Test of the Dimopoulos-Hall-Raby Ansatz for Fermion Mass Matrices

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By evolution of fermion mass matrices of the Fritzsch and the Georgi-Jarlskog forms from the supersymmetric grand unified scale, Dimopoulos, Hall, and Raby obtained predictions for the quark masses and mixings. Using Monte Carlo methods we test these predictions against the latest determinations of the mixings, the CP-violating parameter ϵ_K, and the B_d^0-$\bar{B}_d^0$ mixing parameter r_d. The acceptable solutions closely specify the quark mixings, but lie at the edges of allowed regions at 90% confidence level.

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One of the outstanding problems in particle physics is that of explaining the fermion masses and mixings. In the standard model (SM) the six quark masses, three charged lepton masses, the three quark mixings, and the CP-violating phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix are introduced as phenomenological parameters. Over the years various models have been proposed to reduce the number of these free parameters [1], of which the best known is the Fritzsch model [2]. Recently Dimopoulos, Hall, and Raby (DHR) have proposed an ansatz for fermion mass matrices [3] in the framework of minimal supersymmetric (SUSY) grand unified theories (GUTs). The DHR approach is based on the observation that some discrete symmetries present at the grand unification scale are broken in the low-energy theory. Thus some elements of the fermion mass matrices that vanish at the GUT scale are nonzero at the electroweak scale, and their low-energy values are calculable from the renormalization-group equations. The fermion masses and mixings at the electroweak scale can thereby be expressed in terms of a smaller number of input parameters at the GUT scale. DHR work in the massless neutrino limit and relate the thirteen SM parameters and a SUSY parameter $\tan\beta$ (discussed below) to eight input parameters, leading to six predictions that include an allowed range of 147–187 GeV for the top-quark mass (m_t). In comparison the Fritzsch approach gives $77 \le m_t \le 96$ GeV [1], which is nearly excluded in the SM by the Collider Detector at Fermilab (CDF) experiment [4] at a 90% confidence level (C.L.).

The DHR quark mass matrices at the scale m_t are

$$\mathcal{M}_u = \begin{pmatrix} 0 & C & 0 \\ C & \delta_u & B \\ 0 & B & A \end{pmatrix} \frac{v\sin\beta}{\sqrt{2}}, \quad \mathcal{M}_d = \begin{pmatrix} 0 & Fe^{i\phi} & 0 \\ Fe^{-i\phi} & E & \delta_d \\ 0 & 0 & D \end{pmatrix} \frac{v\cos\beta}{\sqrt{2}}, \tag{1}$$

where all the parameters are real, $\tan\beta = v_2/v_1$ in terms of the Higgs doublet vacuum expectation values, and $v = 246$ GeV. The charged lepton mass matrix $\mathcal{M}_e$ is obtained from the above form of $\mathcal{M}_d$ by the substitutions $\phi = 0$, $\delta_d = 0$, $E \to -3E'$, $D \to D'$, $F \to F'$. At the SUSY-GUT scale, the parameters δ_u and δ_d vanish and $D = D'$, $E = E'$, $F = F'$, so the input mass matrix $\mathcal{M}_u$ is of the Fritzsch form [2] and $\mathcal{M}_d$ and $\mathcal{M}_e$ are of the Georgi-Jarlskog form [5], giving the GUT scale mass relations $m_b = m_\tau$, $m_s \simeq m_\mu/3$, $m_d \simeq 3m_e$ between quarks and leptons. The mass ratio prediction

$$(m_d/m_s)(1 - m_d/m_s)^{-2} = 9(m_e/m_\mu)(1 - m_e/m_\mu)^{-2} \tag{2}$$

holds at all scales.

The Wolfenstein parametrization [6] of the CKM matrix determined from the unitary matrices that diagonalize the DHR mass matrices can be expressed in terms of four angles (θ_i) and a complex phase (ϕ) as follows:

$$\lambda = (s_1^2 + s_2^2 + 2s_1 s_2 \cos\phi)^{1/2} = |V_{cd}| = |V_{us}|, \tag{3a}$$

$$\lambda^2 A = s_3 - s_4 = |V_{cb}|, \tag{3b}$$

$$\lambda(\rho^2 + \eta^2)^{1/2} = s_2 = |V_{ub}/V_{cb}| = (m_u/m_c)^{1/2}, \tag{3c}$$

$$\eta = s_1 s_2 \sin\phi/\lambda^2, \tag{3d}$$

with $s_i = \sin\theta_i$, $c_i = \cos\theta_i$ ($i = 1,2,3,4$), where θ_2, θ_3 are the angles that diagonalize the matrix $\mathcal{M}_u$, and θ_1, θ_4 are those for $\mathcal{M}_d$ [3]; only three of these angles are independent. These mixing angles are related to the quark masses and other parameters by

$$s_1 \simeq (m_d/m_s)^{1/2}, \quad s_2 \simeq (m_u/m_c)^{1/2}, \quad s_3 \simeq |B/A|, \quad s_4 \simeq s_3 - |V_{cb}|. \tag{4}$$

The evolution based on the SUSY renormalization-group equations (RGE) from the GUT scale to the appropriate fermion mass scales, taking all SUSY particles and the second Higgs doublet degenerate at the scale of m_t [3], gives the following relations:

$$m_t = \frac{m_b m_c}{m_\tau |V_{cb}|^2} \frac{x}{\eta_b \eta_c \eta^{1/2}}, \quad m_s - m_d = \tfrac{1}{3} m_\mu \eta_s \eta^{1/2}/x, \tag{5a}$$

$$\sin\beta=\frac{m_t}{\pi v}\left(\frac{3I}{2\eta}\right)^{1/2}[1-y^{12}]^{-1/2},\quad s_3=\frac{|V_{cb}|m_b}{\eta^{1/2}\eta_b m_\tau}x\,, \tag{5b}$$

where

$$x=(\alpha_G/\alpha_1)^{1/6}(\alpha_G/\alpha_2)^{3/2},\quad y=x(m_b/m_\tau)\eta^{-1/2}\eta_b^{-1}\,, \tag{6a}$$

$$\eta(\mu)=\prod_{i=1,2,3}(\alpha_G/\alpha_i)^{c_i/b_i},\quad I(\mu)=\int_\mu^{M_G}\eta(\mu')d\ln\mu'\,. \tag{6b}$$

The RGE parameters b_i, c_i are given in Ref. [3]. In these equations the couplings α_1 and α_2 are evaluated at the scale m_t. The mass parameters are defined as $m_q(\mu=m_q)$ for quarks heavier than 1 GeV, and the lighter quark masses m_s, m_d, m_u are calculated at the scale $\mu=1$ GeV.

Starting from the well-determined values [7], $\alpha^{-1}(M_Z)=127.9$, $\sin^2\theta_W(M_Z)_{\overline{\rm MS}}=0.2326$, and evolving at one-loop level to the intersection of α_1 and α_2 determines the GUT scale $M_G=1.2\times10^{16}$ GeV and the GUT coupling constant $\alpha_G=1/25.0$. Evolving backwards, the strong coupling constant $\alpha_s(M_Z)=0.111$ is obtained, consistent with the result $\alpha_s=0.118\pm0.008$ from experiments at the CERN e^+e^- collider LEP [8]. Also the values $\alpha_1(m_t)=0.017$ and $\alpha_2(m_t)=0.033$ are determined, as well as the factors $\eta(m_t)=10.3$ and $I(m_t)=113.8$. We have used a top-quark threshold of 170 GeV in the RGE, consistent with our output determination. In evolution below the electroweak scale we include three-loop QCD and one-loop QED effects in the running masses to obtain the evolution factors $\eta_b=1.47$, $\eta_c=1.89$, and $\eta_s=2.10$, where $\eta_q=m_q(m_q)/m_q(m_t)$ for $q=b,c$ and $\eta_s=[m_s(1\text{ GeV})/m_\mu(1\text{ GeV})]/[m_s(m_t)/m_\mu(m_t)]$. Quark and lepton thresholds were handled by demanding that the couplings and running masses be continuous. The number of active flavors in the β functions and in the anomalous dimensions was changed as each successive fermion was integrated out of the theory.

Following DHR, we take the following eight relatively better-known parameters as inputs: m_e, m_μ, m_τ, m_c, m_b, m_u/m_d, $|V_{cb}|$, and $|V_{cd}|$. We generate random values for all inputs within 90% C.L. (1.64σ) ranges. The input mass values [9,10] are $m_\tau=1784.1^{+2.7}_{-3.6}$ MeV, $m_c(m_c)=1.27\pm0.05$ GeV, $m_b(m_b)=4.25\pm0.1$ GeV, where 1σ errors are quoted. We also impose the theoretical constraint $0.2\le m_u/m_d\le0.7$ [11]. We next calculate m_d and m_s from Eqs. (2) and (5a) (obtaining $m_d=6.58$ MeV and $m_s=162.5$ MeV), s_1 and s_2 from Eq. (4), s_3 from Eq. (5b) for the input of $|V_{cb}|$, s_4 from Eq. (4), and ϕ from $|V_{cd}|$ of Eq. (3a). Using these values we evaluate the magnitudes of all elements of the CKM matrix. We retain only those Monte Carlo events that satisfy the following ranges from the 1992 Review of Particle Properties [9],

$$|V_{\rm CKM}|=\begin{pmatrix}0.9747\text{–}0.9759 & 0.218\text{–}0.224 & 0.002\text{–}0.007\\ 0.218\text{–}0.224 & 0.9735\text{–}0.9751 & 0.032\text{–}0.054\\ 0.003\text{–}0.018 & 0.030\text{–}0.054 & 0.9985\text{–}0.9995\end{pmatrix}, \tag{7}$$

as well as the ratio $0.051\le|V_{ub}/V_{cb}|\le0.149$ [9]. Figure 1 shows a scatter plot of $\sin\beta$ vs m_t obtained from our Monte Carlo analysis; one sees that only a narrow wedge of the space is permissible. However, we note that the m_t values are reasonably sensitive to the input parameters α^{-1} and $\sin^2\theta_W$ at scale M_Z.

From $\sin\beta\le1$ in Eq. (5), $|V_{cb}|$ must satisfy the inequality

$$|V_{cb}|\gtrsim\left[\frac{x}{\pi v}\left(\frac{3I}{2(1-y^{12})}\right)^{1/2}\frac{m_b m_c}{m_\tau\eta\eta_b\eta_c}\right]^{1/2}\gtrsim0.050\,, \tag{8}$$

which is just at the edge of the 90% C.L. allowed range. Calculating m_t from Eq. (5a) and $\sin\beta$ from Eq. (5b) and requiring that $|V_{cb}|$ be within its allowed range, we find

$$160<m_t<187\text{ GeV},\quad \sin\beta>0.847\ (\tan\beta>1.6)\,. \tag{9}$$

This top-quark mass determination is consistent with estimates from the electroweak radiative corrections [7,8] but is much more restrictive. The predicted value of $\tan\beta$ is large, which may have significant phenomenological implications for Higgs boson searches at colliders [12].

Next we include the constraints from the measured values

$$|\epsilon_K|=(2.259\pm0.018)\times10^{-3}\ [9]\,,\qquad r_d=0.181\pm0.043\ [13]\,, \tag{10}$$

of the CP-violating parameter ϵ_K and the B_d^0-$\bar B_d^0$ mixing parameter r_d. The theoretical formulas, including QCD corrections, can be found in Eqs. (2.1) and (2.10) of Ref. [14]. In our Monte Carlo analysis we allow variations of the bag factors and B-decay constant over the following ranges [14]:

$$0.33\le B_K\le1.5,\quad 0.1\le B_B^{1/2}f_B<0.2\text{ GeV}\,, \tag{11}$$

taking $f_K=160$ MeV and $\Delta M_K=3.521\times10^{-15}$ GeV. The solutions so obtained closely specify the CKM matrix to be

$$|V_{\rm CKM}|=\begin{pmatrix}0.9748\text{–}0.9759 & 0.2185\text{–}0.2236 & 0.0026\text{–}0.0033\\ 0.2185\text{–}0.2236 & 0.9735\text{–}0.9747 & 0.0500\text{–}0.0540\\ 0.0101\text{–}0.0109 & 0.0490\text{–}0.0529 & 0.9985\text{–}0.9987\end{pmatrix}, \tag{12}$$

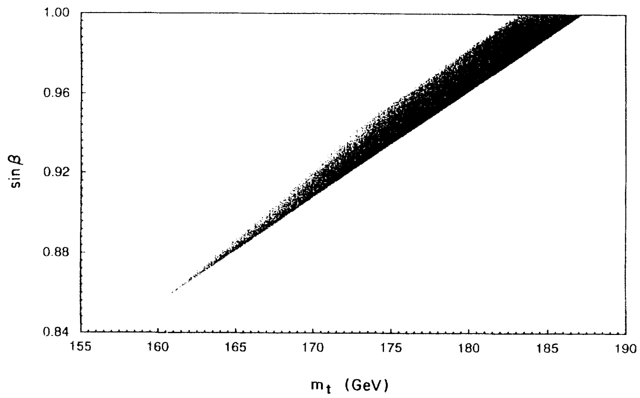

FIG. 1. Scatter plot of $\sin\beta$ vs m_t from our Monte Carlo analysis of the DHR model, imposing the constraints of input masses and present values of CKM matrix elements. For different input couplings at the Z-mass scale the location of the band would be shifted.

and predict the CP-violating phase to be in the range

$$70° < \phi < 82° , \tag{13}$$

which is at the upper end of the 90% C.L. obtained by DHR; our tighter constraint on ϕ is mainly due to fitting ϵ_K and r_d. The inclusion of ϵ_K and r_d almost uniquely determines the values of $|V_{td}|$ and $|V_{ts}|$. Since $|V_{cb}|$ is near its allowed upper limit, $|V_{ub}|$ is pushed to its lower end by the unitarity condition. The output value of the ratio

$$0.051 < |V_{ub}/V_{cb}| < 0.062 \tag{14}$$

is at the low end of the allowed range. Improved experimental determinations of $|V_{cb}|$ and $|V_{ub}/V_{cb}|$ will test the DHR ansatz. In terms of the Wolfenstein parameters [6], we find

$$\begin{aligned} &0.2185 < \lambda < 0.2236, \quad 1.01 < A < 1.13 , \\ &0.194 < \eta < 0.259, \quad 0.104 < \rho < 0.134 , \\ &0.220 < (\rho^2+\eta^2)^{1/2} < 0.292 . \end{aligned} \tag{15}$$

The output values of the mass ratios of the light quarks are

$$\begin{aligned} &0.47 < m_u/m_d < 0.70, \quad m_d/m_s = 0.0405 , \\ &0.019 < m_u/m_s < 0.028 , \end{aligned} \tag{16}$$

giving $3.07 < m_u < 4.60$ MeV. These light-quark masses and their ratios are consistent with those obtained in Refs. [10,11], but do not agree as well with some other recent studies [15], in which $m_u/m_d \lesssim 0.3$ was obtained. Another interesting result is restrictive ranges for the constants B_K and f_B,

$$0.33 < B_K < 0.50, \quad 0.13 < B_B^{1/2} f_B < 0.18 \text{ GeV} , \tag{17}$$

on which theoretical uncertainties have been problematic [16].

We conclude with some brief remarks. From Eq. (5a), m_t is inversely proportional to $\eta_b \eta_c \eta^{1/2}$, and the theoretical uncertainty in this quantity could somewhat enlarge or close the window in m_t (and correspondingly the window in $|V_{cb}|$). The m_t output is sensitive to the inputs for α^{-1} and $\sin^2\theta_W$ at scale M_Z and possibly also to the matching across quark thresholds. Allowing for these uncertainties, the top-quark mass prediction may change by about 10 GeV, and there are corresponding small shifts in the CKM matrix elements.

The DHR analysis assumes dominance of the top-quark Yukawa couplings in the RGE evolution. Since the output $\tan\beta$ may be large, the effects of fully including λ_b and λ_τ in the evolution may not be negligible; this question deserves further study. When λ_b and λ_τ contributions are included, an upper bound of $\tan\beta$ will be obtained. Two-loop renormalization-group equations between M_Z and $M_{\rm GUT}$ should eventually be incorporated.

We have studied the charged Higgs boson effects on ϵ_K and r_d. With $M_{H^\pm}$ degenerate with m_t, as assumed in the model, we found no significant changes in our results. This is due to the fact that $H^\pm$ effects are smaller at large $\tan\beta$ for the K and B systems.

In summary, the DHR ansatz for fermion mass matrices is consistent with all current experimental constraints at 90% C.L. All of our results are compatible with those obtained by DHR but the parameter ranges are more restrictive due to the inclusion of ϵ_K and B-mixing constraints and updated CKM matrix elements. The DHR model leads to closely specified values for quark mixings and a constrained range for m_t which make it an interesting target for future experiments.

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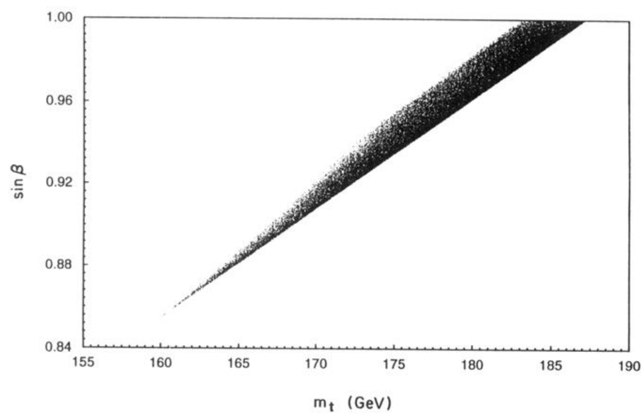

FIG. 1. Scatter plot of sinβ vs m_t from our Monte Carlo analysis of the DHR model, imposing the constraints of input masses and present values of CKM matrix elements. For different input couplings at the Z-mass scale the location of the band would be shifted.