Euclidean Proton Response in Light Nuclei

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Green's-function Monte Carlo methods are used to determine the Euclidean proton response of light nuclei from a realistic Hamiltonian containing two- and three-nucleon potential models. Final-state interactions are exactly included in this approach. The calculated Euclidean proton response functions for the ^{4}He nucleus are found to be in excellent agreement with those obtained from an analysis of the longitudinal response functions measured in (e,e') experiments at the Bates and Saclay laboratories.

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Inclusive and exclusive electron scattering experiments performed on ^{3}H, ^{3}He, and ^{4}He [1-5] in the last decade have unequivocally demonstrated that the plane-wave impulse approximation (PWIA) is too naive a framework for describing the nucleon knockout process in the quasielastic regime at intermediate energies. Indeed, PWIA calculations of the longitudinal and transverse response functions predict far more strength than is experimentally observed in the quasielastic peak region, particularly at low momentum transfer [6].

Progress beyond the PWIA has been made in the theoretical description of the three- and four-body nuclei electromagnetic response by a number of different approaches, such as continuum Faddeev ($A=3$) [7], real-time path integral Monte Carlo (PIMC) ($A=4$) [8], or orthogonal correlated states (OCS) ($A=3$ and 4) [9,10] methods. However, it should be emphasized that all of these techniques involve approximations, of varying degrees of severity. For example, the Faddeev or PIMC calculations are based upon simple interaction models, such as the central Malfliet-Tjon potential [11], while in the OCS method the final-state interactions (FSI) affecting the knocked-out nucleon are accounted for phenomenologically through an optical potential.

The present work is an exact Green's-function Monte Carlo (GFMC) simulation of the Euclidean (or imaginary-time) proton response function of the ^{4}He nucleus, based upon the realistic Hamiltonian

$$H=\sum_{i=1}^{A}\frac{\mathbf{p}_i^2}{2m}+\sum_{i<j=1}^{A}v_{ij}+\sum_{i<j<k=1}^{A}V_{ijk}\,,$$

where v_{ij} and V_{ijk} are the Argonne v_8 two-nucleon [12] and Urbana model-VIII three-nucleon [13] interaction models, respectively. The v_8 model, a simplification of the Argonne v_{14} potential [14], reproduces deuteron properties and S- and P-wave phase shifts up to energies of 400 MeV in the laboratory. The above Hamiltonian overbinds the α particle by about 1 MeV in exact GFMC ground-state calculations [15]. The Euclidean proton response function $E_p(k,\tau)$ is defined as

$$E_p(k,\tau)=\frac{e^{-\tau\omega_{el}}}{Z}\langle 0|\rho_p^\dagger(\mathbf{k})e^{-\tau(H-E_0)}\rho_p(\mathbf{k})|0\rangle - E_{p,el}(k,\tau)\,,$$

where $|0\rangle$ represents the A-nucleon ground state with energy E_0, and $\rho_p(\mathbf{k})$ is the nuclear charge density operator in the impulse approximation,

$$\rho_p(\mathbf{k})=\sum_{i=1}^{A}P_{p,i}e^{i\mathbf{k}\cdot(\mathbf{r}_i-\mathbf{R}_{c.m.})}\,.$$

Here $P_{p,i}$ is a projection operator requiring that particle i be a proton, and $\mathbf{R}_{c.m.}$ denotes the center-of-mass position. The contribution due to the ground state recoiling with momentum $\mathbf{k}$ and elastic energy $\omega_{el}=k^2/2Am$ is given by

$$E_{p,el}(k,\tau)=\frac{e^{-\tau\omega_{el}}}{Z}\langle 0|\rho_p(\mathbf{k})|0\rangle^2\,.$$

The Euclidean proton response function is essentially the Laplace transform of the longitudinal response function $R_L(k,\omega)$ measured in inclusive electron scattering experiments,

$$E_p(k,\tau)=\frac{1}{Z}\int_{\omega_{th}}^{\infty}d\omega\, e^{-\tau\omega}S_p(k,\omega)\,,$$

where $R_L(k,\omega)=[G_{E,p}(k,\omega)]^2S_p(k,\omega)$, and $G_{E,p}$ is the proton electric form factor. We assume that the interaction of a longitudinally polarized virtual photon with a nucleus is well approximated by the coupling given by $\rho_p(\mathbf{k})$. In fact, a study of the $^4\mathrm{He}(e,e'p)^3\mathrm{H}$ reaction suggests that two-body components in the nuclear charge operator, such as those associated with pion production, only lead to a small quenching [roughly (1-2)%] of the longitudinal strength in the momentum transfer range of interest here ($k\le 500$ MeV/c) [10].

In order to obtain $E_p(k,\tau)$, one must integrate $S_p(k,\omega)$ from threshold up to $\omega=\infty$, including contributions from both spacelike ($\omega<k$) and timelike ($\omega>k$) regions. In practice, $S_p(k,\omega)$ can be measured up to some energy $\omega_{max}<k$ by inelastic (e,e') scattering experiments. The unobserved strength contributes roughly

(5–10)% to the Coulomb sum $[E_p(k,\tau=0)]$ [16], but this contribution decreases rapidly for finite τ due to the exponential damping factor $\exp(-\tau\omega)$. Precisely this factor allows one to compare the calculated Euclidean response directly to data, without the ambiguities associated with comparisons of Coulomb or energy-weighted sums [16].

GFMC and related methods [17] have been successfully used to study the ground-state properties of a wide variety of strongly interacting many-boson and many-fermion systems [18]. In particular, methods suitable for the nuclear many-body problem, which allow the complicated spin-isospin structure of the two- and three-nucleon interactions to be treated in full, have been developed and discussed in Refs. [15] and [19], and can be easily generalized to evaluate the Euclidean response. In essence, GFMC involves evaluating the imaginary-time propagator by splitting it up into many small steps $\exp(-\tau H)=\prod_{n=1}^{N}\exp[-(n/N)\tau H]$, choosing an accurate approximation to the short-time propagator, and using Monte Carlo techniques to sample the propagator.

Inserting complete sets of position eigenstates into the right-hand side of the equation defining $E_p(k,\tau)$, we obtain

$$E_p(k,\tau)+E_{p,\mathrm{el}}(k,\tau)=\frac{e^{-\tau\omega_{\mathrm{el}}}}{Z}\int d\mathbf{R}\,d\mathbf{R}'\langle 0|\rho_p^\dagger(\mathbf{k})|\mathbf{R}'\rangle\langle\mathbf{R}'|e^{-\tau(H-E_0)}|\mathbf{R}\rangle\langle\mathbf{R}|\rho_p(\mathbf{k})|0\rangle\,,$$

where $\mathbf{R}$ and $\mathbf{R}'$ denote the initial and final positions of the particles relative to the center of mass. To evaluate this expression, we begin with a set of configurations drawn from a probability density proportional to the square of the wave function, summed over all spin-isospin states. The original coordinates of the configuration are stored, and the amplitudes separated into different components, each corresponding to a proton projection operator for a different particle. These projected configurations are propagated using an importance-sampled propagator which incorporates the ratio of the ground-state wave functions $\Psi_0(\mathbf{R}')/\Psi_0(\mathbf{R})$. Each step in the propagation can be used to produce the response at an increasing value of τ. The response is obtained as an average over these configurations:

$$E_p(k,\tau)+E_{p,\mathrm{el}}(k,\tau)=\frac{e^{-\tau\omega_{\mathrm{el}}}}{Z}\sum_{i,j=1}^{A}\frac{1}{M}\sum_{m=1}^{M}j_0(k|\mathbf{r}'_{i,m}-\mathbf{r}_{j,m}|)\langle\Psi_0^\dagger(\mathbf{R}'_m)P_{p,i}|e^{-\tau(H-E_0)}|P_{p,j}\Psi_0(\mathbf{R}_m)\rangle\,,$$

where the sum over m runs over the sampled configurations, and the spherical Bessel function j_0 is obtained by averaging over the solid angle of the momentum transfer. More general expressions are trivially obtained in cases involving polarization, or if one wishes to do a multipole analysis. The latter may be a useful tool for investigating the continuum spectra of light nuclei. In the present work, we have employed a variational wave function [13] for the starting point of our calculation. In order to correct for the most important deficiencies of this approach, we normalize $E_p(k,\tau)$ by the matrix element of $\langle 0|\exp[-\tau(H-E_0)]|0\rangle$, which is just 1 for the exact ground-state wave function. Corrections to this approximation can be determined by using configurations obtained in ground-state Faddeev ($A=3$) or GFMC ($A=4$) calculations as a starting point.

We have computed the Euclidean response functions for deuterium (as a test of the method) and for the α particle. The ^{4}He results are compared both with those of PWIA calculations and with the experimental data. The $E_p(k,\tau;\mathrm{PWIA})$ is obtained from the Laplace transform of

$$S_p(k,\omega;\mathrm{PWIA})=\int d\mathbf{p}\,N_p(p)\,\delta\left[\omega-E_s-\frac{(\mathbf{p}+\mathbf{k})^2}{2m}-\frac{\mathbf{p}^2}{2(A-1)m}\right],$$

where $N_p(p)$ is the proton momentum distribution (normalized to Z) in the ground state. This equation ignores the initial energy distribution of the struck proton, replacing it by an average separation energy E_s. Indeed, the calculation of the true PWIA response requires knowledge of the spectral function $P_p(p,E)$; however, we have taken $P_p(p,E)=N_p(p)\delta(E-E_s)$. This factorization is exact for the deuteron, but only approximate for $A>2$ [we take $E_s=E_0(^3\mathrm{H})-E_0(^4\mathrm{He})$]. As is easily seen, $E_p(k,\tau=0;\mathrm{PWIA})=1$, independent of momentum transfer (and of the factorization approximation). Hence, the PWIA violates the Coulomb sum rule, which at low and intermediate k ($k<500$ MeV/c) is less than 1. This violation leads to the PWIA overprediction of strength in the quasielastic peak.

The experimental $E_p(k,\tau;\mathrm{EXP})$ is obtained from the measured longitudinal response. Obviously, this requires truncating the ω integral at some $\omega=\omega_{\max}$. In order to estimate the contribution for $\omega>\omega_{\max}$, we assume that $S_p(k,\omega>\omega_{\max})$ is proportional to that of the deuteron, which can be accurately calculated. The constant of proportionality is then determined by requiring that the Coulomb sum be satisfied exactly. We have verified that this procedure yields tails which join the experimental data smoothly, and that the energy-weighted sum is within a few percent of that calculated by direct evaluation in the ^{4}He ground state [16]. The parametrization described above is suggested by the proportionality of the calculated energy- and energy-square-weighted sums in the three- and four-body nuclei to those in the deuteron. Indeed, at high ω the response is sensitive to short-range nucleon-nucleon correlations, which are expected to be rather A independent.

In order to assess the reliability of the present method,

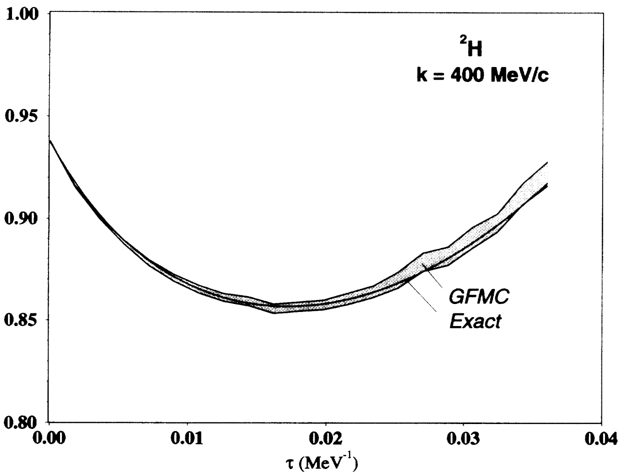

FIG. 1. A comparison of E_p(GFMC) and E_p(EXACT) in ^{2}H at $k=400$ MeV/c. The $E_p(k,\tau)$ is multiplied by $\exp(\tau k^2/2m)$, so that (nonrelativistically) the Euclidean response of a free proton would be 1. The shaded area denotes the GFMC predictions within 1 standard deviation.

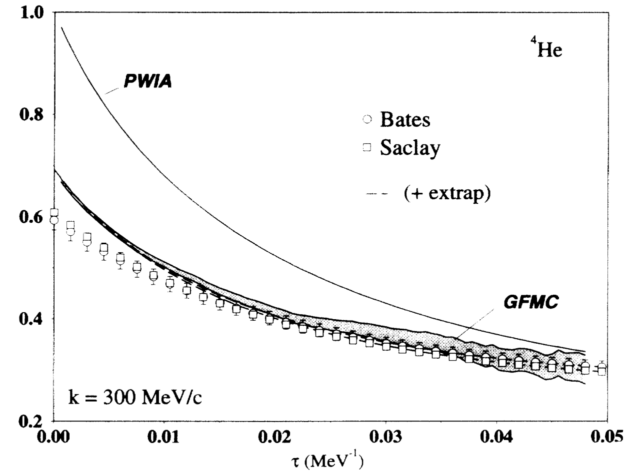

FIG. 2. The ^{4}He Euclidean response functions (scaled as in Fig. 1) at $k=300$ MeV/c. The GFMC and PWIA results are compared with those obtained from the Bates and Saclay data, with and without inclusion of the high ω tail corrections.

we compare the ^{2}H E_p(GFMC) with that obtained by first calculating $S_p(k,\omega)$ with the exact two-nucleon bound and scattering states, and by then evaluating the Laplace transform. The two calculations are in excellent agreement, as shown in Fig. 1.

In Figs. 2 and 3 the ^{4}He $E_p(k,\tau)$ calculated with the GFMC method is compared with those obtained in the PWIA, and by Laplace transforming the Bates [3] and Saclay [4] longitudinal data, both with and without the estimated corrections of the high ω tails. These tail contributions are larger for the Saclay data than for the Bates data, and typically are (5–10)% at $\tau=0$. However, they decrease rapidly with τ, and become negligible for $\tau > 0.02$ MeV^{-1}. At these values of τ, $E_p(k,\tau)$ is really sampling the strength in the quasielastic peak region. The effects of FSI are large, as indicated by the difference between the GFMC and PWIA results, particularly at low k. Note that the FSI are included exactly in the GFMC calculation. At low τ, E_p(GFMC) $< E_p$(PWIA), as expected from sum rule considerations. However, at high τ, the trend is reversed, indicating that FSI enhance the response on the low ω side of the quasielastic peak. This high τ region emphasizes the low ω end of the continuum spectrum, which is strongly affected by collective excitation modes in the system.

We have also explored the possibility of inverting the Laplace transform to obtain $S_p(k,\omega)$. Direct numerical inversion of such a transform is impossible, in general, due to the statistical errors inherent in the calculation. However, methods to obtain dynamic information from Euclidean simulations are currently being developed [20]. In this particular case, we can exploit our knowledge of the dominant features of the response, a large quasielastic peak with a long high-energy tail, to perform a least-squares fit to the E_p(GFMC). We parametrize

$$E_p(k,\tau;\text{FIT}) = E_p(k,\tau;\text{PWIA}) + \sum_{l=1}^{L} a_l(k) F_l(k,\tau).$$

A convenient set of functions F_l is

$$F_l(k,\tau) = e^{-\tau\omega_{\text{th}}}/[\tau+\alpha_l(k)]^2,$$

with $\alpha_l(k)=\alpha(k)/l$. The coefficients a_l are adjusted to minimize the χ^2. In Fig. 4 we display the longitudinal response function obtained by a three-parameter fitting procedure (curve labeled GFMC), which already yields an excellent χ^2, along with the PWIA results, and the Bates and Saclay data. By construction, the R_L(GFMC-FIT) satisfies the Coulomb sum. It is worth noting that a similar strength reduction was obtained in the OCS calculation of the $A=3$ nuclei response func-

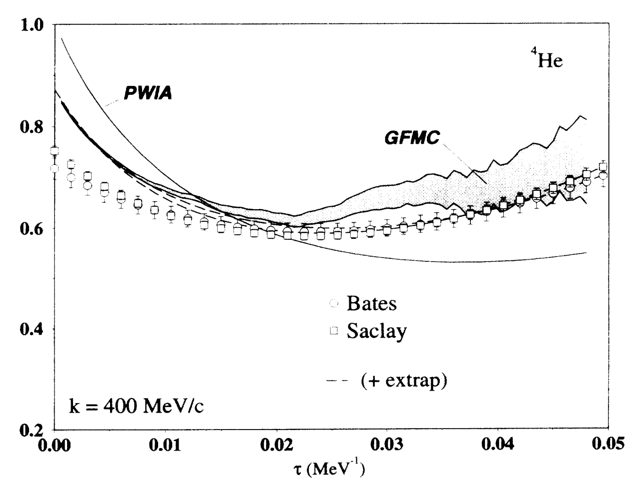

FIG. 3. Same as Fig. 2, but at $k=400$ MeV/c.

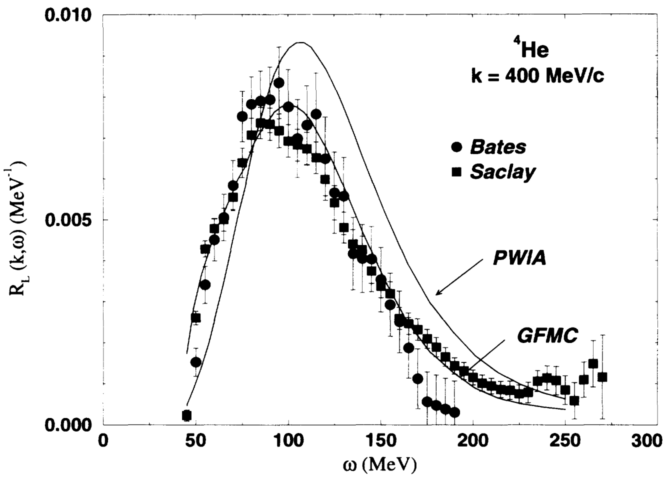

FIG. 4. The ^{4}He longitudinal response functions at $k = 400$ MeV/c obtained from a fit of E_p(GFMC) (curve labeled GFMC) and in PWIA are compared with the Bates and Saclay data.

tions as a consequence of the orthogonality imposed upon the OCS wave functions describing the ground state and two- and three-body breakup channels [9]. The approximate nature of treating the FSI in the OCS approach, however, did not produce a large enough shift of strength towards low ω.

To summarize, GFMC simulations of the ^{4}He proton response in imaginary time have been carried out for a realistic nuclear Hamiltonian. The results of these calculations, in which FSI are included exactly, are in excellent agreement with the experimental data. The present method can be easily generalized to calculate other properties, such as Euclidean transverse response functions with one- and two-body current operators, or Euclidean proton spectral functions, of light nuclei. Because of the special nature of these systems, it appears feasible to obtain reliable estimates of these properties in real time. Work in these areas is being vigorously pursued.

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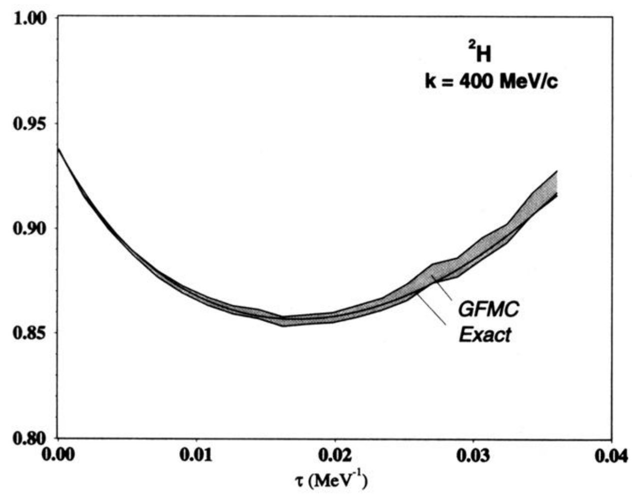
[1] C. Marchand *et al.*, Phys. Lett. **153B**, 29 (1985); K. Dow *et al.*, Phys. Rev. Lett. **61**, 1706 (1988).
[2] E. Jans *et al.*, Phys. Rev. Lett. **49**, 974 (1982); C. Marchand *et al.*, Phys. Rev. Lett. **60**, 1703 (1988).
[3] K. F. von Reden *et al.*, Phys. Rev. C **41**, 1084 (1990).
[4] J. F. Danel *et al.* (unpublished); J. Morgenstern (private communication).
[5] A. Magnon *et al.*, Phys. Lett. B **222**, 352 (1989); J. F. J. van den Brand *et al.*, Phys. Rev. Lett. **66**, 409 (1991).
[6] H. Meier-Hajduk *et al.*, Nucl. Phys. **A395**, 332 (1983); C. Ciofi degli Atti, E. Pace, and G. Salmé, Phys. Lett. **141B**, 14 (1984).
[7] E. van Meijgaard and J. A. Tjon, Phys. Lett. B **228**, 307 (1989); Phys. Rev. C **42**, 74 (1990).
[8] C. Carraro and S. E. Koonin, Nucl. Phys. **A524**, 201 (1991).
[9] R. Schiavilla and V. R. Pandharipande, Phys. Rev. C **36**, 2221 (1987); R. Schiavilla, Phys. Lett. B **218**, 1 (1989).
[10] R. Schiavilla, Phys. Rev. Lett. **65**, 835 (1990).
[11] R. A. Malfliet and J. A. Tjon, Nucl. Phys. **A127**, 161 (1969).
[12] R. B. Wiringa (unpublished).
[13] R. B. Wiringa, Phys. Rev. C **43**, 1585 (1991).
[14] R. B. Wiringa, R. A. Smith, and T. L. Ainsworth, Phys. Rev. C **29**, 1207 (1984).
[15] J. Carlson, Nucl. Phys. **A522**, 185c (1991).
[16] R. Schiavilla, A. Fabrocini, and V. R. Pandharipande, Nucl. Phys. **A473**, 290 (1987); R. Schiavilla, V. R. Pandharipande, and A. Fabrocini, Phys. Rev. C **40**, 1484 (1989).
[17] M. H. Kalos, Phys. Rev. **128**, 1791 (1962).
[18] M. A. Lee, K. E. Schmidt, M. H. Kalos, and G. V. Chester, Phys. Rev. Lett. **46**, 728 (1981); D. M. Ceperley and E. L. Pollock, Phys. Rev. Lett. **56**, 351 (1986).
[19] J. Carlson, Phys. Rev. C **38**, 1879 (1988).
[20] R. N. Silver, D. S. Sivia, and J. E. Gubernatis, Phys. Rev. B **41**, 2380 (1990); J. E. Gubernatis, M. Jarrell, R. N. Silver, and D. S. Sivia, Phys. Rev. B **44**, 6011 (1991).



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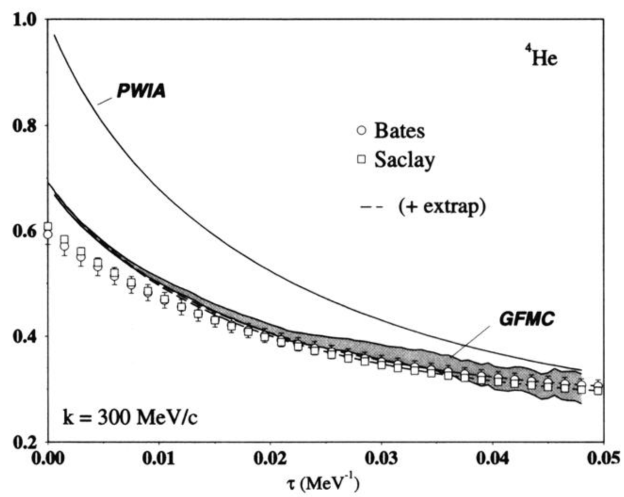

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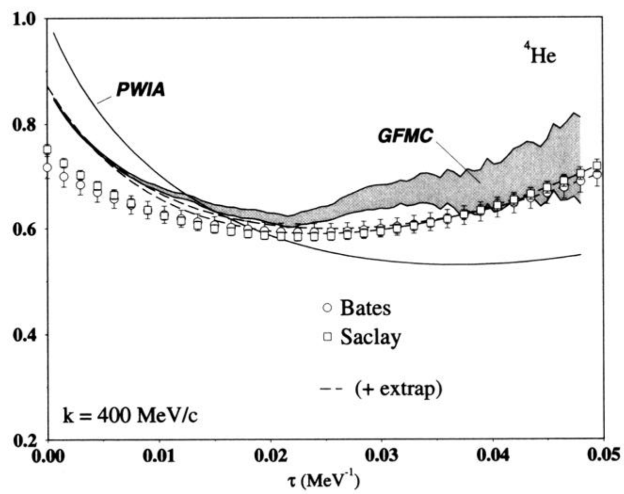

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