Sub-Shot-Noise Lasers without Inversion

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We present an analysis of the quantum fluctuations of a three-level Λ atomic system which gives rise to lasing without inversion. We find that the intensity fluctuations in the laser output may be reduced 50% below the shot noise limit.

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There have recently been several different models proposed for lasing without inversion [1–10]. These publications have mainly focused on the conditions for the onset of lasing action. An exception is the recent paper by Agarwal [11] who has calculated the quantum phase fluctuations of the laser and derived the linewidth. He showed that for weak laser fields a laser without inversion may have a narrower linewidth than conventional lasers. His treatment, however, is incomplete in as far as no self-consistent equation for the laser field is solved.

In this Letter we present some new results for the quantum intensity fluctuations of a Λ-level laser without inversion. The laser model we analyze consists of an ensemble of coherently driven three-level atoms in the Λ configuration. We shall show that well above threshold the laser without inversion has a sub-Poissonian output with intensity fluctuations 50% less than the shot noise of conventional lasers. Thus the laser without inversion joins lasers with pump noise suppression [12–16] as a sub-Poissonian light source. There is a sense in which the laser without inversion is close to the laser with dynamic pump noise suppression [17]. Both models have in common that sub-Poissonian statistics are achieved well above threshold where the Rabi frequency on the lasing transition is much larger than all atomic decay rates. While recycling through many incoherent steps leads to highly regular pumping, in our proposal noise reduction is achieved by the coherence between the two ground-state levels enabling the medium inside the cavity to compensate for intensity fluctuations through counteracting emission or absorption processes. It is essential to employ two independent light modes providing the coherent links from the excited state to the two ground states. While fluctuations are reduced in one of the modes they are increased in the other one. This is why a recent treatment of a single-mode quantum beat laser with injected ground-state coherence [18] could not predict sub-Poissonian statistics.

Before we proceed we would like to remark that lasing "without inversion" in general only refers to the bare atomic basis. Whether or not inversion in a dressed basis is necessary for the onset of lasing is typically a feature of the particular model under consideration. It has been shown by Imamoğlu, Field, and Harris [7] that in the two-mode Λ model treated in this publication no inversion in any basis set is required.

An example of the opposite situation can be found in a recent work by Narducci *et al.* [10] on a four-level model of a Raman-driven laser without inversion. As stated in Ref. [10], the details of the gain mechanism and hence also the question of inversion are sensitive to the specific nature of the model.

We consider the model of Imamoğlu, Field, and Harris [7] for a laser without inversion as depicted in Fig. 1. N three-level atoms are contained within an optical cavity resonant with the lasing transition between levels 0 and 2. The atoms are coherently pumped by a field with intensity I_p (Rabi frequency squared) on the nonlasing transition 1-2. They are also incoherently pumped on the lasing transition 0-2 with pump rate Λ. γ_2 and γ_1 are the spontaneous decay rates for the lasing and the auxiliary transition, respectively. We shall present the final results only. Details of their derivation will be discussed in a subsequent paper.

All results were derived within the framework of Ito quantum stochastic differential equations. The noise properties far above threshold are described accurately by linearization around the semiclassical solutions. Compact analytical results can be obtained by assuming the steady-state intensity to be large as compared with other system parameters.

The steady-state laser light intensity.— We denote the coupling constant between the laser mode and the atoms by g_c and define the cooperativity $C = N|g_c|^2/(\gamma_2 - \Lambda)\Gamma_c$,

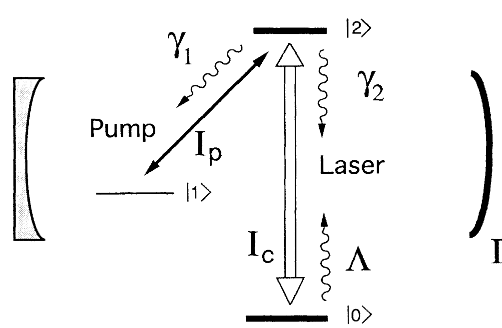

FIG. 1. Λ-model three-level atom inside an optical cavity. The cavity is resonant with the laser transition |0⟩-|2⟩. The cavity damping rate associated with the transmissivity of the mirror is denoted by Γ_c.

where Γ_c is the empty cavity damping rate. For zero detunings we are able to derive a self-consistent equation for the steady-state laser "intensity" $I_c = |g_c|^2 \langle N_c \rangle$, with N_c the photon number operator for the laser mode, well above threshold. We find I_c to be the real, positive root of the polynomial

$$I_c^2 + \left[\Lambda(\Lambda + 2\gamma_1 + 2\gamma_2) + \frac{I_p}{\gamma_1}(\gamma_1 + \gamma_2 + 2\Lambda) \right] I_c + \left[\Lambda(\gamma_1 + \gamma_2 + 2I_p) + \frac{\gamma_2}{\gamma_1} I_p \right] [\Lambda(\Lambda + \gamma_1 + \gamma_2) + I_p] - C(\gamma_2 - \Lambda)^4 p = 0 , \tag{1}$$

where we have introduced the convenient abbreviation $p = (I_p/\gamma_1)\Lambda(\gamma_1 - \gamma_2 + \Lambda)(\gamma_2 - \Lambda)^3$ and assumed that the cavity damping rate Γ_c is much smaller than the atomic rates γ_1, Λ. For the laser without inversion an equivalent cooperativity C' can be defined as

$$C' = C \frac{(\gamma_2 - \Lambda)^2 p}{[\Lambda(\gamma_1 + \gamma_2) + (I_p/\gamma_1)(2\Lambda + \gamma_2)][\Lambda(\Lambda + \gamma_1 + \gamma_2) + I_p]} , \tag{2}$$

such that the familiar condition $C' > 1$ is the requirement for lasing action. We shall derive a solution for Eq. (1) in the limit of large cooperativities, for these are going to yield the interesting case of large intensities. We find the laser intensity to be proportional to the square root of the cooperativity C and the pump factor p,

$$I_c \simeq \left[\frac{(\gamma_2 - \Lambda)}{\gamma_1} \Lambda(\gamma_1 - \gamma_2 + \Lambda) I_p C \right]^{1/2} = (\gamma_2 - \Lambda)^2 \sqrt{pC} . \tag{3}$$

We thus find for the mean number of photons in the cavity,

$$\langle N_c \rangle = n_s \sqrt{pC} , \tag{4}$$

where $n_s = N(\gamma_2 - \Lambda) C^{-1} \Gamma_c^{-1}$ is the saturation photon number.

Light amplification by coherence (LAC).— In this part of the Letter we briefly explain the origin of gain in terms of dressed states for a Λ-configuration three-level atom. A thorough understanding of how lasing action comes into existence is vital in order to understand intensity noise suppression in the output fields. In terms of bare states one finds that the atomic two-photon coherence σ_{01} linking the two nondegenerate ground states $|0\rangle$ and $|1\rangle$ is responsible for lasing action. In a recent publication Agarwal [9] showed that the coherence between the two dressed states for the coherently driven transition $|1\rangle$-$|2\rangle$ was responsible for gain. Although this is correct we think an explanation in terms of dressed states [10,18,19] for the complete atom provides more insight into the physics. In Fig. 2 we depict the energy levels of a Λ atom in terms of bare and dressed states. We realize that incoherent pumping from the ground state at a rate of Λ to the excited level $|2\rangle$ will excite a superposition of the dressed states $|r\rangle$ and $|s\rangle$. Provided that the decay rate γ_1 into the auxiliary transition is larger than the decay rate $\gamma_2 - \Lambda$ into the lasing transition, electrons will predominantly decay to level $|1\rangle$. Of those only the ones ending up in the "trapped" state $|t\rangle$ are relevant since they can be recycled to the ground state $|0\rangle$ under absorption of a photon from the coherent pump field and subsequent emission of one photon into the laser light field inside the cavity. Each of these *coherently recycled* electrons gives rise to the gain of one photon. So we realize that it is the coherence between the state $|t\rangle$ and a superposition of the states $|r\rangle$ and $|s\rangle$, being just another representation of the bare state $|2\rangle$, i.e., the excited state, which is responsible for amplification. In the case of small detunings the coherences between states $|r\rangle$ and $|s\rangle$ play only a minor role because of their destructive interference. With growing cavity intensity the population in level $|1\rangle$ increases and in turn the incoherent pumping of "quasitrapped" electrons in the ground state to the upper lasing level becomes less efficient. This interdependence then eventually leads to a finite value for the cavity intensity I_c. Please note that for large I_c level $|1\rangle$ will be inverted with respect to level $|0\rangle$ and the system behaves similarly to a Raman laser.

The noise properties.— We will now derive the photon statistics of the output laser light. A suitable quantity to characterize intensity fluctuations is the following normally ordered variance measuring deviations from Poissonian statistics defined as $Q = [\langle N_c(N_c - 1)\rangle - \langle N_c \rangle^2]/$

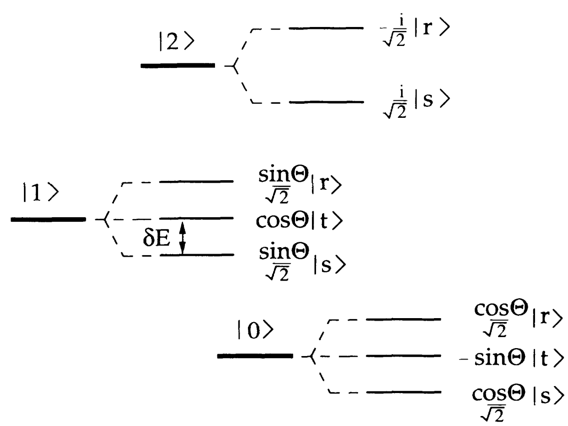

FIG. 2. Bare and dressed states for the Λ atom. For zero detunings each of the bare states can be expressed as a simple superposition of the dressed states $|r\rangle$, $|s\rangle$, and $|t\rangle$. The angle Θ and the energy level shift δE are related to I_p and I_c by $\delta E = \hbar(I_p^2 + I_c^2)^{1/2}$ and $\tan^2\Theta = I_p/I_c$.

$\langle N_c \rangle$, with N_c the photon number operator of the cavity mode. The output intensity fluctuation spectrum of the laser light is related to Q [12,19] by

$$S^{\text{out}}(\omega)=\int_{-\infty}^{\infty}\frac{\langle I^{\text{out}}(\tau)I^{\text{out}}(0)\rangle}{\langle I^{\text{out}}\rangle^2}e^{-i\omega\tau}d\tau = 1+\left(Q\frac{2\Gamma_c}{\lambda}\right)\frac{2\lambda^2}{\lambda^2+\omega^2}, \quad (5)$$

with the factor 1 being the shot noise. λ^{-1} denotes the intensity correlation time of the laser.

As can be seen from Eq. (5) the width of the intensity fluctuation spectrum will ultimately determine the best achievable noise reduction in the output intensity. In standard laser theory the ratio $2\Gamma_c/\lambda$ approaches unity in the case of large cavity intensities. In the high-field limit it is possible to calculate an approximate analytical expression for Q. We find

$$Q=-\frac{1}{2}+\frac{1}{2}\frac{\Lambda+(\gamma_2-\Lambda)r}{\gamma_1+\Lambda-\gamma_2}, \quad (6)$$

where the abbreviation $r=(\gamma_2-\Lambda)/\gamma_1$ is used. We realize that for much faster spontaneous decay on the auxiliary transition $\gamma_1 \gg \gamma_2,\Lambda$ the normally ordered variance Q will approach its optimum value inside the cavity $Q \to -\frac{1}{2}$, as shown in Fig. 3. This corresponds to optimum intracavity noise suppression.

The physical reason for reduced photon number fluctuations is clear from the discussion of the origin of gain above. The fact that the laser field itself is involved in the pumping process through the coherent recycling process makes a reduction of intensity noise possible. The scheme makes use of the fact that there is a stable steady state for the laser intensity which necessarily implies that any fluctuation in the system resulting in an intensity fluctuation must result in a counteracting response by the field. Such negative backaction can reduce Poissonian pump fluctuations since the laser itself plays an active role in the pump process. This leads to a highly regular pumping of the LAC laser with the major remaining degradation of photon number statistics coming from direct spontaneous decays from level $|2\rangle$ to the ground state $|0\rangle$. For any given γ_1 there is an upper bound for pumping rates compatible with noise reduction since too strong pumping also enhances spontaneous emission by creating a larger population on the excited level which can only be compensated for by increasing the decay rate γ_1 into the auxiliary transition $|2\rangle$-$|1\rangle$.

In the case of large cooperativities and hence large cavity intensities we may even obtain a relatively simple analytical term for the intensity width,

$$\lambda \approx 2\Gamma_c\{1+[(\gamma_2-\Lambda)^4/I_c^2]pC\}. \quad (7)$$

Insertion of the asymptotic result for I_c into Eq. (7) reveals that λ approaches a value of $4\Gamma_c$ thereby limiting the intensity reduction in the output field to at best 50%. The overall behavior of the intensity width is to start from its threshold value $\lambda=0$ for which our linearized analysis obviously cannot hold anymore and then to approach a saturation value of $4\Gamma_c$. For small ratios I_c/I_p we find in agreement with Refs. [11,20–22] a reduction of spontaneous emission noise and consequently narrower spectral linewidths in coherently driven three-level systems than in conventional lasers.

In conclusion, we have shown that an amplification principle other than stimulated emission, namely, amplification by coherence, can be responsible for the buildup of a strong coherent light field inside an optical cavity. Evaluation of the Mandel Q parameter shows that optimum noise suppression ($Q \to -\frac{1}{2}$) inside the cavity is possible at least in principle. We were able to identify the mechanism for noise reduction as the self-regularization of the pumping process together with an almost total suppression of spontaneous emission into the laser mode. This is accomplished by the introduction of a more likely second decay path from the upper laser level to an auxiliary level from which electrons are recycled back to the ground state by coherent two-photon processes involving absorption of a photon from the coherent pump and subsequent emission of a photon into the cavity mode.

Sub-Poissonian statistics in the output light is also limited by the width of the spectrum of the intensity fluctuations. In contrast to standard laser theory the intensity width of the laser light coming out of this device will approach twice the empty cavity width for ideal intracavity noise suppression. We therefore may only obtain up to 50% squeezing in the output light of the laser. The main reason for this is the fact that the intensity of this laser

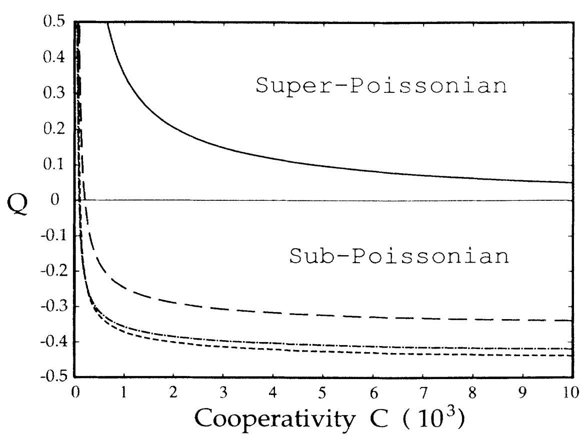

FIG. 3. Q as a function of the cooperativity C for different ratios $(\gamma_2-\Lambda)/\gamma_1=\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16}$. With γ_1 increasing Q approaches $-\frac{1}{2}$ for $C \gg 1$. Q was obtained without an adiabatic elimination of the atomic variables for $\Lambda=\frac{1}{4}$, $I_p^{1/2}=2.74$, and $\Gamma_c=0.0075$ (in units of γ_2). Adiabatic elimination yields results which are slightly better; the difference is of the order of Λ/Γ_c, which is $\approx 3\%$ in this case.

grows proportionally to the square root of the cooperativity $\sqrt{C}$ resulting in a finite broadening of intensity fluctuations even for arbitrarily large cooperativities. We want to note that the best achievable noise reduction in the central mode of the output laser light agrees with that found by Ritsch *et al.* [17] for an incoherently pumped standard three-level laser. The total squeezing, however, in this new device is almost twice as large and can at least in principle be perfect. In a more realistic model inhomogeneous broadening as well as fluctuations in the number of atoms and decay out of the three-level system must be taken into account. It is to be expected that these features of real physical systems will deteriorate the efficiency of LAC and increase the amount of fluctuations in the output intensity.

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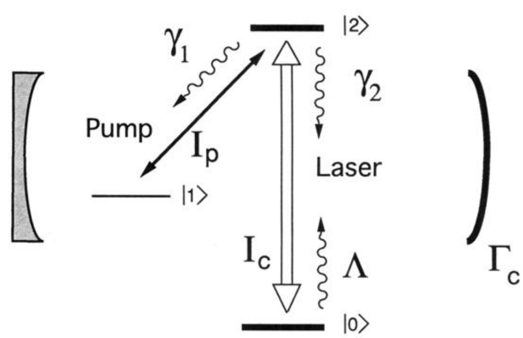

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