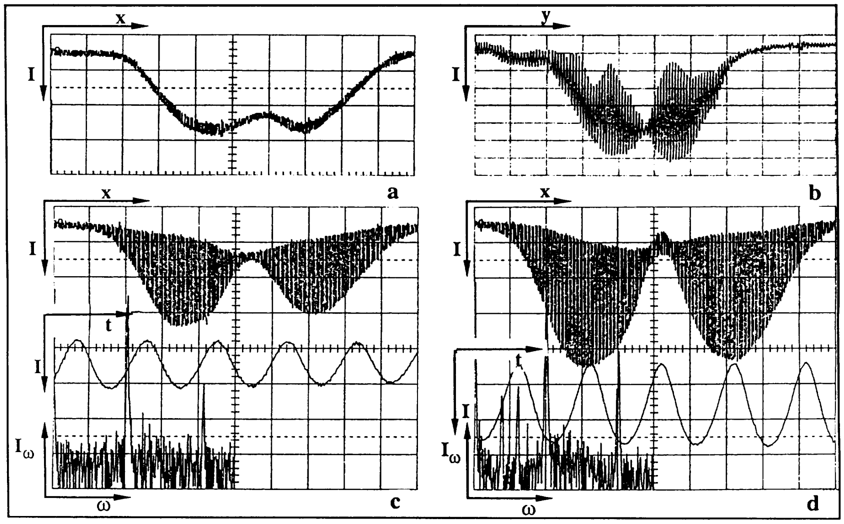

FIG. 5. Intensity across different patterns in a laser with the central region active. (a) Standing wave solution. (b),(c) Cylindrically symmetric+(SW+TW) solution. (d) Cross section of a MW solution. Two frequencies are observed in the power spectrum.

has towards an O(2) symmetry. The appearance of SW, SW+TW, and MW is documented in Fig. 5.

In conclusion, we demonstrate that experimental imperfections to the general symmetry of the system play a determinant role in the stability of spatiotemporal patterns in spatially extended systems. Theoretical models that will not contain this correction may generate not only quantitative but also qualitative features that are not experimentally reproducible even if the corrections are infinitesimal. On the other hand, we also prove that a simple generalization of the bifurcation theory applied to the underlying symmetry group is able to predict the structures of the "real" patterns. The results of this generalization can be applied to any physical system possessing the same symmetry. In the case of laser systems the qualitative behavior of the transition from symmetric to asymmetric patterns is parameter independent. Finally we believe that symmetries are frequent in nature. Therefore we usually forget how nongeneric they are in mathematical terms. Small perturbations that break the symmetry of a system can have a dramatic influence in its solutions.

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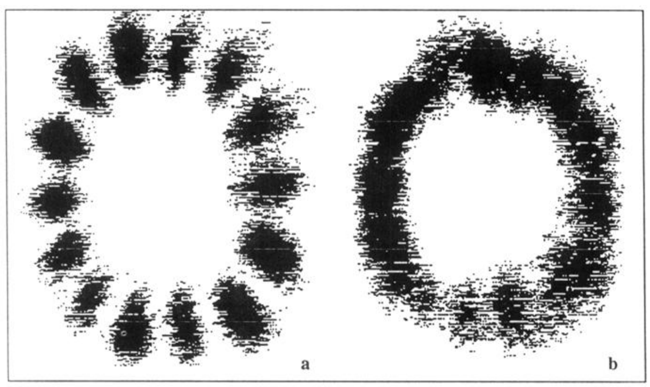

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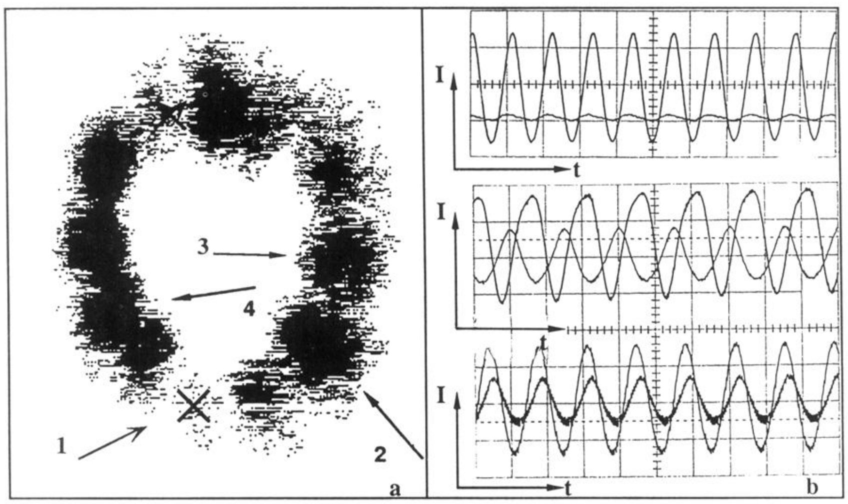

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