Limits on the *CP*-Even Higgs-Boson Masses in the Minimal Supersymmetric Model

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We calculate corrections to the masses of the two *CP*-even Higgs bosons in the minimal supersymmetric model. We find an upper bound for the mass of the lighter Higgs scalar, and a lower bound for the mass of the heavier Higgs scalar. In our analysis we consider all possible variations of superparticle masses between 0.1 and 1 TeV. By requiring the light Higgs boson to be greater than the current experimental bound we rule out a region of the $\tan\beta$–top-mass parameter space, where $\tan\beta$ is the ratio of vacuum expectation values of the two Higgs fields. We make our formalism explicit to elucidate the treatment of mass thresholds.

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We address two questions of current phenomenological interest. In the minimal supersymmetric model (MSSM) [1] there are two *CP*-even Higgs bosons. These particles, which we refer to as h^0 and H^0, are respectively lighter and heavier than the Z^0 boson at tree level. We utilize the effective potential to determine the heaviest possible h^0 mass and the lightest possible H^0 mass in the MSSM to one-loop order.

At tree level h^0 is constrained to be less massive than $M_Z|\cos 2\beta|$, where $\tan\beta = v_2/v_1$ is the ratio of vacuum expectation values of the two Higgs fields. This bound is saturated for $m_A \gg M_Z$. Similarly, H^0 satisfies $m_H \geq M_Z$ at tree level and this inequality saturates when $m_A = 0$. We calculate the leading logarithmic one-loop corrections to these saturated inequalities. Corrections to the masses of the Higgs bosons have appeared in several papers [2]. This work has been further elaborated on in Ref. [3]. Corrections to the charged-Higgs-boson masses have been studied [4], and corrections to Higgs-boson mass sum rules have been calculated [5].

The two Higgs doublets in this model have the charge structure

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}, \tag{1}$$

and these acquire vacuum expectation values $(1/\sqrt{2})\binom{v_1}{0}$ and $(1/\sqrt{2})\binom{0}{v_2}$. We choose v_1 and v_2 to be real and positive. Writing $H_1^0 = (1/\sqrt{2})(S_1 - iP_1)$, $H_2^0 = (1/\sqrt{2})(S_2 + iP_2)$, we have the tree-level potential for the fields S_1 and S_2,

$$V_{\rm tree} = \tfrac{1}{2} m_1^2 S_1^2 + \tfrac{1}{2} m_2^2 S_2^2 - m_3^2 S_1 S_2 + \frac{g^2 + g'^2}{32}(S_1^2 - S_2^2)^2 . \tag{2}$$

The coefficient of the quartic term is a combination of g and g', the SU(2) and U(1) coupling constants, respectively. This is in contrast to the standard model where the coefficient of the quartic term in the Higgs potential is arbitrary. The masses of the *CP*-even Higgs bosons are given by the eigenvalues of the mass matrix

$$m_{ij}^2 = \frac{\partial^2 V}{\partial S_i \partial S_j}, \tag{3}$$

where V is the scalar potential. We define v_1 and v_2 to be the vacuum expectation values of H_1^0 and H_2^0 by requiring

$$\left.\frac{\partial V}{\partial S_1}\right|_{v_1,v_2} = 0 = \left.\frac{\partial V}{\partial S_2}\right|_{v_1,v_2} . \tag{4}$$

At tree level, we can use Eqs. (3) and (4) to obtain the mass relation

$$m_{h,H}^2 = \tfrac{1}{2}\{m_A^2 + M_Z^2 \mp [(m_A^2 + M_Z^2)^2 - 4M_Z^2 m_A^2 \cos^2 2\beta]^{1/2}\}, \tag{5}$$

where $M_Z^2 = \frac{1}{4}(g'^2 + g^2)(v_1^2 + v_2^2)$ and $m_A^2 = m_3^2(\tan\beta + \cot\beta)$. At one-loop level the potential $V^{(1)} = V_{\rm tree} + \Delta V^{(1)}$ can be explicitly modified so that v_1 and v_2 receive no corrections. To do this we simply add to $\Delta V^{(1)}$ terms proportional to S_1^2 and S_2^2 by redefining the tree parameters m_1 and m_2. We have

$$V^{(1)} = V'_{\rm tree} + \Delta V^{(1)\prime}, \tag{6}$$

$$\Delta V^{(1)\prime} = \Delta V^{(1)} + aS_1^2 + bS_2^2 , \tag{7}$$

where the primes indicate the redefined potentials. We determine a and b by requiring

$$\left.\frac{\partial \Delta V^{(1)\prime}}{\partial S_1}\right|_{v_1,v_2} = 0 = \left.\frac{\partial \Delta V^{(1)\prime}}{\partial S_2}\right|_{v_1,v_2} . \tag{8}$$

Hence,

$$a = -\frac{1}{2v_1}\left.\frac{\partial \Delta V^{(1)}}{\partial S_1}\right|_{v_1,v_2}, \quad b = -\frac{1}{2v_2}\left.\frac{\partial \Delta V^{(1)}}{\partial S_2}\right|_{v_1,v_2}, \tag{9}$$

and the correction to the mass matrix is given by

$$\Delta m_{ij}^2 = \left.\frac{\partial^2 \Delta V^{(1)}}{\partial S_i \partial S_j}\right|_{v_1,v_2} - \delta_{i1}\delta_{j1}\frac{1}{v_1}\left.\frac{\partial \Delta V^{(1)}}{\partial S_1}\right|_{v_1,v_2} - \delta_{i2}\delta_{j2}\frac{1}{v_2}\left.\frac{\partial \Delta V^{(1)}}{\partial S_2}\right|_{v_1,v_2} . \tag{10}$$

The one-loop potential has three contributions,

$$\Delta V^{(1)} = \Delta V^{(1)}_{\text{eff pot}} + \Delta V^{(1)}_{\substack{\text{gauge}\\\text{renorm}}} + \Delta V^{(1)}_{\substack{\text{wave}\\\text{function}}}\,, \tag{11}$$

and we discuss these three contributions in turn. We should remark on the renormalizations in Eq. (11). Some authors utilizing the effective potential in calculations do not explicitly include wave function and gauge coupling renormalization in their procedure. Instead, they introduce a renormalization scale which mimics the effect of including these renormalizations. In our approach all three terms in Eq. (11) are cutoff dependent. However, this dependence cancels in the sum leaving behind definite mass thresholds, i.e., the arguments of the logarithms are ratios of particle masses; no other scales are introduced. Although the two procedures are conceptually distinct, in this case the difference is numerically small. As a simplification, we keep only leading logarithms whose argument is the ratio of a supersymmetric (SUSY) particle mass to a weak scale mass. The only terms we ignore which may be important have as the argument of the logarithm the ratio of the two top squark masses [3]. Thus, we restrict our analysis to the class of MSSM mass spectrum scenarios wherein the two top squark masses are nearly degenerate.

Effective potential.—The first correction in Eq. (11), $\Delta V^{(1)}_{\text{eff pot}}$, is due to the one-loop effective potential [6],

$$\Delta V^{(1)}_{\text{eff pot}} = (1/64\pi^2)\,\text{Str}\,\mathcal{M}^4 \ln(\mathcal{M}^2/\Lambda^2)\,, \tag{12}$$

where Λ is an ultraviolet cutoff and $\mathcal{M}^2$ is the field-dependent squared mass matrix for all of the spin 0, $\frac{1}{2}$, and 1 particles in the model. The supertrace is defined as usual for any function f by $\text{Str} f(\mathcal{M}^2) = \sum_i (-1)^{2J_i}(2J_i+1) f(m_i^2)$, where m_i^2 is the ith squared mass eigenvalue of the mass matrix $\mathcal{M}^2$, for a particle of spin J_i.

As we are only interested in the logarithmic corrections, we do not differentiate the logarithms in $\Delta V^{(1)}_{\text{eff pot}}$. Hence, we can evaluate them at the vacuum. Expanding $\Delta V^{(1)}_{\text{eff pot}}$ in powers of S_1 and S_2, we see from Eq. (10) that all terms proportional to S_1^2 and S_2^2 do not contribute to Δm^2. Additionally, terms proportional to $S_1 S_2$ can be absorbed into a redefinition of the tree parameter m_3. This procedure leaves terms proportional to S_1^4, S_2^4, and $S_1^2 S_2^2$. It is then straightforward to determine $\Delta V^{(1)}_{\text{eff pot}}$ by calculating the mass matrices for all of the particles in the MSSM. We then obtain the contribution to the mass matrix using Eq. (10).

Gauge coupling renormalization.—The second contribution in Eq. (11), $\Delta V^{(1)}_{\text{gauge renorm}}$, is due to gauge coupling renormalization. We must include this contribution to renormalize the mass of the Z^0. The part of the tree potential which depends on the gauge couplings is the quartic piece, $V_{\text{tree}}^{\text{quartic}} = \frac{1}{32}(g^2+g'^2)(S_1^2 - S_2^2)^2$. We relate the renormalized coupling g_R to the unrenormalized coupling g_U through the relation $g_R = g_U - \Delta g$. Writing the tree potential in terms of the renormalized couplings gives a contribution to the potential,

$$\Delta V_{\text{gauge renorm}} = \tfrac{1}{16}(g\Delta g + g'\Delta g')(S_1^2 - S_2^2)^2\,, \tag{13}$$

and from Eq. (10) we determine the contribution to the mass matrix. We obtain the MSSM renormalizations of the gauge couplings [7],

$$\Delta g' = \frac{g'^3}{32\pi^2}\left\{\frac{17}{36}\left[\ln\left(\frac{\Lambda^2}{m_{\tilde t}^2}\right) + 2\ln\left(\frac{\Lambda^2}{m_t^2}\right)\right] + \frac{103}{36}\left[\ln\left(\frac{\Lambda^2}{M_{\tilde Q}^2}\right) + 2\ln\left(\frac{\Lambda^2}{M_Z^2}\right)\right] + \frac{1}{3}\left[\ln\left(\frac{\Lambda^2}{M_H^2}\right) + 2\ln\left(\frac{\Lambda^2}{M_{1/2}^2}\right)\right]\right\},$$

$$\Delta g = \frac{g^3}{32\pi^2}\left\{\frac{1}{4}\left[\ln\left(\frac{\Lambda^2}{m_{\tilde t}^2}\right) + 2\ln\left(\frac{\Lambda^2}{m_t^2}\right)\right] + \frac{7}{4}\ln\left(\frac{\Lambda^2}{M_{\tilde Q}^2}\right) + \frac{1}{3}\ln\left(\frac{\Lambda^2}{M_H^2}\right) + 2\ln\left(\frac{\Lambda^2}{M_{1/2}^2}\right) - \frac{23}{6}\ln\left(\frac{\Lambda^2}{M_Z^2}\right)\right\}. \tag{14}$$

We write a common mass $M_{1/2}$ for all the Higgsinos and gauginos, a mass $M_{\tilde Q}$ for the squarks and sleptons (except the top squark mass $m_{\tilde t}$), and a mass M_H for the Higgs bosons. We discuss these masses in the last section.

Wave-function renormalization.—The third correction in Eq. (11), $\Delta V^{(1)}_{\text{wave function}}$, arises due to wave-function renormalization. Renormalizing the fields via $H_{iR} = Z_i^{-1/2} H_{iU}$ $(i=1,2)$, where H_U denotes an unrenormalized field and H_R denotes a renormalized one, we get a correction to the potential

$$\Delta V_{\text{wave function}} = \tfrac{1}{16}(g^2+g'^2)(S_1^2 - S_2^2)(\Delta Z_1 S_1^2 - \Delta Z_2 S_2^2)\,, \tag{15}$$

where the fields are renormalized with $Z_i = 1 + \Delta Z_i$. (We only need to consider the quartic part of the potential here as well, since the terms proportional to S_1^2 and S_2^2 do not contribute to Δm^2 and the terms proportional to $S_1 S_2$ can be absorbed by the tree parameter m_3.) As the field H_2 couples to the top quark, it receives an additional renormalization compared with H_1. We have the MSSM wave-function renormalization [7]

$$\Delta Z_1 = \frac{3g^2+g'^2}{16\pi^2}\left[\frac{1}{4}\ln\left(\frac{\Lambda^2}{M_H^2}\right) - \frac{1}{2}\ln\left(\frac{\Lambda^2}{M_{1/2}^2}\right) + \frac{1}{2}\ln\left(\frac{\Lambda^2}{M_Z^2}\right)\right],\quad \Delta Z_2 = \Delta Z_1 - \frac{3}{16\pi^2}\lambda_t^2 \ln\left(\frac{\Lambda^2}{m_t^2}\right), \tag{16}$$

where λ_t is the top quark Yukawa coupling (we neglect the others) and the top quark mass is given by $m_t = \lambda_t v_2/\sqrt{2}$. Again, the correction to the mass matrix is obtained from Eq. (10).

Results.—Combining the corrections in Eq. (11), we find that the cutoff dependence cancels. We stress that we must include all three of these contributions in order to have a physical, finite result. The logarithmic corrections to the CP-even Higgs-boson mass matrix are

$$\Delta m_{\tilde{1}1}^2 = \cos^2\beta \left(\frac{g^2 M_Z^4}{96\pi^2 M_W^2}\right)\left[(1+4s_W^2+2s_W^4)\ln\left(\frac{M_H^2}{M_{1/2}^2}\right)+(53-112s_W^2+62s_W^4)\ln\left(\frac{M_Z^2}{M_{1/2}^2}\right)\right.$$
$$\left.+\left(21-42s_W^2+\frac{166}{3}s_W^4\right)\ln\left(\frac{M_{\tilde{Q}}^2}{M_Z^2}\right)+\left(3-6s_W^2+\frac{26}{3}s_W^4\right)\ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)\right], \tag{17}$$

$$\Delta m_{\tilde{1}2}^2 = \sin\beta\cos\beta \left(\frac{g^2 M_Z^4}{96\pi^2 M_W^2}\right)\left[(17-28s_W^2+10s_W^4)\ln\left(\frac{M_H^2}{M_{1/2}^2}\right)+(1+40s_W^2-26s_W^4)\ln\left(\frac{M_Z^2}{M_{1/2}^2}\right)\right.$$
$$\left.-\left(21-42s_W^2+\frac{166}{3}s_W^4\right)\ln\left(\frac{M_{\tilde{Q}}^2}{M_Z^2}\right)+\left(\frac{9m_t^2}{M_Z^2\sin^2\beta}-3+6s_W^2-\frac{26}{3}s_W^4\right)\ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)\right], \tag{18}$$

$$\Delta m_{22}^2 = \left(\frac{3g^2 M_Z^4}{16\pi^2 M_W^2}\right)\left[\frac{2m_t^4}{M_Z^4\sin^2\beta}-\frac{m_t^2}{M_Z^2}\right]\ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)+\tan^2\beta\,\Delta m_{\tilde{1}1}^2, \tag{19}$$

where $s_W = \sin\theta_W$.

In order to approximate the effects of all possible superparticle masses, including mixing and nondegeneracies, we independently vary the common mass parameters $M_{1/2}$, $M_{\tilde{Q}}$, and M_H appearing in our formulas from 0.1 to 1 TeV. The extrema of this variation corresponds, over most of the $\tan\beta$-top-quark-mass parameter space considered, to the case where the SUSY fermions are light and the SUSY scalars are heavy, or vice versa. The variation in our results represents the theoretical uncertainty due to the lack of information on the superparticle spectrum. In keeping with the effective potential approximation, we must have some large logarithms in order for our results to be trustworthy. In particular, our result is not trustworthy for cases where the top quark can give a large contribution to the mass, and $m_{\tilde{t}} \simeq m_t$, i.e., for cases where the logarithm multiplying the top quark contribution becomes small. Hence, we always set the top squark mass to 1 TeV.

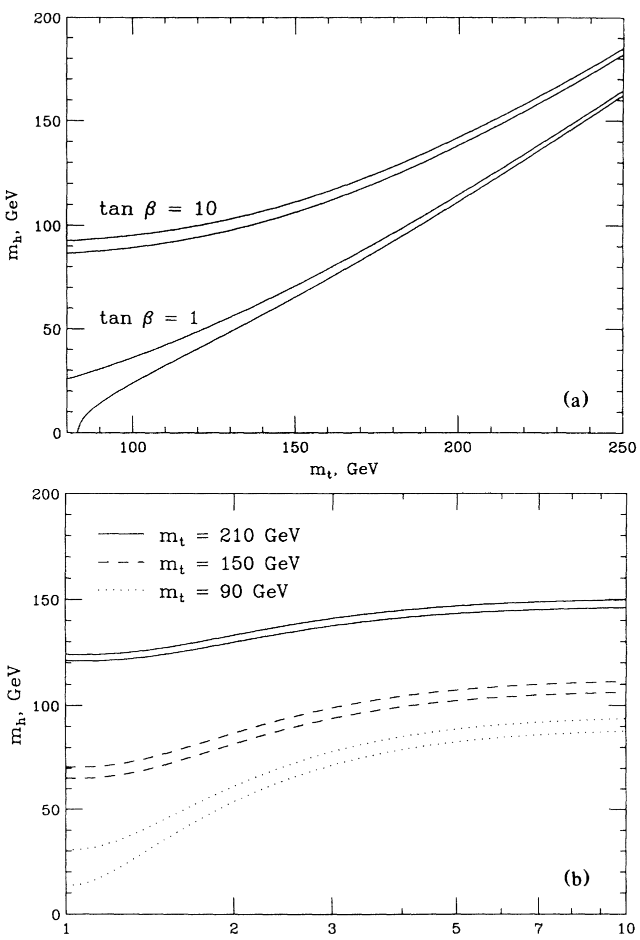

FIG. 1. The heaviest possible light-Higgs-boson mass at one-loop level. The two lines in both (a) and (b) show the extrema of the result when SUSY particle masses are allowed to vary independently between 100 GeV and 1 TeV. The top squark mass is 1 TeV.

In Figs. 1(a) and 1(b) we plot the heaviest possible h^0 mass as a function of the top quark mass and $\tan\beta$, respectively. Hence we set m_A equal to 1 TeV. The two curves in the plots correspond to the extrema of the h^0 particle mass when the common mass parameters $M_{1/2}$, $M_{\tilde{Q}}$, and M_H vary independently between 0.1 and 1 TeV. We find that the uncertainty in the superparticle spectrum typically gives us an uncertainty of 3 to 5 GeV in the mass of the Higgs bosons. The difference in the mass for $\tan\beta = 10$ and $\tan\beta = \infty$ is less than a few percent. If the top squark mass is less than 1 TeV the light Higgs boson becomes lighter than shown in Fig. 1.

At tree level the lightest possible heavy-Higgs-boson mass is M_Z and this occurs when $m_A = 0$. At one-loop level, however, if $m_A = 0$ and if $\tan\beta$ is near 1 we find that the light Higgs boson can become lighter than the current experimental lower bound [8] of 41 GeV. Hence we should increase the parameter m_A until the mass of the light Higgs boson reaches this lower bound. (m_A does not correspond to the mass of the *CP*-odd Higgs boson at one-loop level, as it does at tree level. However, in the limit of no squark mixing the difference is small [3].) We can then use this value of m_A to evaluate the mass of the heavy Higgs boson. In Figs. 2(a) and 2(b) we plot the lightest possible heavy-Higgs-boson mass consistent with the bound on the light Higgs boson. The two curves in

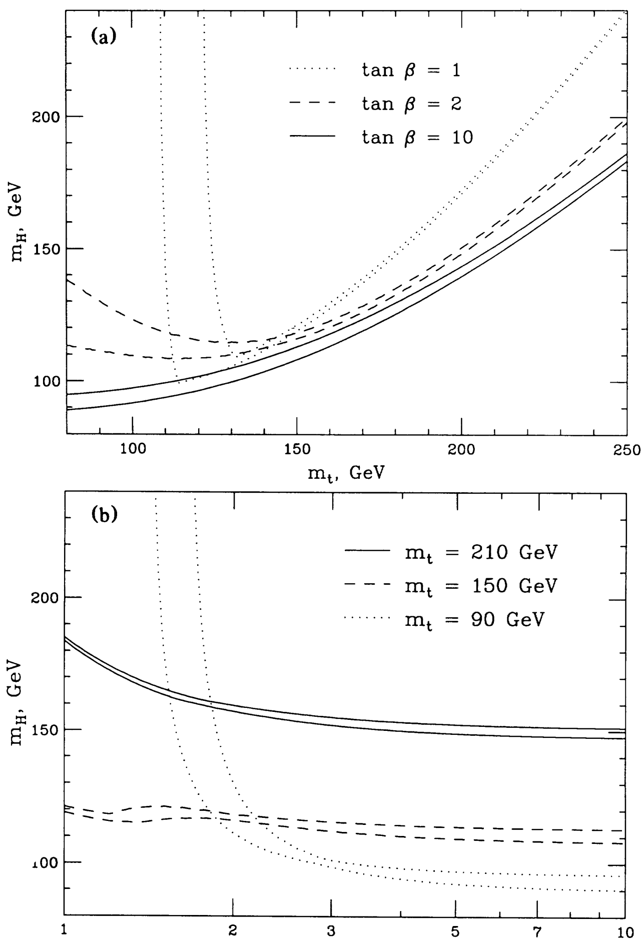

FIG. 2. The lightest possible heavy-Higgs-boson mass at one-loop level consistent with the experimental bound $m_h \gtrsim 41$ GeV. The two lines correspond to the extrema of the result when the superparticle masses are varied as in Fig. 1. The top squark mass is 1 TeV.

the plots show the maximal variation of the heavy-Higgs-boson mass while letting all of the SUSY mass parameters (except the top squark mass) vary independently between 100 and 1000 GeV. At tree level for $\tan\beta = 1$ the light Higgs boson is massless, independent of m_A. At one-loop level, if $m_t \lesssim 100$ GeV, the top quark contribution is not big enough to increase the mass of the light Higgs boson above the experimental bound. Hence, we see in Fig. 2(a) with m_t below around 100 GeV and $\tan\beta = 1$ that $m_H \gg M_Z$. Similarly in Fig. 2(b) we see for the $m_t = 90$ GeV curve that we have no lower bound on the heavy-Higgs-boson mass for values of $\tan\beta \lesssim 1.5$. In Fig. 3 we show the excluded region in the $\tan\beta$-m_t plane for various values of m_A. Note that for small m_A a large portion of the parameter space is excluded. This is because the tree contribution to m_h is small and thus a very large m_t is needed to meet the experimental bound.

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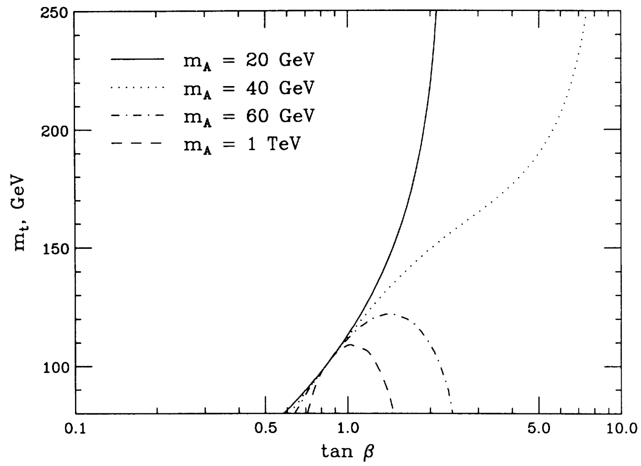

FIG. 3. Excluded region of top-quark-mass–$\tan\beta$ parameter space, found by requiring m_h to be greater than the current limit of 41 GeV from experiments at the CERN e^+e^- collider LEP. For $m_A = 20$ and 40 GeV, the region to the right of the curves is excluded. For $m_A = 60$ GeV and 1 TeV, the region below the curves is excluded.

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