Anomalous Ion Heating via Parametric Resonance in rf-Driven Plasma Sheaths

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A novel mechanism for strong ion heating inside magnetized sheaths, formed at plasma-wall interfaces in radio-frequency-driven plasmas, is studied. The ion gyration frequency is modulated by the oscillating charge density inside the sheath, resulting in a Hill-type parametric equation of motion. The gyration velocity is unstable when the time-averaged gyrofrequency $\hat{\Omega}$ is near a subharmonic $n\omega/2$ of the rf frequency. Large energy absorption occurs within only a few gyrations because of the initial exponential growth. Nonlinear saturation occurs at kinetic energies higher than the time-averaged potential difference across the sheath.

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Sheath formation at plasma-boundary interfaces occurs in a variety of situations, ranging from dc electric discharges, to reactors for plasma processing, to microwave heating of tokamak plasmas. Sheaths generated by rf waves around the Faraday shield bars have been the subject of renewed interest, because of their importance during ion cyclotron plasma heating (ICH). The steep electrostatic gradients inside the sheaths can accelerate ions to energies sufficient to cause sputtering of the shield material [1,2]. This results in impurity influx and deterioration of the plasma energy confinement time during ICH, observed in the JET, TFTR, and Phaedrus tokamaks. Although the sputtering problem is alleviated with the use of low-sputtering-yield coatings (Be,B), the parasitic energy absorption at the plasma edge can considerably reduce the efficiency of the energy deposition in the plasma core. The observed energy losses can be as high as 30% in JET [3] and 50% in Phaedrus [4], when the magnetic-field geometry and antenna configuration favor sheath formation. Energy deposition and energetic particle production inside the sheaths are also important in plasma etching and plasma processing.

The studies so far on sheath dynamics greatly simplify the ion motion by considering only the acceleration *parallel* to the magnetic-field lines in the time-averaged sheath potential $\Phi_0(x) \equiv \langle \Phi(x,t) \rangle$. However, *transverse acceleration* by the oscillating rf fields inside the sheaths can be equally important. This is particularly true in the case of two-dimensional sheaths and magnetic lines almost perpendicular to the potential gradients. There the ion guiding-center (GC) motion is mainly an $\mathbf{E}\times\mathbf{B}$ drift around equipotential surfaces, with only a small fraction of the potential difference going to acceleration parallel to the magnetic lines. This paper studies the transverse ion motion in magnetized rf sheaths in considerable detail. It is shown, by employing the "moving-plate capacitor" model to describe the electric fields, that the ion gyromotion obeys a nonlinear Hill-type parametric equation. The time modulation in the ion gyration frequency $\tilde{\Omega}(t) = [\Omega^2 - \tilde{\omega}_s^2(t)]^{1/2}$, induced by the oscillating sheath plasma frequency $\tilde{\omega}_s^2(t) = (e/m_i) dE(x,t)/dx$, yields the parametric dependence. In the linear regime, this equation demonstrates the well-known *exponential* instability in the ion velocity when the time-averaged gyration frequency $\hat{\Omega} \equiv \langle \tilde{\Omega}(t) \rangle$ is near a (sub)harmonic of the rf frequency. Of particular importance is the lowest parametric resonance $\omega/2 = \hat{\Omega}$, characterized by a broad instability band. This condition is met near the plasma edge because the frequency ω for ion cyclotron heating at the plasma core is higher than the ion cyclotron frequency near the edge, $\omega > \Omega > \hat{\Omega}$. Nonlinear saturation occurs at gyration energies that can be higher than the dc sheath potential and much higher than the thermal energy κT of the ambient plasma. Because of the original exponential growth, transverse acceleration allows strong energy absorption by ions within only a few gyroperiods, usually within the sheath transit time.

The following approximations are introduced in order to simplify the analysis and relate with previous studies: (a) The one-dimensional electrostatic model is used for the sheath fields. (b) A separation in time scales is assumed in the ion motion. The one-dimensional sheath model [5,6] yields a good approximation for the local sheath structure across x when the sheath thickness Δ in the direction of the electrostatic fields is much shorter than the sheath length L determined by the boundary dimensions. The electrostatic treatment is justified when Δ is much shorter than the wavelength of the main plasma mode launched by the antenna, $\Delta \ll v_A/\omega$, where v_A is the Alfvén speed. The temporal scale separation assumes that the gyromotion proceeds much faster than the GC motion across the sheath potential. Since the GC velocity across the sheath is determined by the tilting of the magnetic lines in the x direction, $\dot{X} \sim \sin\theta (e\Phi_0/m_i)^{1/2}$ where $\theta = \tan^{-1}(B_x/B_z)$, the approximation is satisfied for $\theta \ll 1$. The above simplifications preserve the essential features of the ion dynamics and are valid for the operational parameters of large tokamaks.

The moving-plate capacitor model is shown in Fig. 1. An ion column of density n_i fills the space between the "unperturbed" plasma at $x=0$ and the sheath boundary at $x=\Delta$. An electron column of density n_e is driven back and forth between 0 and Δ. The rectified rf potential is contained in the space between $x=\Delta$ and the moving electron-ion interface at $x=s(t)$. The model assumes adiabatic electron response, implying local thermodynam-

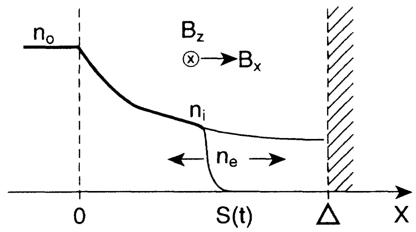

FIG. 1. Illustration of density profiles in rf-induced sheaths.

ic equilibrium with $\Phi(x,t)$ and abrupt decay of the electron density within distance $\lambda_D \ll \Delta$ at the position $s(t)$. The ion density is determined by the guiding-center flow along the magnetic lines under the time-averaged potential Φ_0. The acceleration in the gyrovelocity does not shift the guiding center across the magnetic lines and so does not directly affect the ion density profile. The conditions for the applicability of the moving-plate capacitor model stay valid as long as $\rho\cos\theta \ll \Delta$. However, if the ion Larmor radii expand to $\rho \sim \Delta$ within a sheath transit time, enhanced ion transport will definitely affect the density profile and the sheath voltage gradients.

The analytic expressions [5] for the time-averaged sheath potential $\Phi_0(x)$ and the ion sheath density $n_i(x)$ are parametrized by the ratio of the potential difference to the thermal energy $H=(eV_s/\kappa T)^{1/2}$, where $V_s \equiv \Phi_0(\Delta) - \Phi(0)$. The same parameter equals the ratio of the ambient plasma density $n_0 = n_i(0)$ to the sheath density at the wall $n_i(\Delta)$. For our purposes we mimic the monotonic ion density profile by

$$n_i(x) = \frac{n_0}{(\alpha x/\Delta + 1)} . \quad (1)$$

Numerical studies using the general profile $n_i(x) = n_0(\alpha_\zeta x/\Delta + 1)^{-1/\zeta}$ with $\zeta = \frac{1}{2}, \frac{1}{3}$, 2, 3, 4, and 100 demonstrated similar behavior as the case $\zeta=1$, Eq. (1), by choosing $\alpha_\zeta = H^\zeta - 1$ to ensure the same sheath potential to thermal energy ratio H^2. The oscillating electron boundary is given by a triangular pulse

$$s(t) = \Delta(1 - |t|/\tau), \quad -\tau/2 < t < \tau/2 , \quad (2)$$

where $\tau = 2\pi/\omega$ is the rf period. The above approximations are easy to follow analytically and preserve the essential underlying physics.

Using Poisson's equation, and Eqs. (1) and (2) for the charge distributions of Fig. 1, the electric field $E(x,t) = -\partial\Phi(x,t)/\partial x$ is given by

$$E(x,t) = \begin{cases} \dfrac{m_i}{e}\omega_{pi}^2 \ln\left(\dfrac{\alpha x/\Delta + 1}{\alpha s(t)/\Delta + 1}\right), & x \ge s(t), \\ 0, \quad x < s(t), \end{cases} \quad (3)$$

where the ambient ion plasma frequency $\omega_{pi}^2 = 4\pi e^2 n_0/m_i$. The ion equations of motion are

$$\ddot{x} + \Omega\dot{y} = \cos\theta F(x,t) , \quad (4a)$$

$$\ddot{y} - \Omega\dot{x} = \sin\theta F(x,t) , \quad (4b)$$

where we have defined $F(x,t) \equiv (e/m_i)E(x,t)$. The left-hand side of (4b) is the rate of change $\dot{X}$ of the guiding-center location. For small θ, $\dot{X}$ is much slower than the characteristic time scale of (4a), and X is treated as an adiabatic invariant $X \equiv -(\dot{y} - \Omega x)/\Omega = X_0$. The force is split into a dc and an oscillating part,

$$F(x,t) = F_0(x) + \tilde{F}(x,t) , \quad (5a)$$

$$\tilde{F}(x,t) = \sum_{m=1}^{m=\infty} F_m(x)\cos(\omega t) . \quad (5b)$$

The Fourier coefficients obtained from (3) are given by

$$\frac{\partial F_0}{\partial x} = \omega_s^2(x) = \omega_{pi}^2 \frac{\bar{x}}{\alpha\bar{x} + 1} , \quad (6a)$$

$$\frac{\partial F_m}{\partial x} = 2\omega_{pi}^2 \frac{\sin(m\pi\bar{x})}{m\pi} \frac{\bar{x}}{\alpha\bar{x} + 1} , \quad (6b)$$

where $\bar{x} = x/\Delta$. A strong F_0 results inside the sheath from the rectification of the harmonic driving potential, and the oscillating part $\tilde{F}$ contains only cosine terms [5] from the even symmetry in $s(t)$. Combining Eqs. (3) and (4), expanding the fields (5) around the GC location X, and shifting the origin of the coordinate system by $\Delta X = F_0/(\Omega^2 - F_0')$ yields

$$\ddot{u} + [\hat{\Omega}^2 - \tilde{F}'(X,t)]u = \tilde{F}(X,t) + \tfrac{1}{2}[F_0''(X) + \tilde{F}''(X,t)]u^2 , \quad (7)$$

where $u = x - X$ and $(') \equiv d/dx|_{x=X}$. The expansion is valid for transverse excursions u smaller than the scale length Δ of the field F, true for the injected-in-the-sheath ions before acceleration takes place. In the absence of the oscillating terms $\tilde{F}$, the homogeneous equation

$$\ddot{u} + \hat{\Omega}^2 u = 0 \quad (8)$$

describes the ion gyromotion inside the time-averaged sheath potential. The gyrofrequency

$$\hat{\Omega} = (\Omega^2 - \omega_s^2)^{1/2} \quad (9)$$

is modified by the time-averaged electric-field gradient $\omega_s^2(X) = dF_0/dX$ at the GC location [7]. The ion motion is an ellipse with minor-to-major axis ratio $\Delta X/\Delta Y = (1 - \omega_s^2/\Omega^2)^{1/2}$, around a guiding center located at $X = X_0$ and drifting along $Y = Y_0 + (cF_0'/\Omega)t$. The x excursion is enhanced by a factor $(1 - \omega_s^2/\Omega^2)^{-1}$ over the Larmor radius $\rho_0 = u_0/\Omega$. Note that for $\omega_s^2 \ge \Omega^2$ the orbit in the time-averaged potential is unbounded, and the ions are accelerated across the magnetic lines without completing a rotation. We will focus on the case $\omega_s^2 < \Omega^2$, of more relevance to the tenuous edge plasmas.

To study the full, driven response, Eq. (7) is recast in nondimensional variables, normalizing time to $\hat{\Omega}^{-1}$, length to the original Larmor radius ρ_0, and mass to the ion mass m_i, obtaining

$$\ddot{u} + \delta^2(t)u = g(u,t) . \quad (10)$$

Equation (10) is the driven, nonlinear Hill equation for an oscillator with time-varying eigenfrequency

$$\delta^2(t)=1-\epsilon\sum_{m=1}^{m=\infty} f_m'(X)\cos(m\nu t)\,, \qquad (11)$$

where $\nu=\omega/\hat{\Omega}$. The eigenfrequency modulation comes from the oscillating field gradients at the GC location, $\partial\tilde{F}(X,t)/\partial X$, expressed in terms of the rescaled Fourier coefficients $f_m\equiv F_m(X)/\omega_{pi}^2$ and the modulation amplitude $\epsilon\equiv\omega_{pi}^2/\hat{\Omega}^2$. The local sheath plasma frequency is expressed in terms of the ambient ω_{pi} by $\omega_s^2(X)=\omega_{pi}^2\times f_0'(X)$, and the modified ion cyclotron frequency by $\hat{\Omega}=[\Omega^2-\omega_{pi}^2 f_0'(X)]^{1/2}$. The driving term

$$g(u,t)=\epsilon\sum_{m=1}^{m=\infty}[f_m(X)+\tfrac{1}{2}f_m''(X)u^2]\cos(m\nu t) \qquad (12)$$

contains the field $\tilde{E}(X,t)$ at the GC location plus the lowest-order nonlinear corrections.

The behavior of (10) has been extensively studied as a model equation for parametric resonance phenomena. If only the $m=2$ term is kept inside $\delta(t)$, and when $g(t)=0$, it reduces to the well-known Matheus equation. The general theory for the linear stability [ignoring the quadratic terms u^2 in (12)] relies on the Floquet theory to prove the existence of instability zones at driving frequencies $m\nu^{-1}=2/n$, i.e., $\hat{\Omega}/\omega=mn/2$. The width of the unstable bands depends on the modulation amplitude $\epsilon=\mu/(1-f_0'\mu)^{1/2}$, where $\mu\equiv\omega_{pi}^2/\Omega^2$, and the Fourier amplitude $f_m'(X)$. Each Fourier coefficient f_m creates a sequence of resonances at $\nu^{-1}=mn/2$. Many Fourier coefficients m' contribute to a given resonant $\nu^{-1}=m'n'/2$ with various n'. The dominant contribution for a given ν^{-1} comes from $n=1$, $\nu^{-1}=m$, i.e., the direct subharmonic $\nu^{-1}=m/2$, and has a frequency width of order ϵ. Contributions from $n>1$ scale as ϵ^2. Figure 2 shows the first three unstable frequency bands $\nu^{-1}=\frac{1}{2}$, 1, $\frac{3}{2}$ as functions of ω_{pi}^2. Ten harmonic coefficients f_m have been included in the calculation for each band, following the approach described in Ref. [8]. The boundaries of the parametric instability, without the driving term g on the right-hand side of (10), are given by the solid lines. Inclusion of the driving terms further expands the unstable region.

The parametric resonance $\nu=2$, with the rf frequency ω nearly twice the modified gyrofrequency $\hat{\Omega}$, is of particular importance for ICH. It is easily satisfied in edge plasmas where $\omega>\Omega>\hat{\Omega}$. Typical ion trajectories from the numerical integration of the full equations of motion (2) are plotted in velocity space $v_x\equiv\dot{u}$ and $v_y=\Omega u$ in Figs. 3(a) and 3(b). The sheath parameters are $\Delta/\rho_0=32$, $\omega_{pi}/\Omega=1$, the GC location $X_0=\Delta/2$, and initially $v_{x0}=0$, $v_{y0}=1$. The exponential gyrovelocity growth is evident in the unstable case $\nu=2.0$, Fig. 3(a), and the kinetic energy increases over 200 times within 10 gyrations. The Larmor radius at saturation is comparable to the sheath thickness. In contrast, the trajectories for $\nu=2.15$

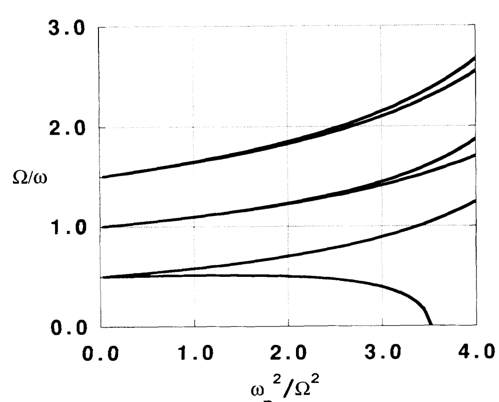

FIG. 2. Instability bands in frequency Ω/ω vs ambient ion plasma frequency for GC located in the middle of the sheath $X=\Delta/2$.

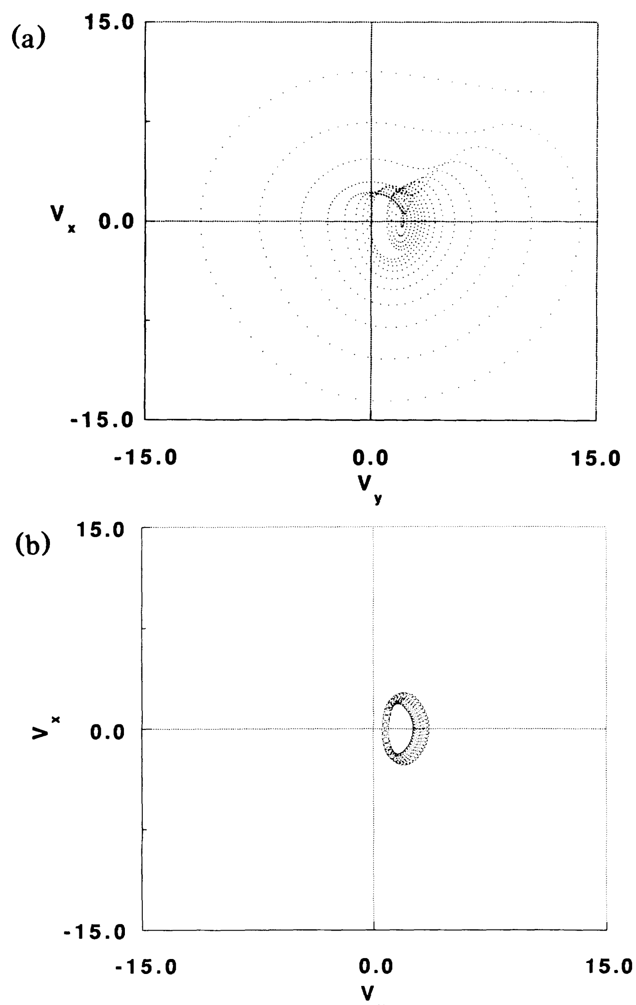

FIG. 3. Typical particle trajectories in velocity space for (a) $\omega=2\Omega$ and (b) $\omega=2.15\Omega$. The GC location is at $X=\Delta/2$ and the elapsed time is about 10 rf periods.

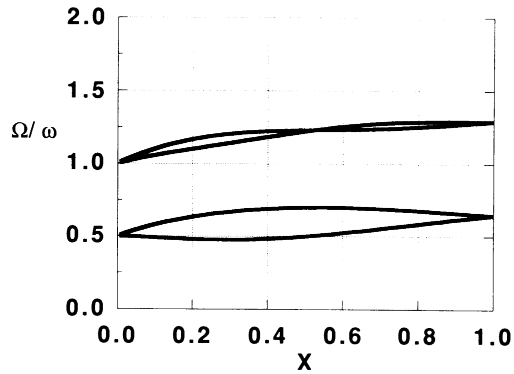

FIG. 4. Instability zones in frequency Ω/ω vs GC location inside the sheath at $\omega_{pi}^2=2\Omega^2$.

outside the unstable band, shown in Fig. 3(b), show no instability. The gyrovelocity $v(t)=(v_x^2+v_y^2)^{1/2}$ oscillates periodically in time. The average $\langle v(t)\rangle$ is comparable to the motion in the time-averaged field, Eq. (8), and no net energy absorption occurs.

The unstable frequency bands are plotted against the GC position in the sheath $\bar{X}=X/\Delta$ in Fig. 4, for ambient plasma density $\omega_{pi}^2=2\Omega^2$. The rf frequency for the lowest resonance $\nu^{-1}=\frac{1}{2}$ lies in the interval $2\Omega(1-\frac{2}{5})^{1/2}<\omega<2\Omega$ as the sheath plasma frequency varies from $\omega_s=0$ at $X=0$ to $\omega_s=\omega_{pi}(\frac{2}{5})^{1/2}$ at $X=\Delta$. When ω falls in the above interval, the resonance condition will be met at some X as the ionic guiding center drifts along the tilting in the x-direction magnetic lines. Actually, because of the large resonance width, shown in Fig. 4, the majority of the sheath ions will spend most of their life in the unstable region when $1.5\Omega<\omega<2\Omega$. The transit time through the resonance is given in terms of the resonance width $\Delta'\sim\Delta$ by $\tau_r\simeq\Delta'/\sin\theta(e\Phi_0/2m_i)^{1/2}$. For typical edge parameters $\Phi_0=400$ V, $B=1$ T, $n_0=10^{10}$ cm^{-3}, and $\Delta'\sim\Delta=3\times10^{-1}$ cm, the transit time is larger than a gyroperiod for $\sin\theta<0.30$. Because of the exponential growth, a significant energy boost occurs within only a few gyrations in the unstable regime. The importance of rf effects in parasitic edge plasma heating is strengthened by two-dimensional effects in the sheath structure. It has been observed numerically and argued theoretically [9] that for small θ the ion GC executes a cross-field drift near equipotential surfaces, rather than flowing along the magnetic lines. Only a small fraction of the sheath dc potential difference can be absorbed into acceleration parallel to the magnetic lines in this case. Most of the observed energy absorption must be attributed to the rf interactions.

The initial exponential growth for small u is eventually reduced by the nonlinear terms. A two-time scale approach can be used to analyze Eq. (10) in the nonlinear regime letting $u=U\cos t$ with U slowly varying. Multiplying both sides by $\cos t$ and averaging over the fast time scale shows that nonlinear saturation $\langle f(u,t)\cos t\rangle\simeq 0$ occurs at $U\sim[2\tilde{f}(X)/\tilde{f}''(X)]^{1/2}\sim\Delta$. The dc potential of the sheath is estimated from (6a) to be less than $\frac{1}{2}\omega_{pi}^2\Delta^2/\alpha$. The gyration energy, using $v_y=\Omega\Delta$ at $y=0$, is $\frac{1}{2}\Omega^2\Delta^2$. Particle energies can exceed the dc sheath potential when $\alpha>\omega_{pi}^2/\Omega^2$. The initial exponential growth indicates that the thermal velocity region lies in the neighborhood of an unstable fixed point. The orbits follow the unstable manifold (separatrix) around large size island(s) ($\delta u\sim\Delta\Omega$). The motion at large U may still be nonintegrable, as (10) lacks an obvious time invariant. However, the energy absorption takes place in a slower, random-walk manner, which is not as effective within the sheath transit time as the initial exponential acceleration.

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