Thermal Stimulated Brillouin Scattering in Laser-Produced Plasmas

R. W. Short and E. M. Epperlein

Laboratory for Laser Energetics, University of Rochester, 250 East River Road, Rochester, New York 14623-1299

(Received 3 February 1992; revised manuscript received 16 April 1992)

A new form of the stimulated Brillouin scattering (SBS) instability in laser-produced plasmas is analyzed in the context of recent advances in thermal transport theory. Inverse-bremsstrahlung heating rather than the usual ponderomotive force provides the driving force for the instability. It is found that in low-temperature, high-density, high-Z plasmas this form of the instability dominates and can lead to significantly higher growth rates for SBS.

PACS numbers: 52.40.Nk, 52.35.Mw, 52.35.Nx

An electromagnetic wave propagating through a plasma can be scattered by an ion-acoustic wave, transferring energy to the sound wave and the scattered light wave in such a way that these waves grow exponentially in time. This instability, known as stimulated Brillouin scattering (SBS), is a potentially serious energy-loss mechanism for laser-driven fusion, and so has been studied extensively both experimentally [1-3] and theoretically [4-6]. Previous theoretical analyses of SBS in laser-produced plasmas have generally proceeded on the assumption that the plasma is isothermal, this assumption being justified by the fact that the wavelength of the sound wave is very small (about half the laser wavelength), so that using classical (Spitzer-Harm [7]) thermal transport theory the time for thermal equilibration across this distance is found to be much smaller than the sound wave period. In this model the driving term for the instability is the ponderomotive force: Plasma tends to be driven out of regions of high electromagnetic field intensity arising from the interference between the incident and scattered light waves, so that the interference pattern intensifies the sound wave.

Recently, however, numerical studies of thermal transport in laser-fusion plasmas using the Fokker-Planck (FP) equation have shown that classical transport theory is inadequate to treat phenomena occurring over short distances, even if the local temperature scale length ($T/|\nabla T|$) is much longer than the electron mean free path [8,9]. In particular, it is found that thermal conduction can be greatly reduced for temperature variations with wavelengths shorter than the mean free path of the electrons which classically carry the bulk of the heat flow. These electrons, which have velocities near $3.7v_t$ (where v_t is the electron thermal velocity), rapidly become uniformly distributed and decoupled from the spatial variations in energy density that persist in the slower electrons, which contain most of the thermal energy. Thus these spatial variations persist longer than predicted by classical theory. This effect is especially significant if the energy variations arise from a source which preferentially heats the slower electrons [9]. Inverse bremsstrahlung, the principal heating mechanism in laser-produced plasmas, is such a source, since it arises from thermalization of the electron oscillatory velocity by collisions with ions, which are more frequent for slow electrons.

Since SBS involves ion waves with very short wavelengths, these advances in the understanding of thermal transport make it necessary to question the isothermality assumption in SBS theory. Inverse-bremsstrahlung heating raises the temperature and pressure of the plasma in regions of high electromagnetic field intensity and thus tends to expel plasma from such regions just as the ponderomotive force does. With classical thermal conductivity the resulting temperature variations would be negligible, but in the light of the "nonlocal" transport theory described above we shall find that they are significant, and in high-Z plasmas they can in fact become the dominant driving force for the SBS instability. We note in passing that a similar mechanism has been proposed in the analysis of SBS in ionospheric heating experiments [10]; in that case it is the Earth's magnetic field rather than nonlocal transport effects that provides the necessary reduction in thermal transport.

To analyze the instability we consider a homogeneous equilibrium plasma with electron density n_0 and temperature T_0. The electric field of the laser light is represented by $E = E_0\exp[i(k_0x-\omega_0t)]+\text{c.c.}$, where $\omega_0^2=\omega_p^2+k_0^2c^2$ and ω_p is the electron plasma frequency. For simplicity we consider only backscatter, so that the scattered light field can be represented by $E_1(t)\exp[i(k_1x-\omega_1t)]+\text{c.c.}$, the temperature perturbation by $T_1(t)\exp[i(kx-\omega t)]+\text{c.c.}$, and the density perturbation by $n_1(t)\exp[i(kx-\omega t)]+\text{c.c.}$, where $\omega_1^2=\omega_p^2+k_1^2c^2$, $k_1=k_0-k$, $\omega=\omega_0-\omega_1$, c_s is the ion sound speed, and k is the wave number of the ion sound wave. We assume perfect wave-number matching and look for temporal growth, represented by the slow time dependence of E_1, T_1, and n_1. Using Maxwell's equations and the usual fluid equations for the plasma the derivation of the equations for the perturbed fields and densities is straightforward:

$$2i\omega_1\frac{\partial}{\partial t}E_1^*(t)=-\frac{\omega_1}{\omega_0}\omega_p^2E_0^*\frac{n_1(t)}{n_0}\,, \tag{1}$$

$$\left[\omega^2-k^2c_s^2+2i\omega\frac{\partial}{\partial t}-\frac{\partial^2}{\partial t^2}\right]\frac{n_1(t)}{n_0} = k^2c_s^2\frac{T_1(t)}{T_0}+\frac{Ze^2}{m_em_i\omega_0\omega_1}k^2E_0E_1^*(t)\,. \tag{2}$$

Here c is the speed of light, Z is the average ion charge, and m_e and m_i are the electron and ion masses.

There are two potential sources of energy which can drive the temperature variation T_1: the $P\,dV$ work of the oscillating sound wave and the inverse-bremsstrahlung heating resulting from the light waves. From FP simulations we have found that the former, which adds energy to all electron velocity groups equally, is smoothed so rapidly by thermal conduction that it makes a negligible contribution to the temperature variation. In contrast, inverse bremsstrahlung heats mainly the slow electrons, and the resulting variations in energy density are smoothed much less rapidly than classical models would predict. Balancing inverse-bremsstrahlung heating with thermal diffusion and taking into account the temperature and density dependence of the inverse-bremsstrahlung absorption coefficient we find the following expression for the temperature variation:

$$\left\{\frac{3}{2}\left[\frac{\partial}{\partial t}-i\omega\right]+\frac{\kappa_0^{\mathrm{th}}k^2}{n_0}+\frac{3}{2}\frac{\kappa_0^{\mathrm{IB}}I_0}{n_0T_0}\right\}\frac{T_1(t)}{T_0}=2\frac{\kappa_0^{\mathrm{IB}}I_0}{n_0T_0}\frac{n_1(t)}{n_0}+\frac{c\varepsilon_0^{1/2}\kappa_0^{\mathrm{IB}}}{2\pi n_0T_0}E_0E_1^*(t). \quad (3)$$

Here ε_0 is the homogeneous plasma dielectric constant, κ_0^{IB} is the inverse-bremsstrahlung absorption coefficient, I_0 is the incident laser intensity, and κ_0^{th} is the modified thermal conductivity, to be discussed below. In Eqs. (1)-(3) we have for simplicity neglected the various wave damping mechanisms. It would be possible to include damping in the usual phenomenological way [11] by the replacement $\omega\to\omega+i\nu$ in Eqs. (1) and (2), where ν would be the inverse-bremsstrahlung damping rate for the light wave in (1) and a combination of collisional and Landau damping for the sound wave in (2). A more sophisticated approach would be to write an additional energy equation for the $P\,dV$ work with the conductivity and specific heat modified to model both collisional and kinetic damping [12]. We will not pursue this subject further here since we are primarily interested in determining under what circumstances the thermal driving term significantly enhances the SBS growth rate, and this question is to first order independent of damping. A simple way to estimate the instability threshold due to damping will be discussed below.

The terms in (1)-(3) involving κ_0^{IB}, κ_0^{th}, and the electric fields may be conveniently written in terms of dimensionless parameters γ_{T1}, γ_{T2}, and γ_p (closely related to those introduced by Schmitt [13]):

$$\frac{\kappa_0^{\mathrm{IB}}I_0}{\omega n_0T_0}=\frac{3}{2}\frac{\gamma_{T2}}{\gamma_{T1}},\quad \frac{\kappa_0^{\mathrm{th}}k^2}{\omega n_0}=\frac{3}{2}\frac{1}{\gamma_{T1}}\frac{\kappa_0^{\mathrm{th}}}{\kappa_0^{\mathrm{SH}}},$$

where γ_{T1} is the ratio of the thermal conduction transit time to the ion-acoustic transit time across k^{-1} $(\approx c/2\omega_0\varepsilon_0^{1/2})$,

$$\gamma_{T1}=6.75\times10^{-6}\frac{\ln\Lambda}{T_0^2(\mathrm{keV})\varepsilon_0^{1/2}\lambda_0(\mu\mathrm{m})}\times\frac{Z^*}{\phi(Z^*)}\left(\frac{Z}{A}\right)^{1/2}\left(\frac{n_0}{n_c}\right), \quad (4)$$

where γ_{T2} is the ratio of the inverse-bremsstrahlung heating rate to the thermal conduction cooling rate across k^{-1},

$$\gamma_{T2}=2.24\times10^{-9}\frac{I_0(10^{14}\ \mathrm{W/cm^2})}{T_0^5(\mathrm{keV})\varepsilon_0^{3/2}}\frac{Z^{*2}(\ln\Lambda)^2}{\phi(Z^*)}\left(\frac{n_0}{n_c}\right)^2, \quad (5)$$

and γ_p is the ratio of the ponderomotive pressure to the thermal pressure:

$$\gamma_p=\frac{e^2E_0E_0^*}{m_e\omega_0^2T_0}=9.33\times10^{-3}\frac{\lambda_0^2(\mu\mathrm{m})I_0(10^{14}\ \mathrm{W/cm^2})}{\varepsilon_0^{1/2}T_0(\mathrm{keV})}. \quad (6)$$

In these expressions A is the ion atomic number, $Z^*\equiv\langle Z^2\rangle/\langle Z\rangle$ (where $\langle\ \rangle$ denotes an average over the ion species), $\phi(Z^*)\equiv(Z^*+0.24)/(1+0.24Z^*)$, and $\ln\Lambda$ is the Coulomb logarithm.

The factor $\kappa_0^{\mathrm{th}}/\kappa_0^{\mathrm{SH}}$ represents the ratio of the thermal conductivity for inverse-bremsstrahlung heat to the classical Spitzer-Harm conductivity. It has been shown previously by means of FP simulations [9] that the effects of nonlocal transport on thermal conductivity for the case of an inverse-bremsstrahlung heating perturbation of wave number k are very well approximated by

$$\left(\frac{\kappa_0^{\mathrm{th}}}{\kappa_0^{\mathrm{SH}}}\right)=\frac{1}{1+(ak\lambda_e)^\beta}, \quad (7)$$

where $\lambda_e\equiv T_0^2/4\pi n_0e^4[4.2Z^*/\phi(Z^*)]^{1/2}\ln\Lambda$. The parameters a and β are chosen to fit the numerical results as described in Ref. [9]; for the conditions relevant to SBS, i.e., $10\le k\lambda_e\le1000$, simulations show that the best fit is given by $a\approx21$ and $\beta\approx1.44$. (The effects of ion motion and time-varying heating were included in these simulations but had no significant effect on the values of $\kappa_0^{\mathrm{th}}/\kappa_0^{\mathrm{SH}}$.) Assuming a time dependence for the perturbed quantities of the form $\exp(-i\Omega t)$ and using the thermal conductivity correction factor (7), Eqs. (1)-(3) become simultaneous algebraic equations which may be combined to yield the dispersion relation for the instability as a quartic polynomial in Ω. The roots are readily found numerically and the imaginary part corresponds to the instability growth rate (generally only one root has a positive imaginary part). The growth rates are maximized for values of k near the resonance of the scattered electromagnetic wave

$$k=2k_0-\frac{2\omega_0}{c}\frac{c_s}{c}$$

and this value of k is used in obtaining the results below.

For low-Z plasmas it is found that nonlocal thermal conductivity has a negligible effect, and the ponderomotive force remains the dominant driver for SBS. However, for high-Z plasmas, which provide the x rays in radiatively driven laser-fusion targets [14], the inverse-bremsstrahlung heating becomes significant. Figure 1 shows the growth rate $\mathrm{Im}(\Omega)$ as a function of laser intensity for an Au ($Z=70$) plasma with $n_0/n_c=0.5$ and $T_0=1$ keV. The classical (isothermal) ponderomotive result is shown by the dashed curve. The solid curve results from Eqs. (1)-(3) and (7) and shows an enhancement of more than a factor of 2 throughout the intensity range. The enhancement of growth arises from thermally driven SBS, which dominates when $(\kappa_0^{\mathrm{th}}/\kappa_0^{\mathrm{SH}})^{-1}\gamma_{T2}/\gamma_p>1$. As a check on the fluid model results FP simulations were run at selected values of the intensity, shown by the circles in Fig. 1. The agreement is quite good at all intensities, with the fluid model slightly underestimating the growth rate enhancement.

In a homogeneous plasma the threshold for instability is determined by the damping rates for the daughter waves:

$$\mathrm{Im}(\Omega)>\nu\equiv(\nu_{\mathrm{EM}}\nu_{\mathrm{IA}})^{1/2}, \tag{8}$$

where ν_{EM} is the inverse-bremsstrahlung damping rate for the scattered light wave and ν_{IA} is the damping rate for the ion-acoustic wave. The ion-acoustic wave damping is primarily electron Landau damping for high-Z plasmas with $ZT_e>T_i$ [15]. The inverse-bremsstrahlung damping of the scattered light is given by $\nu_{\mathrm{EM}}=n_0\nu_{ei}/2n_c$, where ν_{ei} is the electron-ion collision frequency [16].

The horizontal line in Fig. 1 indicates the threshold for SBS resulting from Eq. (8). Note that the threshold intensity is lowered by nearly 2 orders of magnitude by thermal effects.

At low intensities γ_{T2} and γ_p become small enough that the dispersion relation resulting from Eqs. (1)-(3) and (7) may be considerably simplified. The resulting approximate expression for the growth rate $\Gamma=\mathrm{Im}(\Omega)$ is

$$\frac{\Gamma^2}{k^2c_s^2}=\frac{1}{8}\frac{n_0}{n_c}\frac{c}{c_s}\frac{1}{\varepsilon_0^{1/2}}\left[\gamma_p+\left(\frac{\kappa_0^{\mathrm{th}}}{\kappa_0^{\mathrm{SH}}}\right)^{-1}\gamma_{T2}\right] \tag{9}$$

and is shown by the dotted line in Fig. 1. The first and second terms in square brackets in Eq. (9) represent the ponderomotive and thermal contributions to the instability, respectively. For given plasma parameters these terms may be evaluated using Eqs. (5)-(7) and provide a convenient guide to the relative importance of the two driving forces. In general the thermal term becomes more important for larger Z and n_0, and for smaller λ_0 and T_0.

The effect of nonlocal thermal conduction on SBS in inhomogeneous plasmas remains to be studied. Current laser-fusion experiments involve plasmas which are sufficiently inhomogeneous that density and velocity gradients may be expected to determine the threshold, and the incident laser light contains "hot spots" which vary widely in intensity. Consequently, it is difficult at present to compare theories of SBS (as well as other parametric instabilities) with existing experimental results. Nevertheless, the above results clearly show that thermal effects must be taken into consideration in modeling SBS in high-Z plasmas, and as experiments approach the long plasma scale lengths and uniform illumination required for reactor target implosions accurate modeling of this instability will become increasingly important.

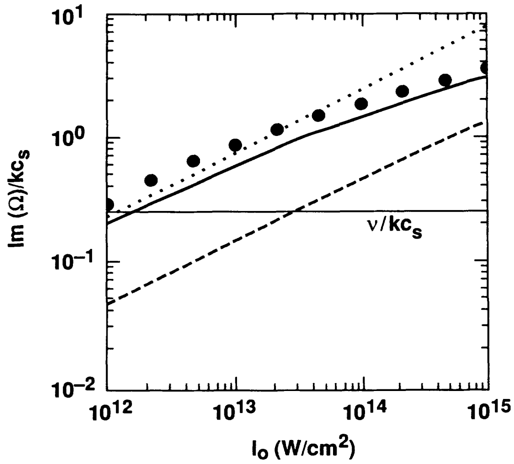

FIG. 1. Growth rates of SBS normalized to kc_S as a function of the incident laser intensity for a gold ($Z=70$) plasma with $n_0/n_c=0.5$, $T_0=1$ keV. The dashed line corresponds to the classical isothermal ponderomotive result, the solid line represents the growth rates calculated from Eqs. (1)-(3) and (7), the solid circles show the results of Fokker-Planck simulations, and the dotted line is the approximate growth rate given by Eq. (9). The horizontal line represents the homogeneous threshold due to damping as given by Eq. (8).

In conclusion, we have studied the impact of recent advances in the understanding of thermal transport on the theory of SBS, and in particular have developed the theory of a new form of the instability, thermal SBS. This instability bears the same relation to the familiar ponderomotive SBS as thermal filamentation does to ponderomotive filamentation, and we have shown that it is the dominant form of the instability for high-Z, low-temperature, high-density plasmas.

This work was supported by the U.S. Department of Energy Division of Inertial Fusion under Agreement No. DE-FC03-85DP40200 and by the Laser Fusion Feasibility Project at the Laboratory for Laser Energetics, which is sponsored by the New York State Energy Research and Development Authority and the University of Rochester.

[1] P. E. Young, K. G. Estabrook, W. L. Kruer, E. A. Williams, P. J. Wegner, R. P. Drake, H. A. Baldis, and T. W. Johnston, Phys. Fluids B 2, 1907 (1990).

[2] K. Tanaka, L. M. Goldman, W. Seka, R. W. Short, and E. A. Williams, Phys. Fluids **27**, 2960 (1984).
[3] H. A. Baldis and C. J. Walsh, Phys. Fluids **26**, 3426 (1983).
[4] W. L. Kruer, *The Physics of Laser Plasma Interactions* (Addison-Wesley, New York, 1988), Chap. 8.
[5] J. F. Drake, P. K. Kaw, Y. C. Lee, G. Schmidt, C. S. Liu, and M. N. Rosenbluth, Phys. Fluids **17**, 778 (1974).
[6] W. M. Manheimer and D. G. Colombant, Phys. Fluids **24**, 2319 (1981).
[7] L. Spitzer, Jr., and R. Harm, Phys. Rev. **89**, 977 (1953).
[8] A. R. Bell, Phys. Fluids **26**, 279 (1983).
[9] E. M. Epperlein, Phys. Rev. Lett. **65**, 2145 (1990); E. M. Epperlein and R. W. Short, Phys. Fluids B **3**, 3092 (1991).
[10] J. A. Fejer, *Generation of Large Scale Aligned Irregularities in Ionospheric Modification Experiments* (AGARD, Neuilly sur Seine, France, 1974); B. L. Cragin and J. A. Fejer, Radio Sci. **9**, 1071 (1974).
[11] Kruer [4], p. 79.
[12] G. W. Hammett and F. W. Perkins, Phys. Rev. Lett. **64**, 3019 (1990).
[13] A. J. Schmitt, Phys. Fluids **31**, 3079 (1988).
[14] E. F. Gabl, B. H. Failor, Gar. E. Busch, R. J. Schroeder, D. Ress, and L. Suter, Phys. Fluids B **2**, 2437 (1990); D. Ress, L. J. Suter, E. F. Gabl, and B. H. Failor, Phys. Fluids B **2**, 2448 (1990); D. R. Kania, W. L. Kruer, P. Bell, S. P. Obenschain, K. Kearney, and A. Mostovych, Phys. Fluids B **3**, 1496 (1991).
[15] C. J. Randall, Phys. Fluids **25**, 2231 (1982).
[16] S. I. Braginskii, in *Reviews of Plasma Physics,* edited by M. A. Leontovich (Consulting Bureau, New York, 1965), Vol. 1, p. 205.